Effect of Truncation on DTFT

– key step in using DFT to approximate the FT

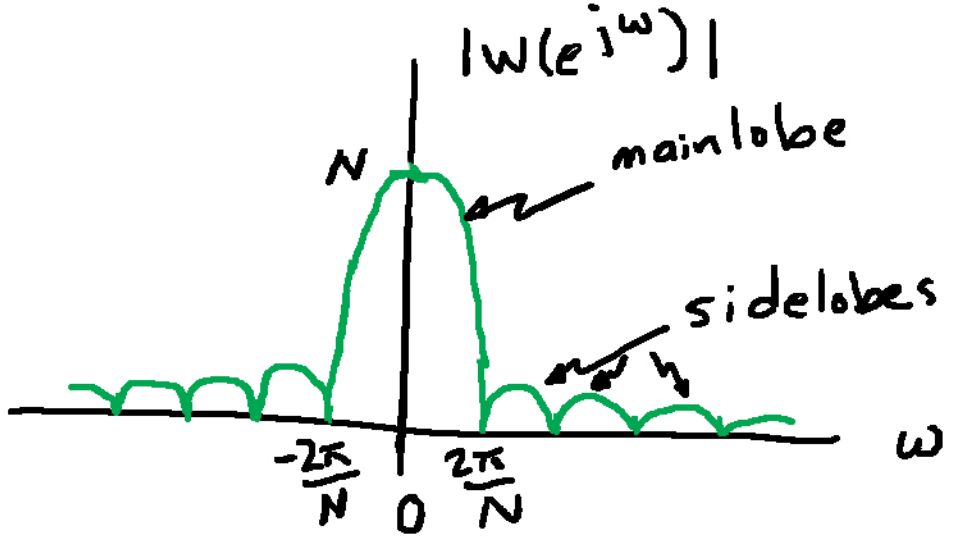
$$z[n] = x[n]\,w[n] \quad \text{where} \quad w[n] = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

$$z[n] \overset{\text{DTFT}}{\longleftrightarrow} Z(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) * W(e^{j\omega})$$

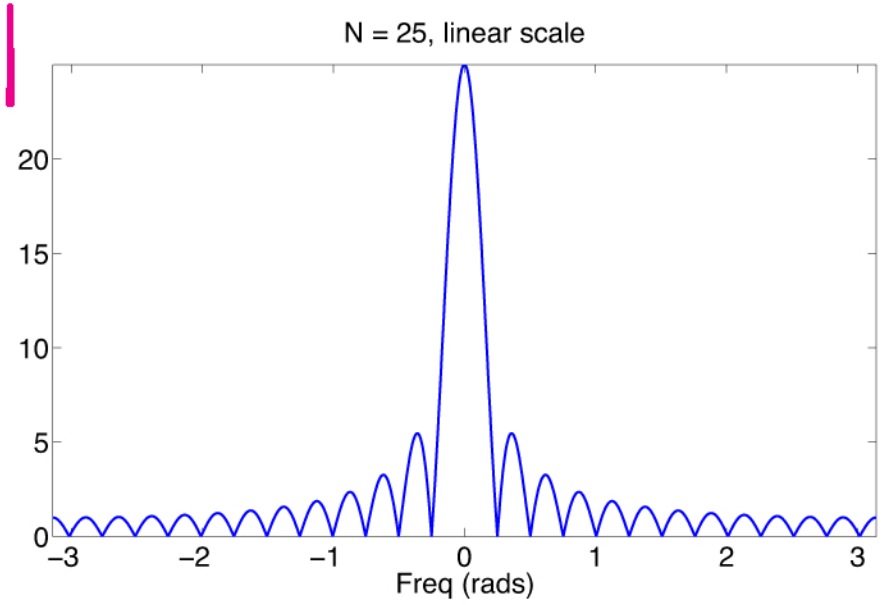
$$W(e^{j\omega}) = e^{-j\frac{(N-1)\omega}{2}} \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin(\omega/2)}$$

