2-D Sampling

$f[m,n] = f(m\Delta x, n\Delta y)$ sampling intervals $\Delta x, \Delta y$ meters

Continuous-space frequency
U, V rads/meter

Discrete-space frequency
u, v rads

$$\boxed{\begin{aligned} u &= U\Delta x \\ v &= V\Delta y \end{aligned}}$$

Sampling frequencies

$U_s = \frac{2\pi}{\Delta x}$ rads/meter

$V_s = \frac{2\pi}{\Delta y}$ rads/meter

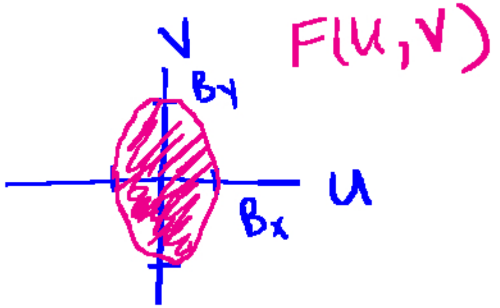
$$f_s(x,y) = \sum_{m,n} f[m,n]\,\delta(x-m\Delta x, y-m\Delta y) \xleftrightarrow{FT} F_s(U,V) = F(U,V) * S(U,V)$$

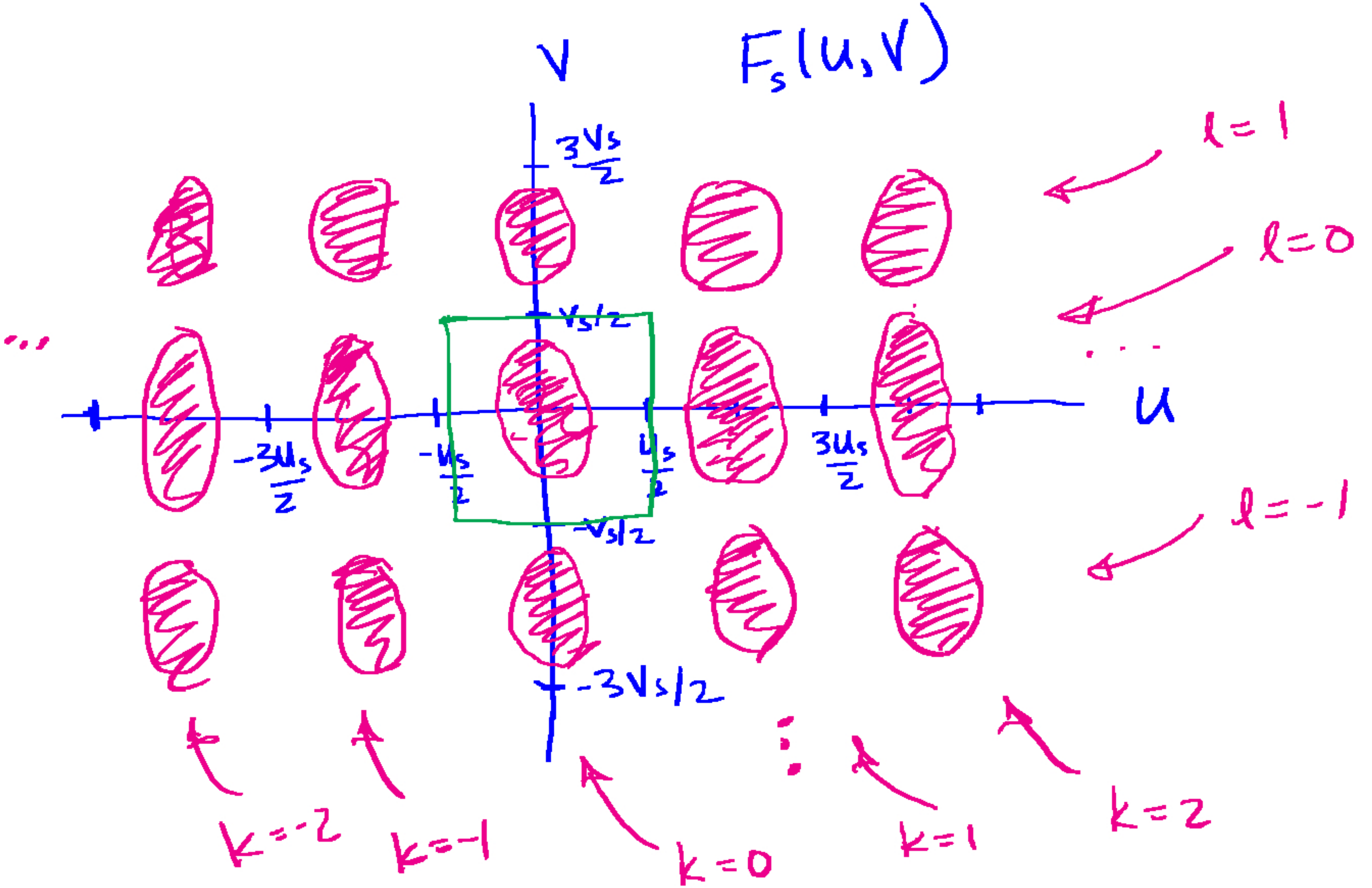
$$= f(x,y) \underbrace{\sum_{m,n} \delta(x-m\Delta x, y-m\Delta y)}_{s(x,y)}$$

$$\propto F(U,V) * \sum_{k,l} \delta(U-kU_s, V-lV_s)$$

$$= \sum_{k,l} F(U-kU_s, V-lV_s)$$

$$f_s(x,y) = \sum_{m,n} f[m,n]\,\delta(x-m\Delta x,\, y-n\Delta y) \xleftrightarrow{FT} F_s(u,v) \propto \sum_{k,l} F(u-kU_s,\, v-lV_s)$$





For no aliasing

$$U_s = \frac{2\pi}{\Delta x} > 2B_x$$

$$V_s = \frac{2\pi}{\Delta y} > 2B_y$$

Nyquist Sampling Theorem (2D)

Let $f(x,y)$ be bandlimited to $-B_x < u < B_x, -B_y < v < B_y$

If $f[m,n] = f(m\Delta x, n\Delta y)$ and $\Delta x < \frac{\pi}{B_x}$, $\Delta y < \frac{\pi}{B_y}$,

then $f(x,y)$ can be uniquely recovered from the samples $f[m,n]$.