Windows: Trading Resolution for Dynamic Range

$$z[n] = x[n]\,w[n]$$ truncates $x[n]$ to interval $0 \le n \le N-1$

$$Z(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) * W(e^{j\omega})$$

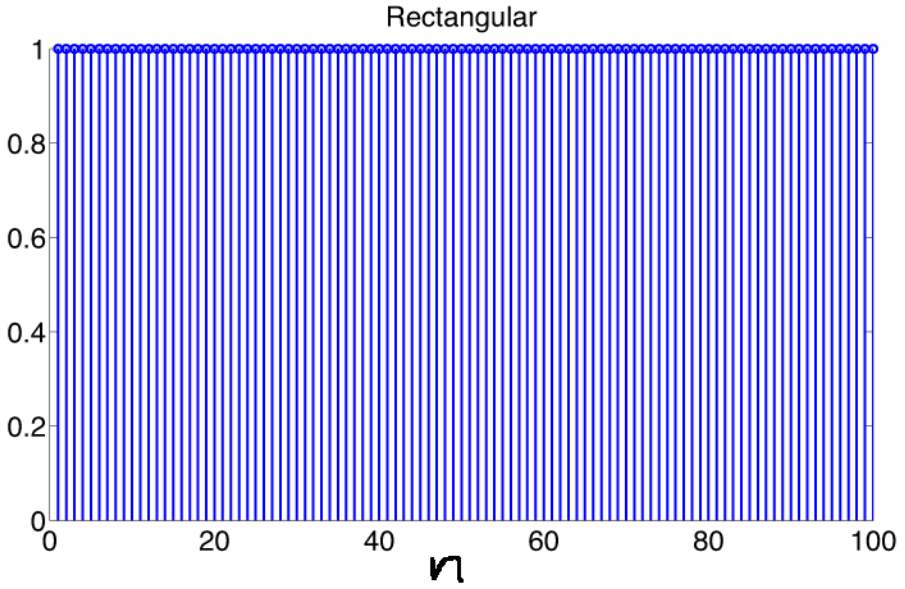
- High sidelobes in $W(e^{j\omega})$ from sharp transition in $w[n]$

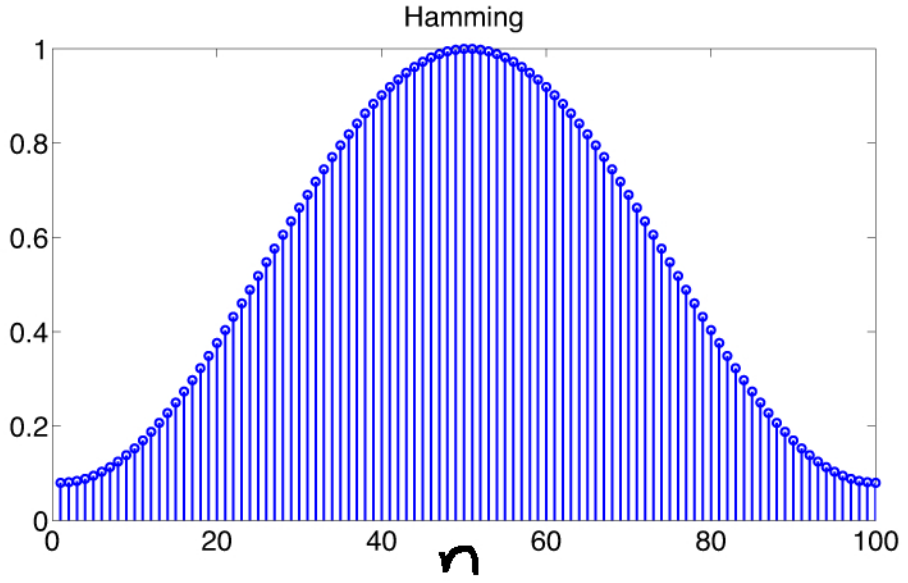
Use different shaped windows (or tapers) to reduce sidelobe height in return for wider mainlobe

