2-D Signal Processing: Discrete Space

Discrete space: 2-D array of numbers, "pixels"

Linear Shift Invariant Systems

$$f[m,n] \longrightarrow \boxed{\begin{matrix}\text{LSI}\\ H\end{matrix}} \longrightarrow g[m,n] = h[m,n] * f[m,n]$$

$$h[m,n] = H\{\delta[m,n]\}$$ < impulse response / point spread function

$m \downarrow$

$f[m,n]$ 

$\xrightarrow{\quad n \quad}$

$$g[m,n] = h[m,n] * f[m,n]$$

$$= \sum_{k=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} h[m-k, n-\ell]\, f[k,\ell]$$

2-D Convolution – Let $f[m,n] = 0 \quad n<0,\ n \geq N$

$$g[m,n] = \sum_{k=0}^{N-1} \sum_{\ell=0}^{N-1} h[m-k, n-\ell]\, f[k,\ell]$$

1) Flip $h[k,\ell]$ horizontally and vertically to get $h[-k,-\ell]$
2) Shift $h[-k,-\ell]$ by $[m,n] \Rightarrow h[0,0]$ is at $[m,n]$
3) Form product $h[m-k, n-\ell]\, f[k,\ell]$ and sum all values

Example:

$$\underline{h} = \begin{bmatrix} h[0,0] & h[0,1] \\ h[1,0] & h[1,1] \end{bmatrix}$$

$\left(h[m,n] = 0 \quad \begin{matrix} m,n > 1 \\ m,n < 0 \end{matrix} \right)$

$$\underline{f} = \begin{bmatrix} f[0,0] & \cdots & f[0,N-1] \\ \vdots & & \vdots \\ f[N-1,0] & \cdots & f[N-1,N-1] \end{bmatrix}$$

$\underline{g} = \underline{h} * \underline{f}$

Steps 1 + 2: $h[m-k, n-\ell]$

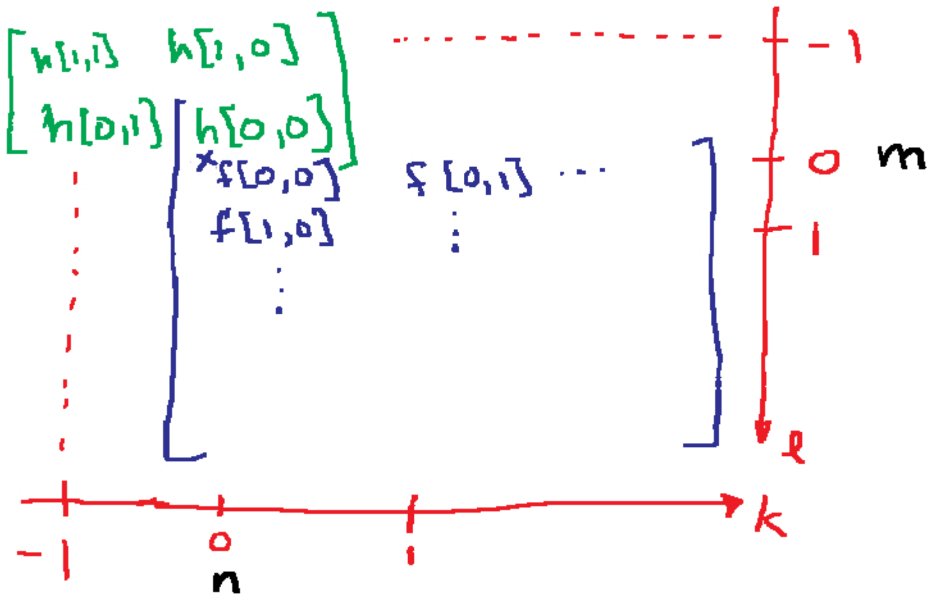
$$\begin{bmatrix} h[1,1] & h[1,0] \\ h[0,1] & h[0,0] \end{bmatrix} \leftarrow \text{loc. } m$$

$\uparrow$ location n

Step 3: multiply and Sum

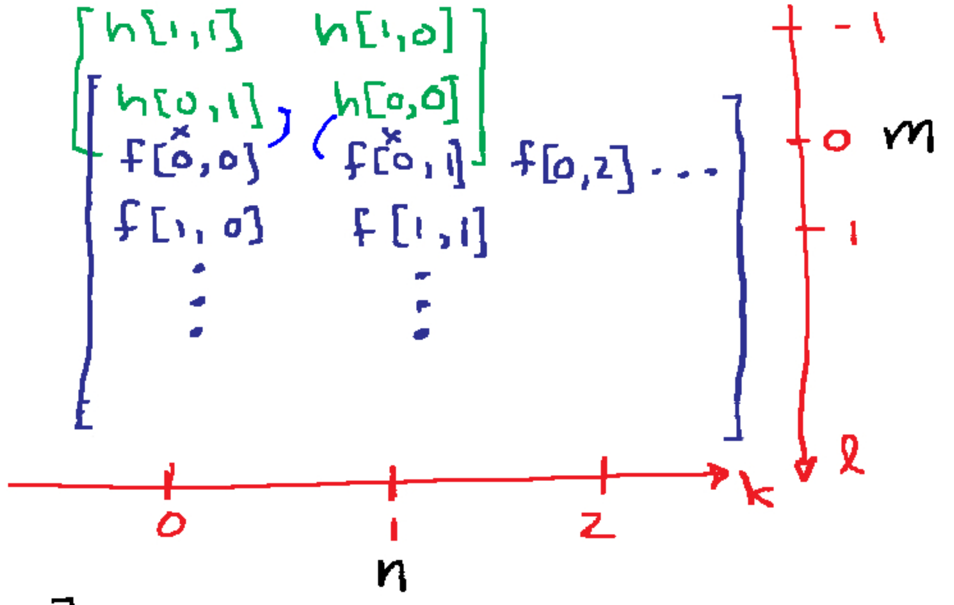
$g[0,0]$:

$m=0, n=0$



$$g[0,0] = h[0,0]f[0,0]$$

$g[0,1]$: $m=0,\ n=1$



$$g[0,1] = h[0,1]f[0,0] + h[0,0]f[0,1]$$

In general —

$$g[m,n] = h[0,0]f[m,n] + h[0,1]f[m,n-1] + h[1,0]f[m-1,n] + h[1,1]f[m-1,n-1]$$

- cols of $\underline{h}$ filter cols of $\underline{f}$ ✓
- rows of $\underline{h}$ filter rows of $\underline{f}$ ✓

Examples for $\underline{h}$

1) $\underline{h} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

blurs image - each pixel replaced by average of 4 pixels

2) $\underline{h} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

blurs vertically
accentuates differences horizontally

3) $\underline{h} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

accentuates differences vertically
blurs horizontally

4) $\underline{h} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

blurs image - more than 1)