Example: DFT Approximation to the FT

$$x(t) = e^{-t/10}\left(\cos(10t)+\cos(12t)\right)u(t) \overset{FT}{\longleftrightarrow} X(\Omega) = \frac{\frac{1}{10}+j\Omega}{\left(\frac{1}{10}+j\Omega\right)^2+100} + \frac{\frac{1}{10}+j\Omega}{\left(\frac{1}{10}+j\Omega\right)^2+144}$$

Desired parameters: 1) Resolution $\frac{2\pi}{5}$ rads/s, sample Ω @ $\pi/40$ $\frac{\text{rads}}{\text{s}}$

2) Resolution $\frac{2\pi}{25}$ rads/s, sample Ω @ $\pi/50$ $\frac{\text{rads}}{\text{s}}$

Assume BW $\approx$ 300 rad/s

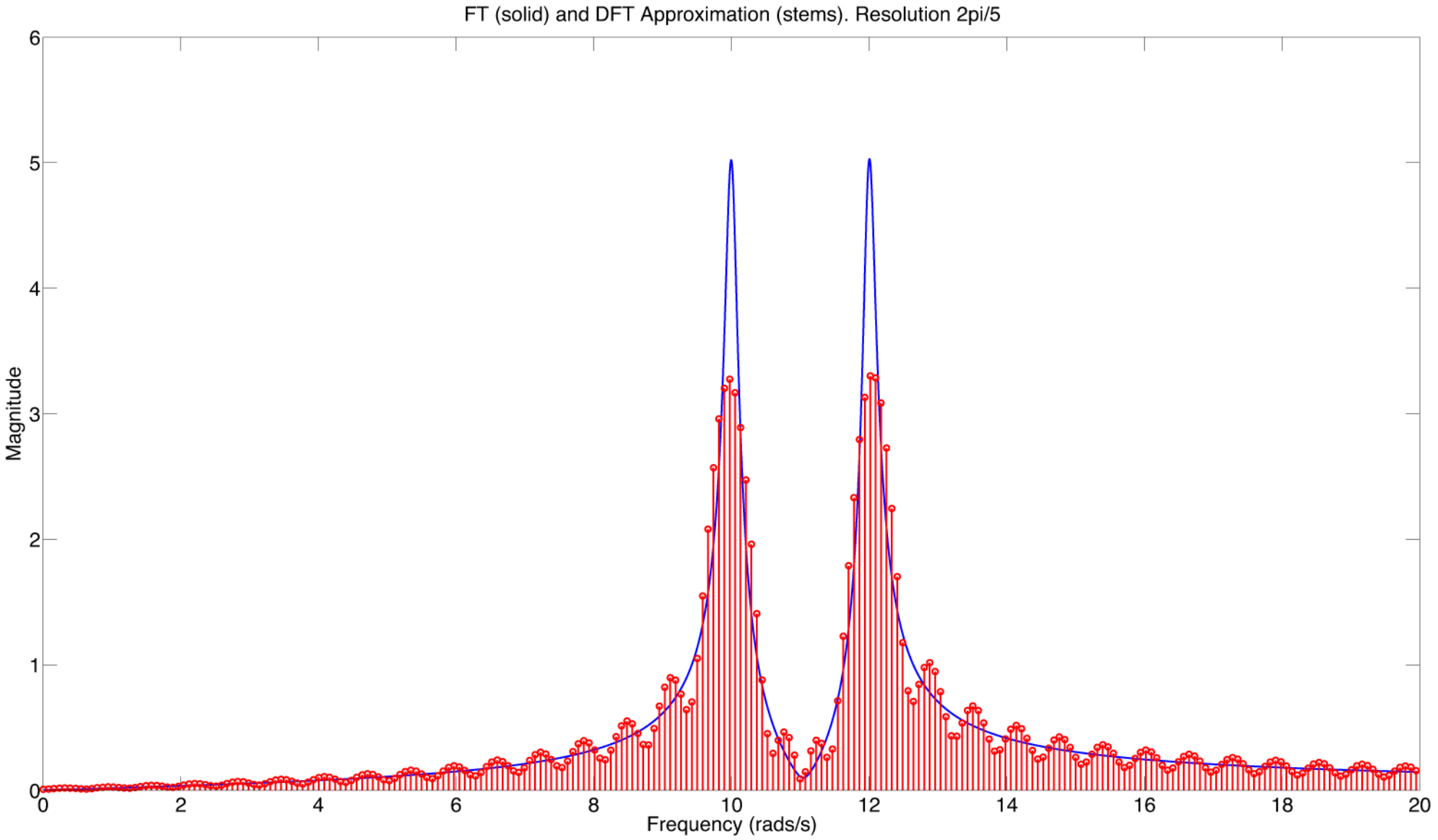
Sampling Thm: choose $\Omega_s = 200\pi \Rightarrow T = 0.01$ sec