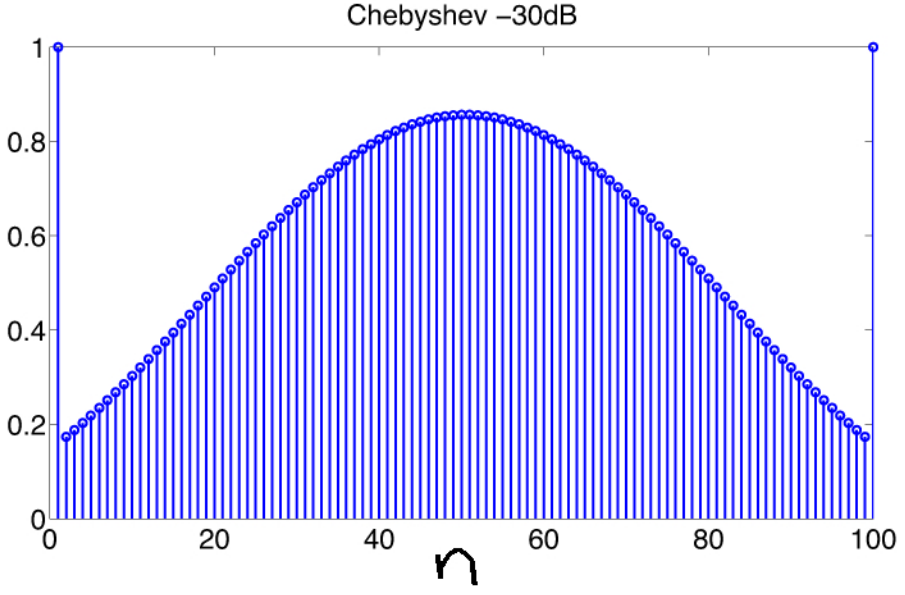
Example: Hamming window

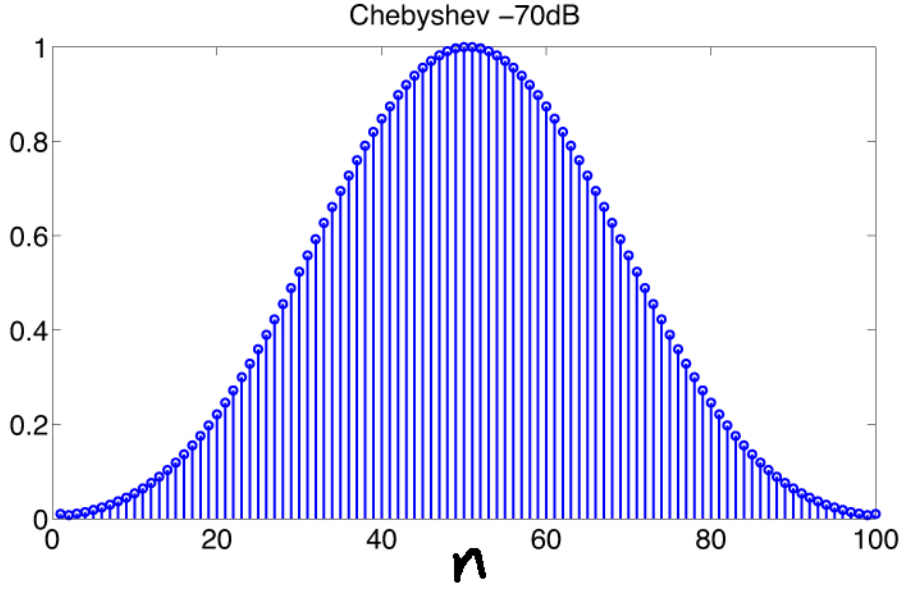
$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

