2-D Signal Processing: Continuous Space

- Imaging
- Extension to 3+D

y

$f(x,y)$

x

2-D Systems

$$f(x,y) \longrightarrow \boxed{H} \longrightarrow g(x,y) = H\{f(x,y)\}$$

Linear, Shift Invariant (LSI) systems

1) $H\{\alpha_1 f_1(x,y) + \alpha_2 f_2(x,y)\} = \alpha_1 H\{f_1(x,y)\} + \alpha_2 H\{f_2(x,y)\}$

2) $H\{f(x-x_0, y-y_0)\} = g(x-x_0, y-y_0)$ where $H\{f(x,y)\} = g(x,y)$

LSI Input-Output : 2-D Convolution

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x-\alpha, y-\beta)\, f(\alpha,\beta)\, d\alpha\, d\beta$$

where $h(x,y) = \mathcal{H}\{\delta(x,y)\}$ - impulse response
- point spread function

2-D Fourier Transform

u: freq vbl in x
v: freq vbl in y

2-D FT

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-jux}\, e^{-jvy}\, dx\, dy$$

$$= \underbrace{\int_{-\infty}^{\infty} \underbrace{\left[\int_{-\infty}^{\infty} f(x,y) e^{-jux}\, dx\right]}_{\text{FT over } x} e^{-jvy}\, dy}_{\text{FT over } y}$$

2-D Inv FT

$$f(x,y) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{jux}\, e^{jvy}\, dx\, dy$$

Convolution - Multiplication Property

$$e^{ju_0x}e^{jv_0y} \longrightarrow \boxed{\begin{matrix}\text{LSI}\\ H\end{matrix}} \longrightarrow H(u_0,v_0)\,e^{ju_0x}e^{jv_0y}$$

2-D complex sinusoid → 2-D complex sinusoid

$$H(u_0,v_0) = H(u,v)\Big|_{\substack{u=u_0\\ v=v_0}}$$

and $H(u,v) = FT\{h(x,y)\}$

$$f(x,y) \longrightarrow \boxed{\begin{matrix}\text{LSI}\\ H\end{matrix}} \longrightarrow g(x,y) = h(x,y) * f(x,y)$$

$$\Big\updownarrow \text{FT}$$

$$\underline{G(u,v) = H(u,v)\,F(u,v)}$$