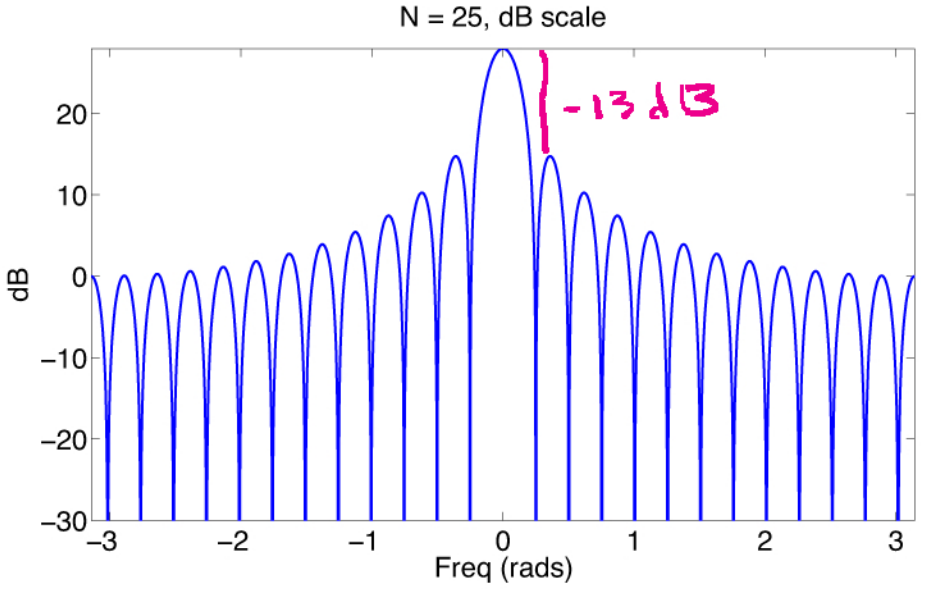
1) Mainlobe blurs detail in $X(e^{j\omega})$
Resolution ~ mainlobe width = $\frac{4\pi}{N}$

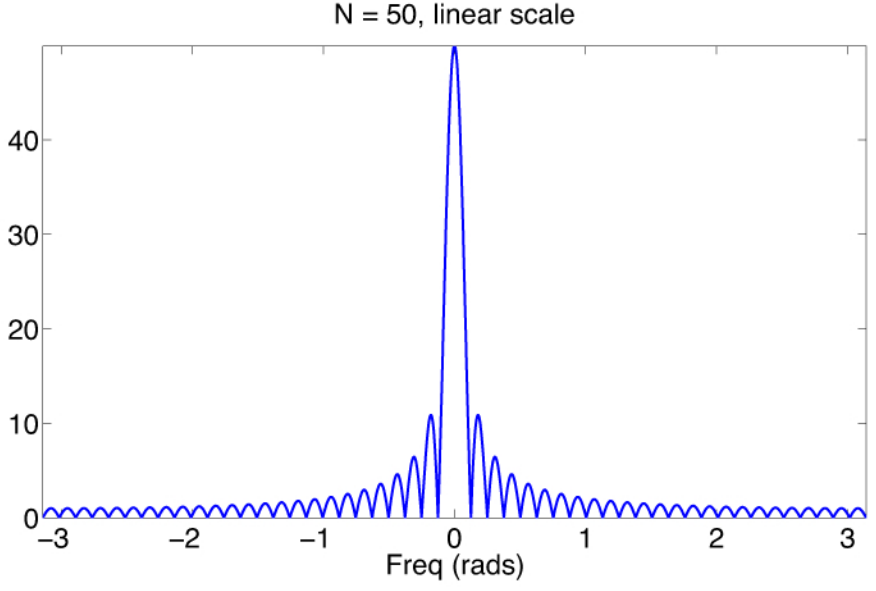
2) Sidelobes limit dynamic range.
Weak components obscured by sidelobes of large components