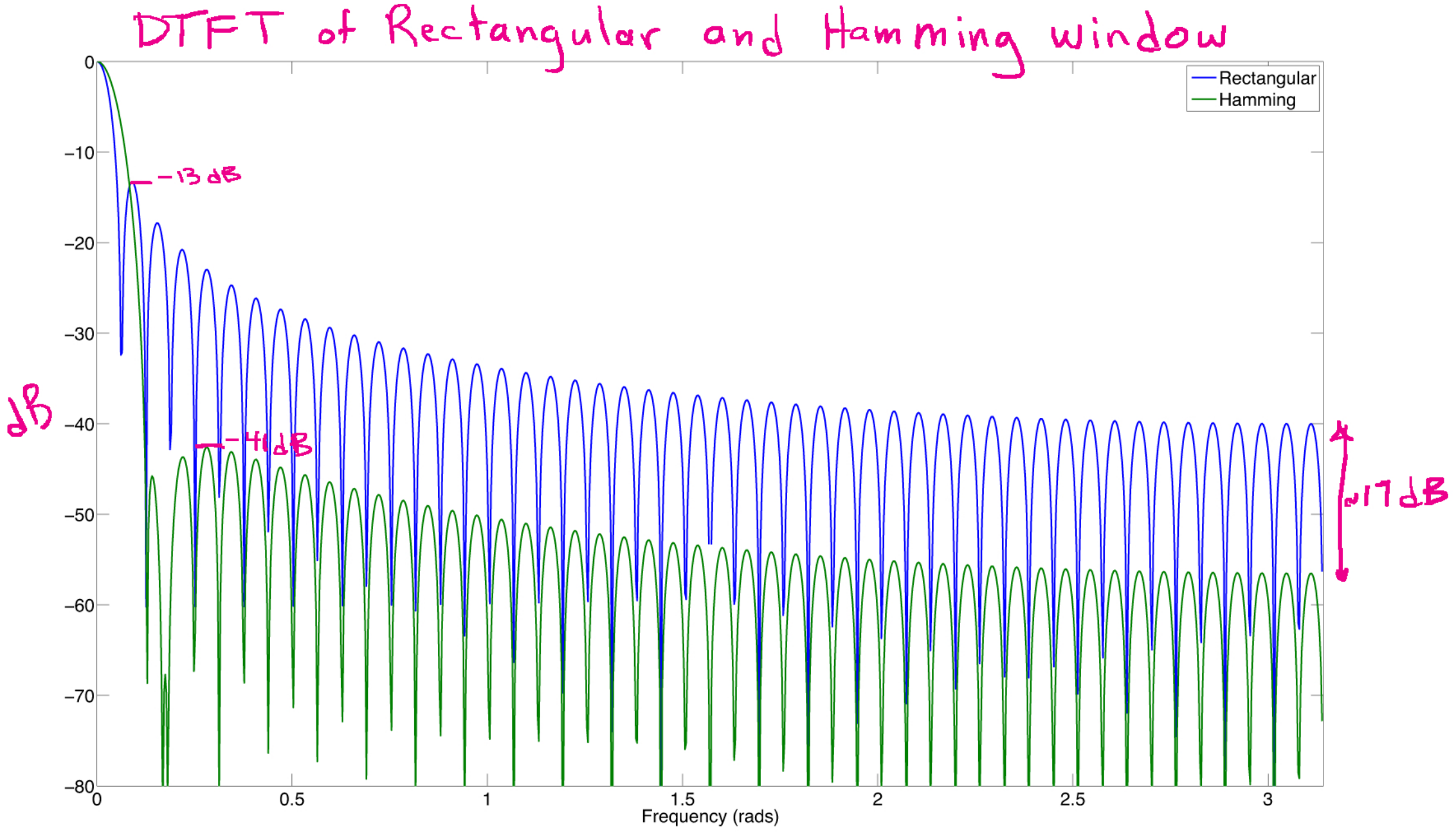
window	mainlobe width	sidelobe height
rectangular	$4\pi/N$	-13 dB
Hamming	$8\pi/N$	-41 dB