Mainlobe width $\frac{4\pi}{N}$ rads $\Rightarrow \frac{4\pi}{NT}$ rads/s

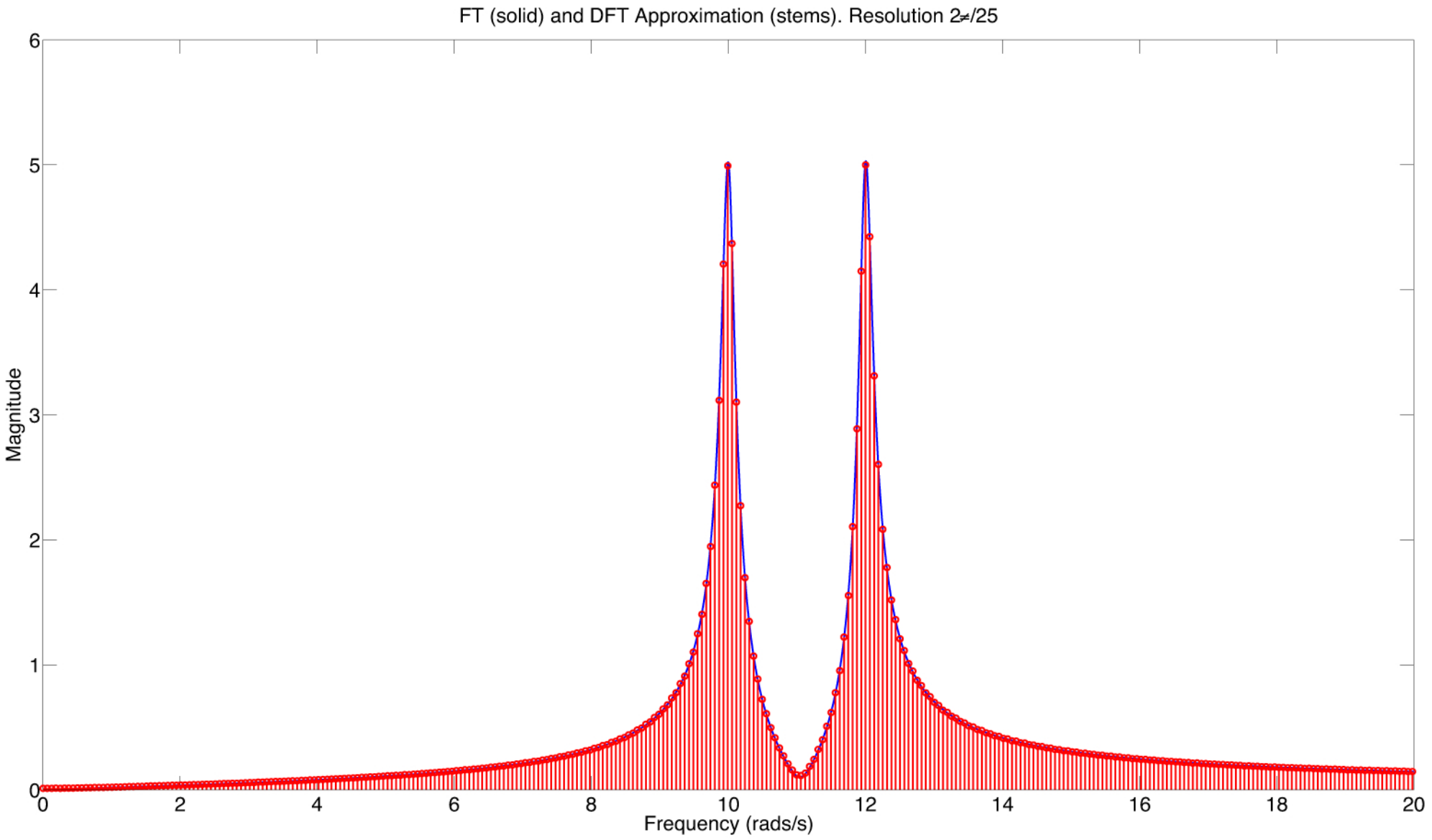
1) $\frac{4\pi}{NT} \le \frac{2\pi}{5} \Rightarrow N \ge 1000$, $\frac{2\pi}{MT} \le \frac{\pi}{40} \Rightarrow M \ge 8000$