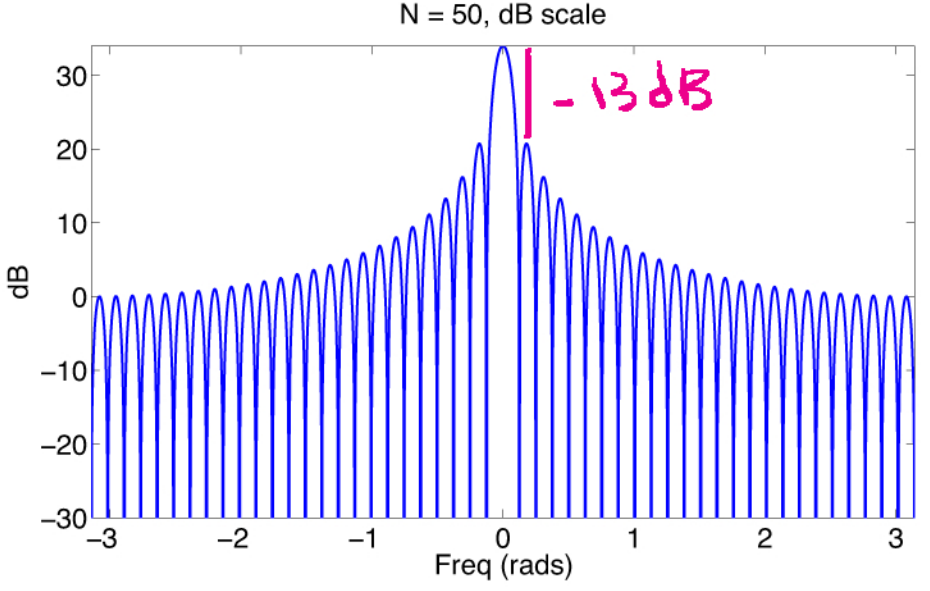


$|W(e^{j\omega})|$


N = 25, linear scale
Freq (rads)

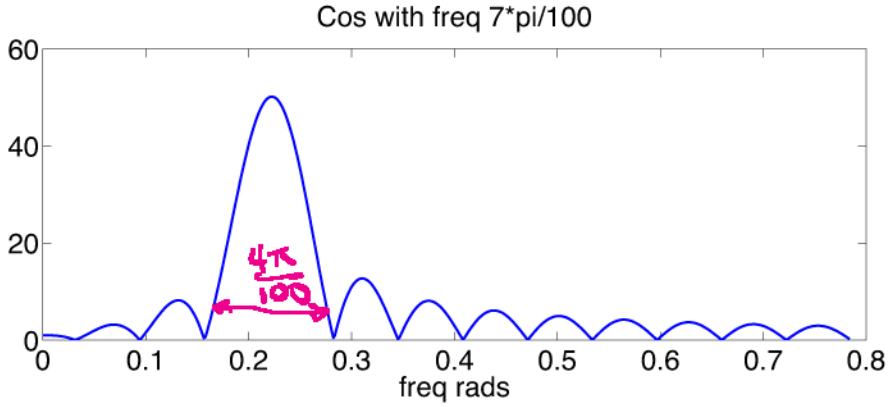

N = 25, dB scale
-13 dB
dB
Freq (rads)

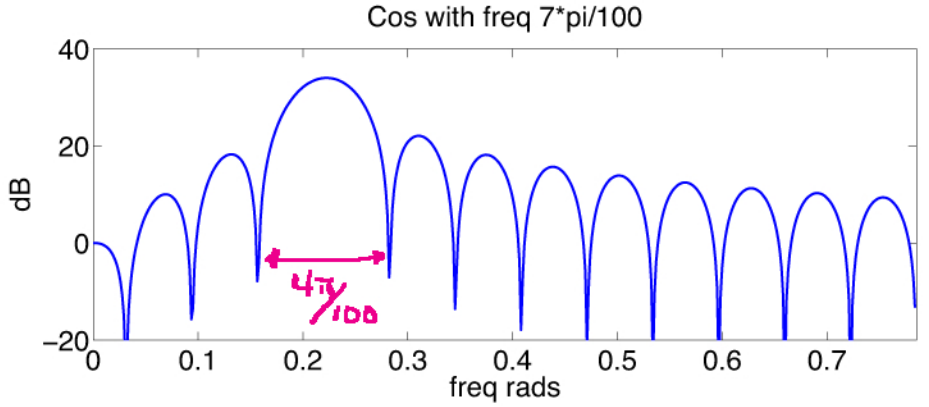

N = 50, linear scale
Freq (rads)