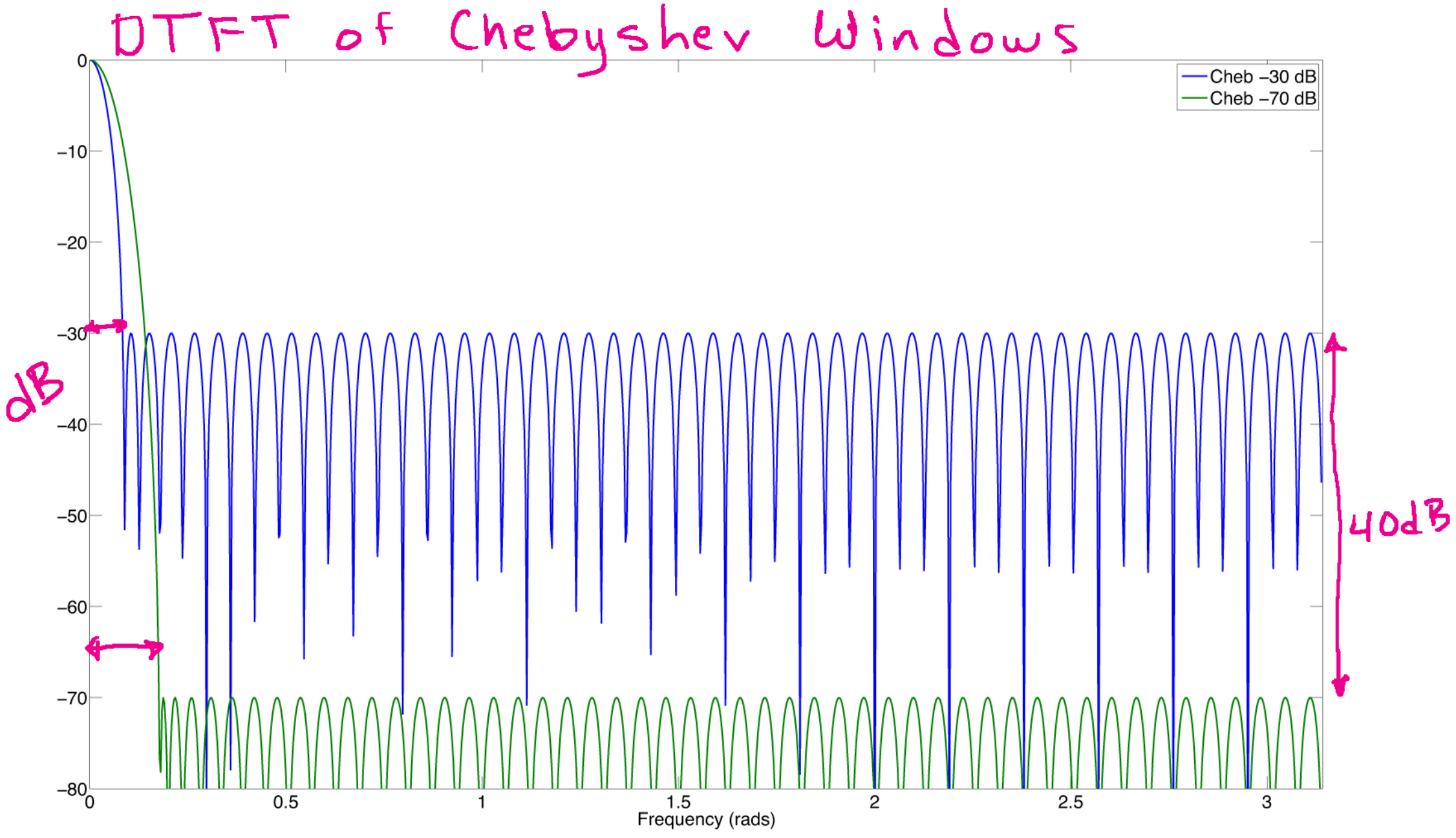

Rectangular
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
n


Hamming
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
n


Chebyshev −30dB
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
n


Chebyshev −70dB
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
n


DTFT of Rectangular and Hamming window
Rectangular
Hamming
dB
0
−10
−20
−30
−40
−50
−60
−70
−80
−13 dB
−41 dB
≈17 dB
0
0.5
1
1.5
2
2.5
3
Frequency (rads)


DTFT of Chebyshev Windows
Cheb –30 dB
Cheb –70 dB
0
–10
–20
–30
–40
–50
–60
–70
–80
0
0.5
1
1.5
2
2.5
3
Frequency (rads)
dB
40dB

Example:

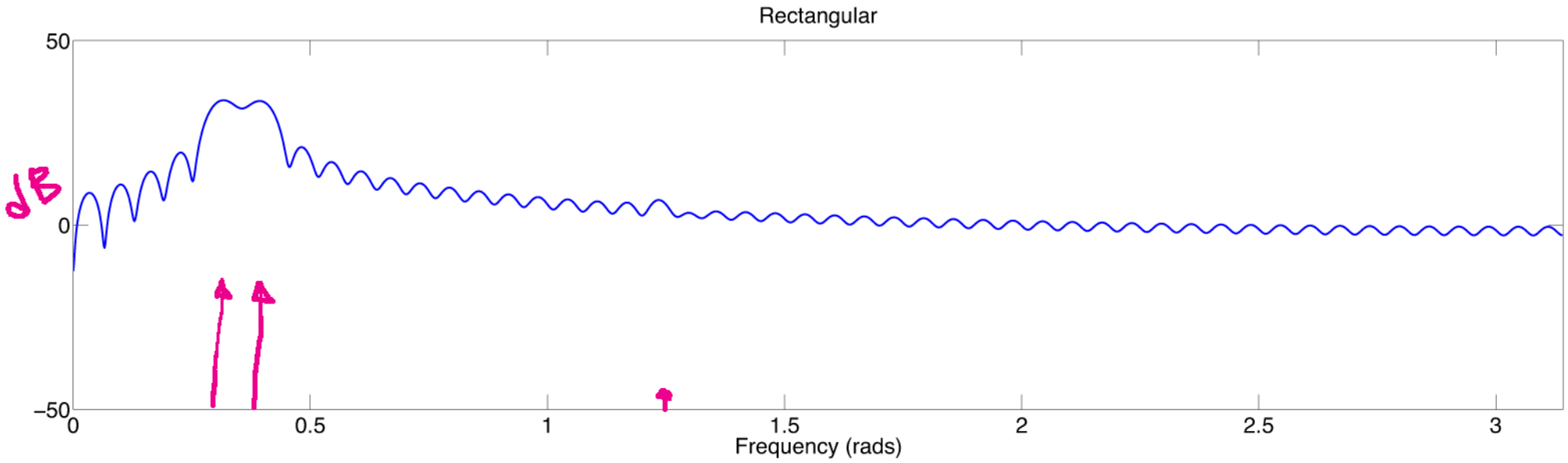
$$x[n] = \cos\left(\frac{\pi}{10}n\right) + \cos\left(\frac{\pi}{8}n + \frac{\pi}{3}\right) + 0.01\cos\left(\frac{4\pi}{10}n - \frac{3\pi}{5}\right)$$