2) $\frac{4\pi}{NT} \le \frac{2\pi}{25} \Rightarrow N \ge 5000$, $\frac{2\pi}{MT} \le \frac{\pi}{50} \Rightarrow M \ge 10{,}000$

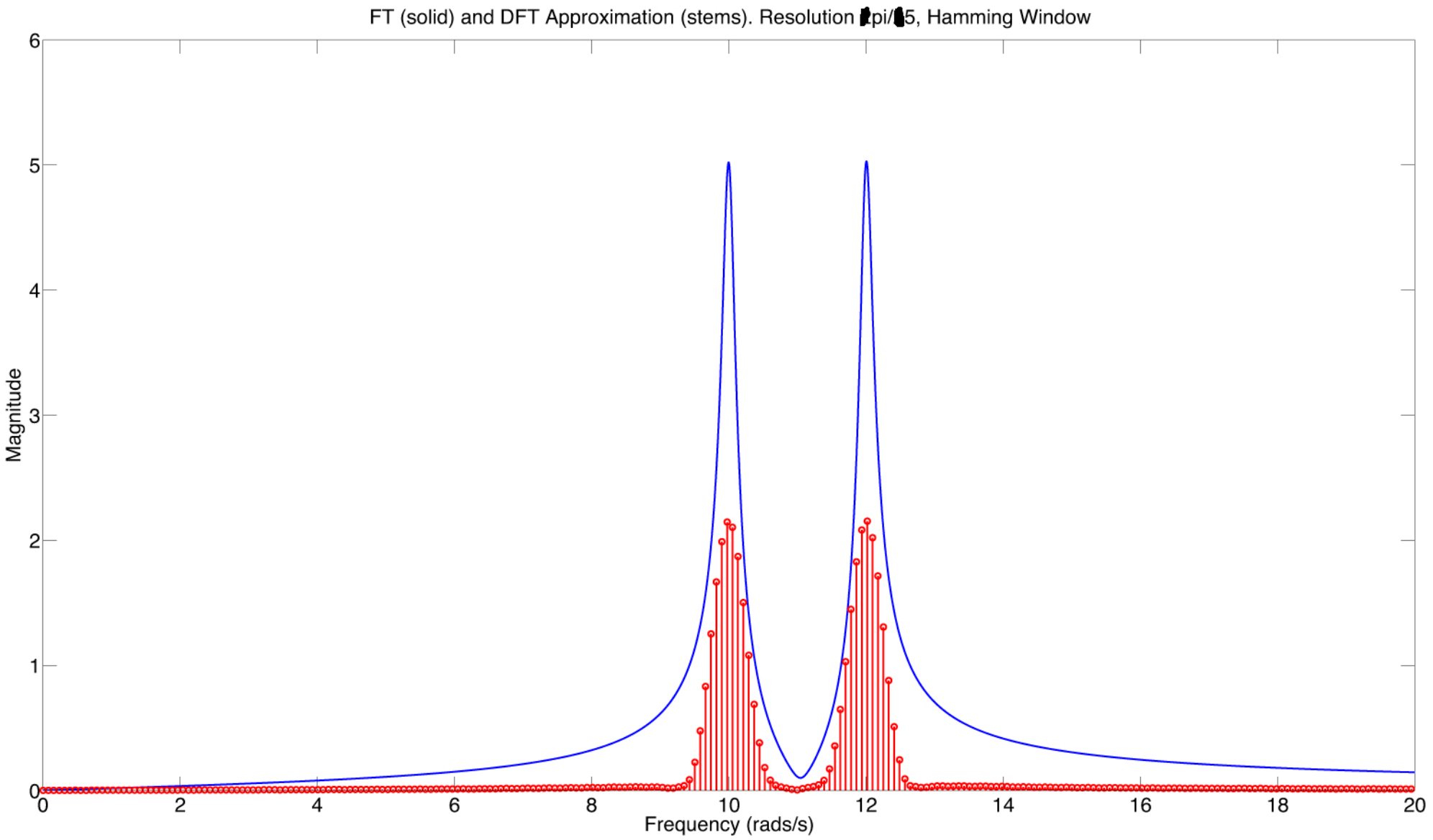
$N = 1000, M = 8000$

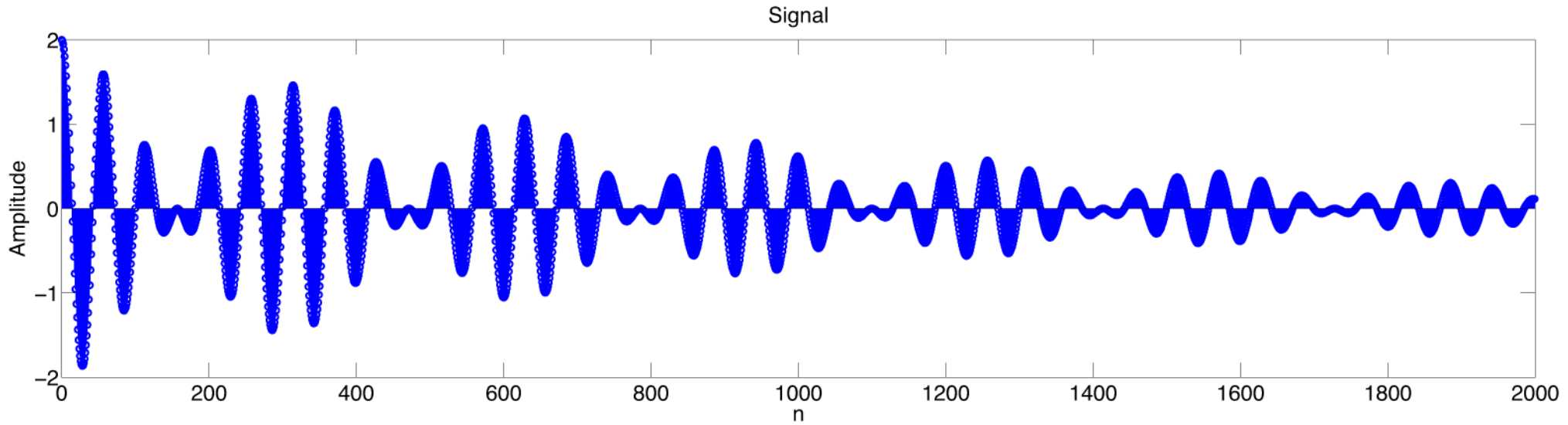

FT (solid) and DFT Approximation (stems). Resolution 2pi/5
Magnitude
Frequency (rads/s)