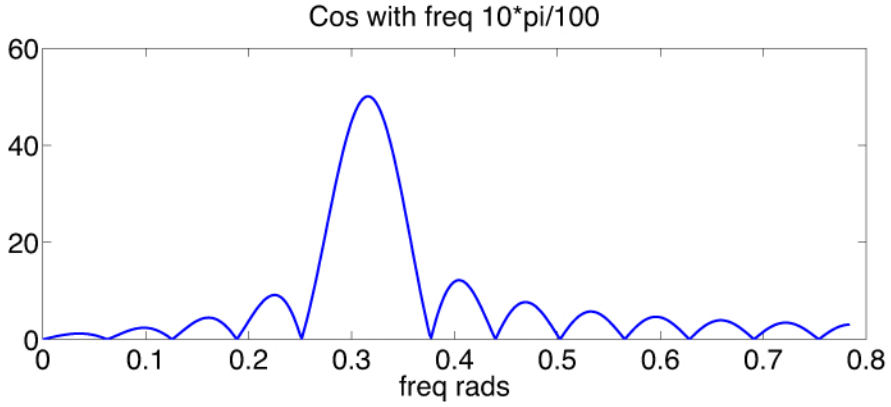

N = 50, dB scale
-13 dB
dB
Freq (rads)

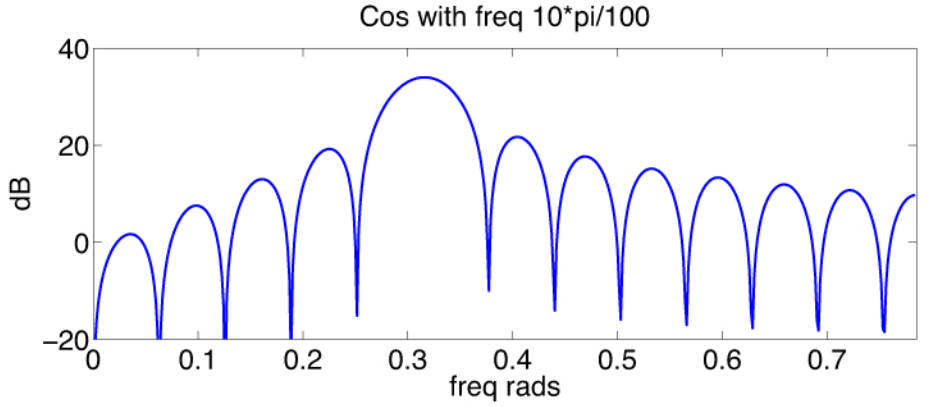
$|Z(e^{j\omega})|$

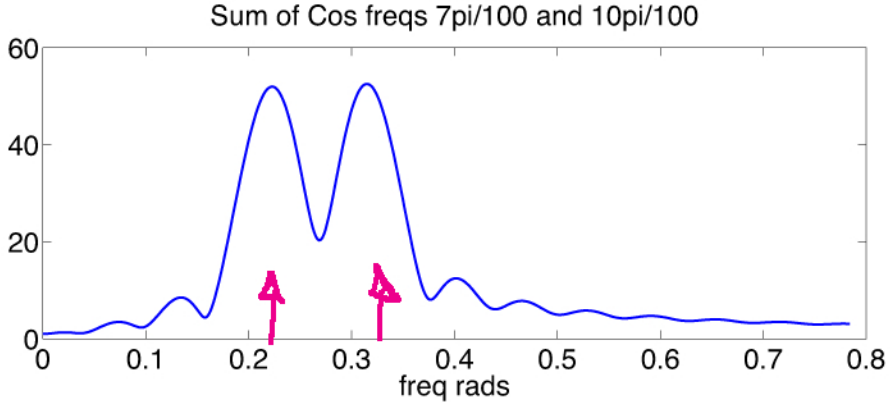