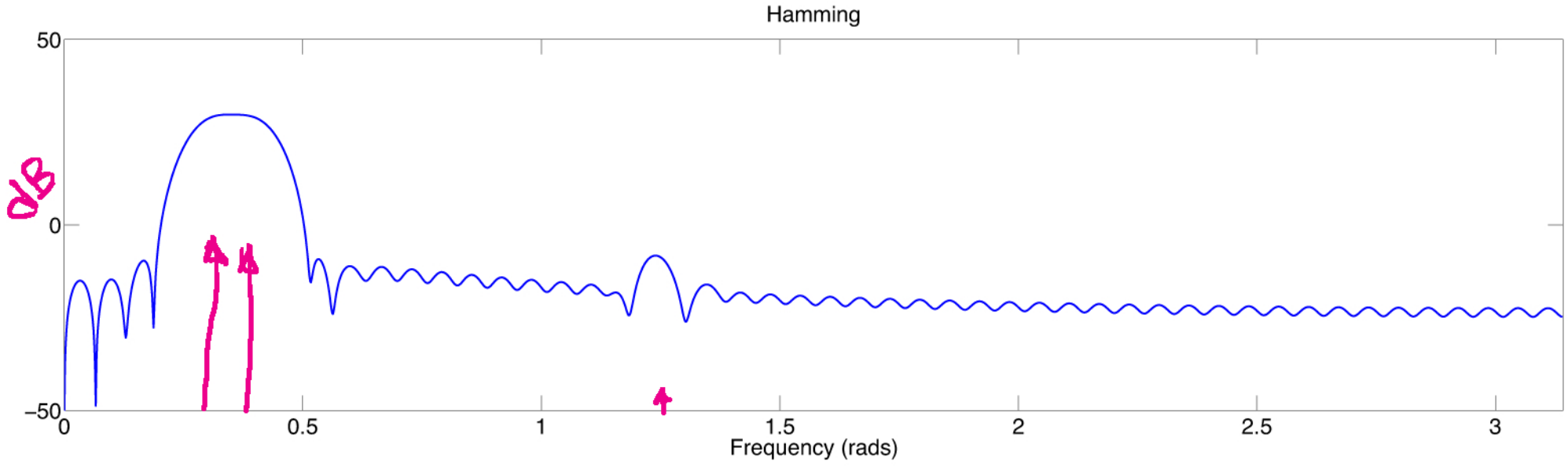
equal amplitude

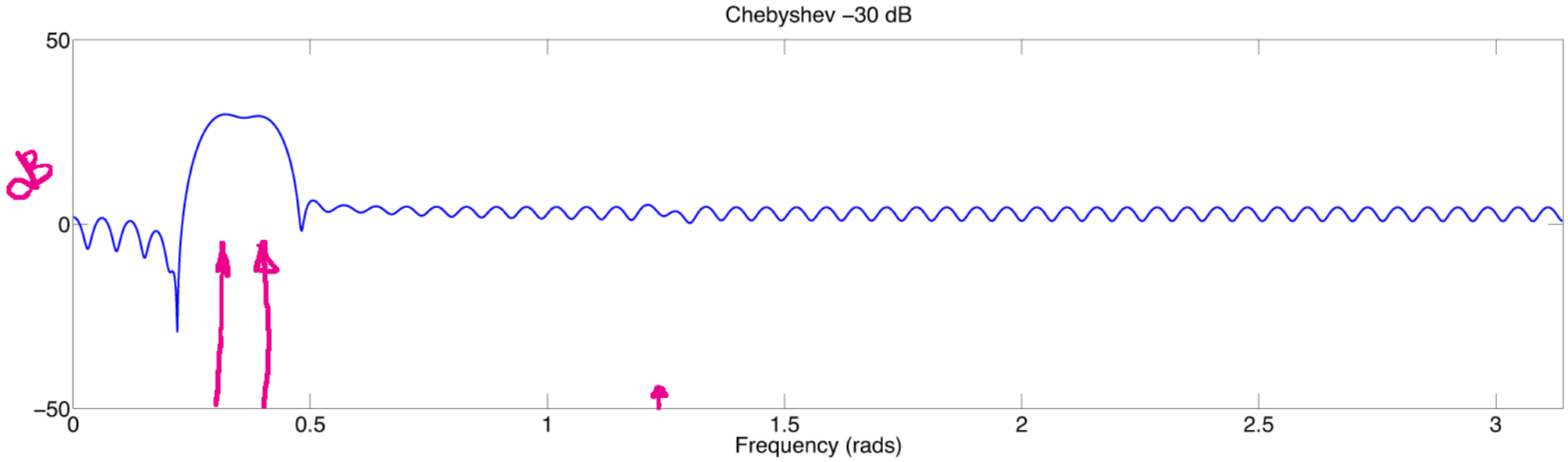
weak amplitude

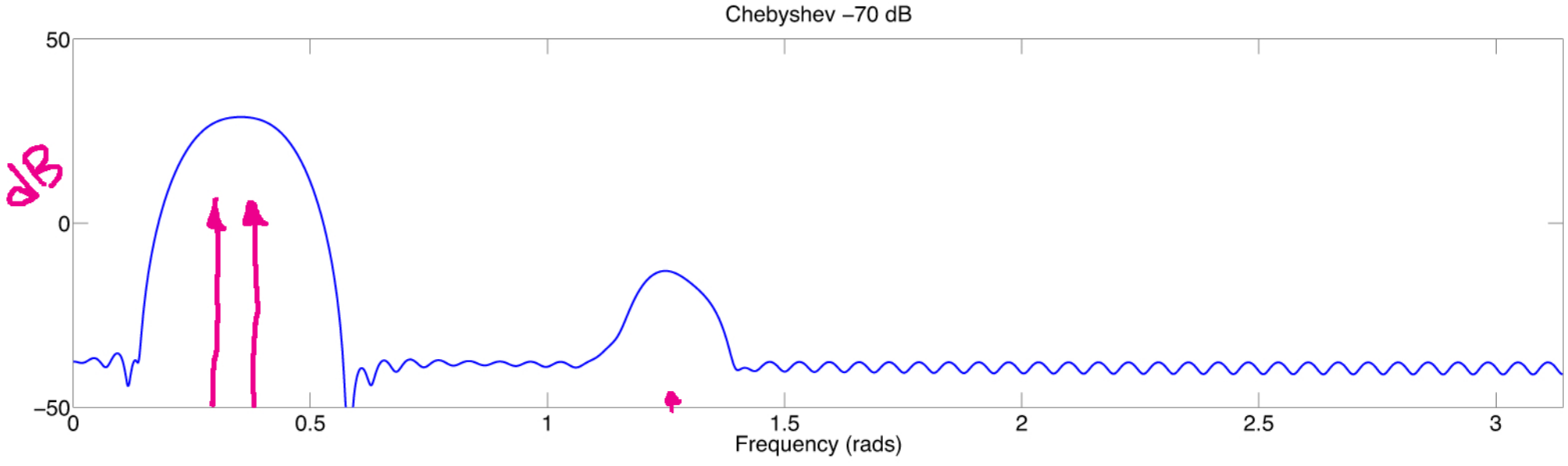
$N = 100$:

1) Rectangular window
2) Hamming window
3) Chebyshev -30 dB
4) Chebyshev -70 dB


Rectangular
50
0
−50
dB
0
0.5
1
1.5
2
2.5
3
Frequency (rads)


Hamming
50
0
−50
dB
0
0.5
1
1.5
2
2.5
3
Frequency (rads)


Chebyshev –30 dB
50
0
–50
dB
0
0.5
1
1.5
2
2.5
3
Frequency (rads)


Chebyshev –70 dB
50
0
–50
dB
0
0.5
1
1.5
2
2.5
3
Frequency (rads)