0
1
2
3
4
5
6
0
2
4
6
8
10
12
14
16
18
20

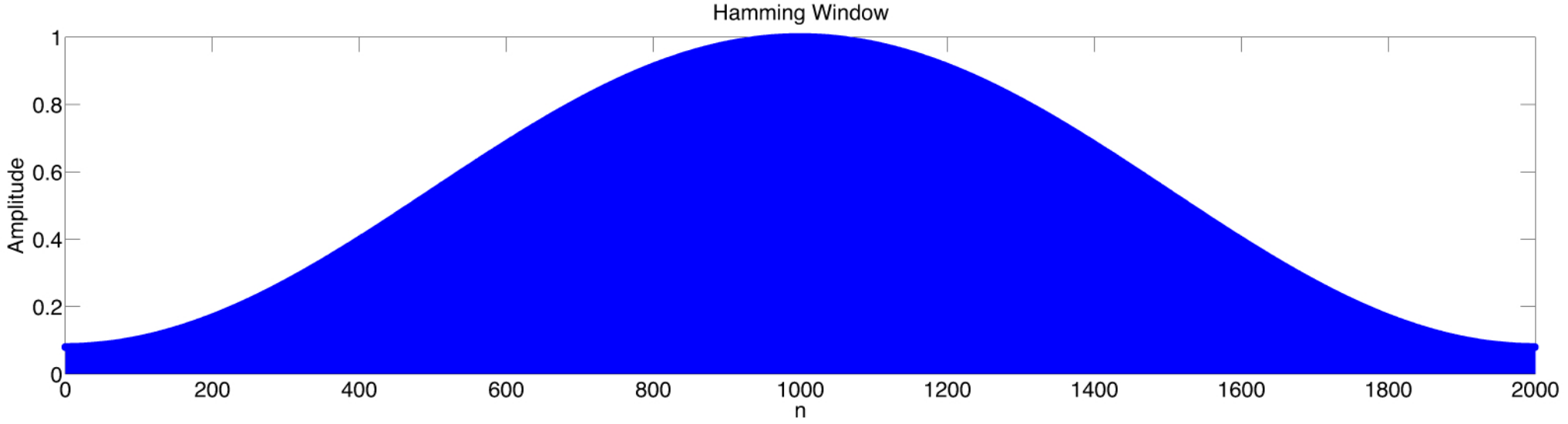
N = 5000 , M = 10000



Hamming Window - N = 2000, M = 8000




Signal
Amplitude
n
2
1
0
−1
−2
0
200
400
600
800
1000
1200
1400
1600
1800
2000


Hamming Window
Amplitude
n
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000