$N = 100$

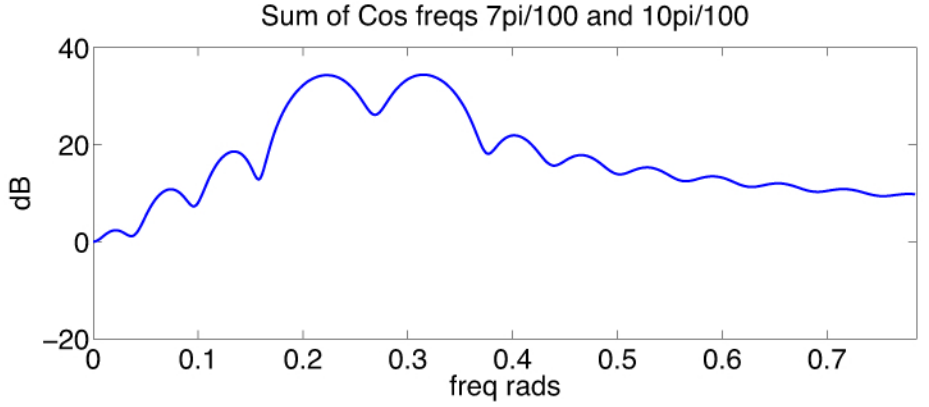








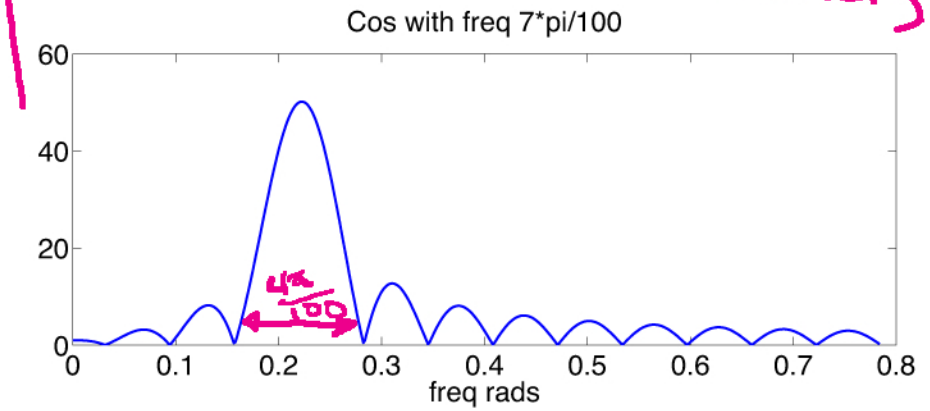


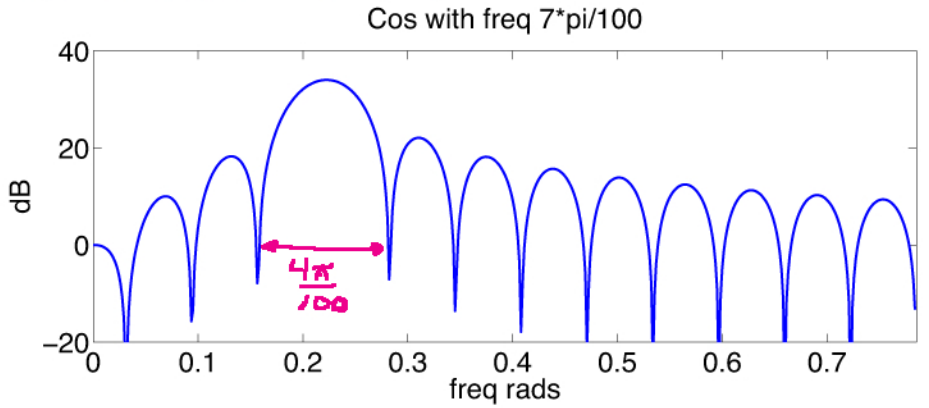


