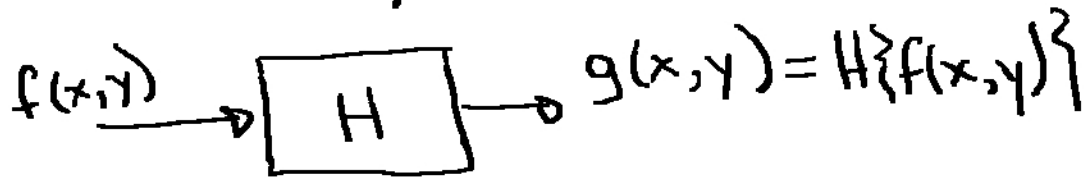


# 2-D Signal Processing: Continuous Space

- Imaging

- Extension to 3+D

2-D Systems



$f(x,y)$



Linear, Shift Invariant (LSI) Systems

$$1) H\{\alpha_1 f_1(x,y) + \alpha_2 f_2(x,y)\} = \alpha_1 H\{f_1(x,y)\} + \alpha_2 H\{f_2(x,y)\}$$

$$2) H\{f(x-x_0, y-y_0)\} = g(x-x_0, y-y_0) \quad \text{where } H\{f(x,y)\} = g(x,y)$$

# LSI Input-Output: 2-D Convolution

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x-\alpha, y-\beta) f(\alpha, \beta) d\alpha d\beta$$

where  $h(x, y) = \mathcal{H}\{\delta(x, y)\}$  - impulse response  
- point spread function

## 2-D Fourier Transform

2-D  
FT

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-jux} e^{-jvy} dx dy$$

$u$ : freq vbl in  $x$   
 $v$ : freq vbl in  $y$

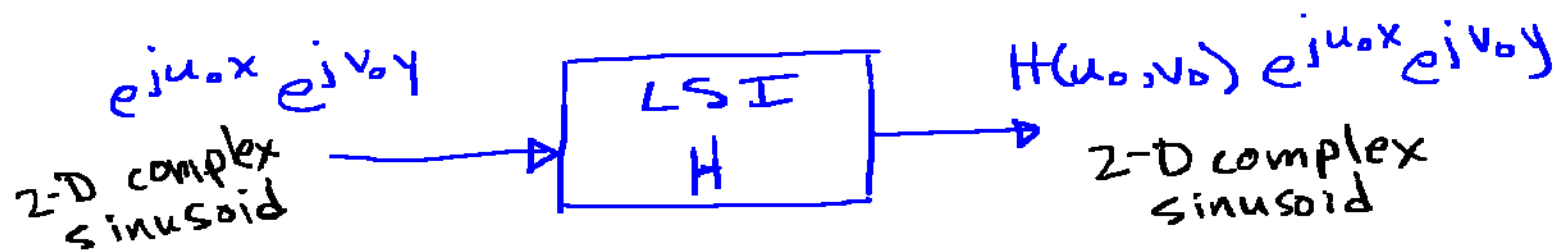
$$= \int_{-\infty}^{\infty} \underbrace{\left[ \int_{-\infty}^{\infty} f(x, y) e^{-jux} dx \right]}_{\text{FT over } x} \underbrace{e^{-jvy} dy}_{\text{FT over } y}$$

2-D  
Inv FT

$$f(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{jux} e^{jvy} dx dy$$

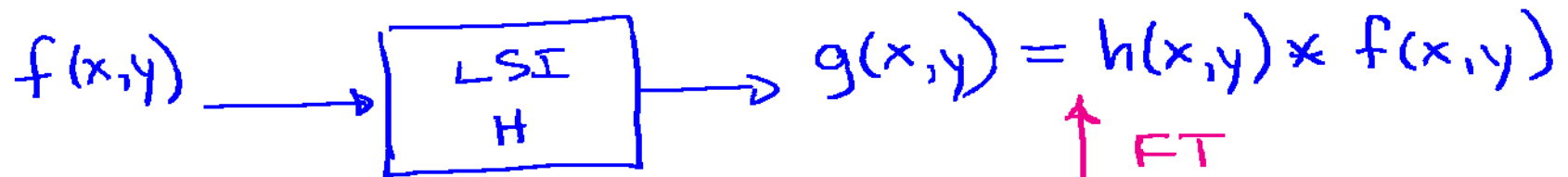
# Convolution - Multiplication Property

3



$$H(u_0, v_0) = H(u, v) \Big|_{\substack{u=u_0 \\ v=v_0}}$$

and  $H(u, v) = \text{FT}\{h(x, y)\}$



FT

$G(u, v) = H(u, v) F(u, v)$

A red double-headed vertical arrow labeled 'FT' connects the two equations. The second equation is underlined in green.