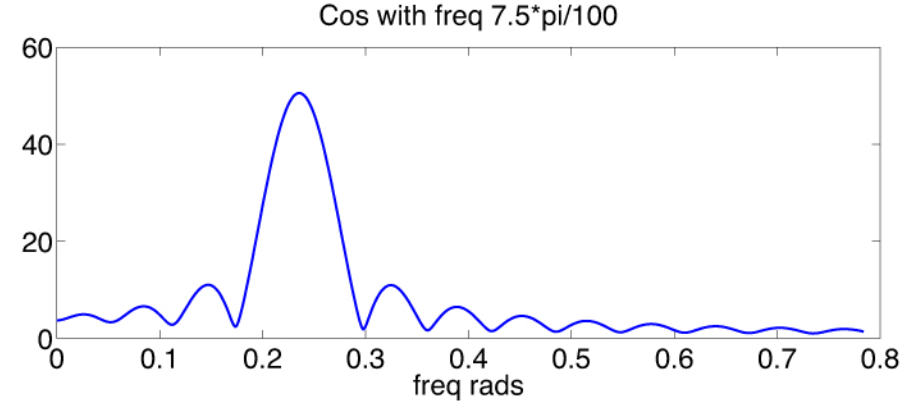
Closely Spaced Sinusoids

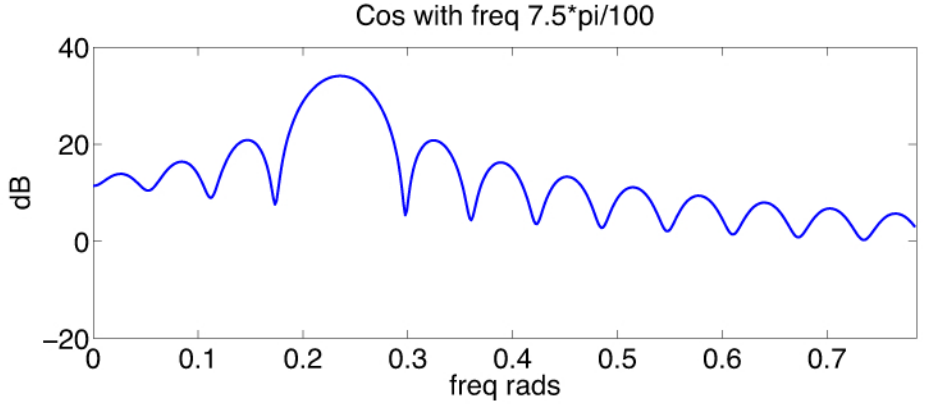
$|Z(e^{j\omega})|$

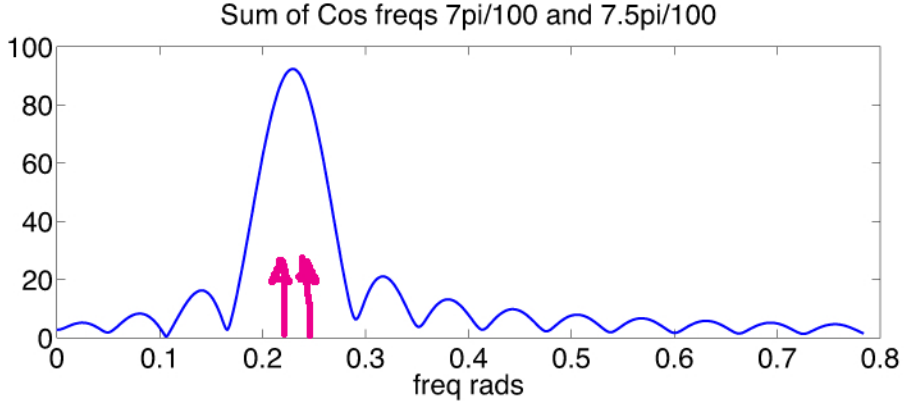
$N = 100$

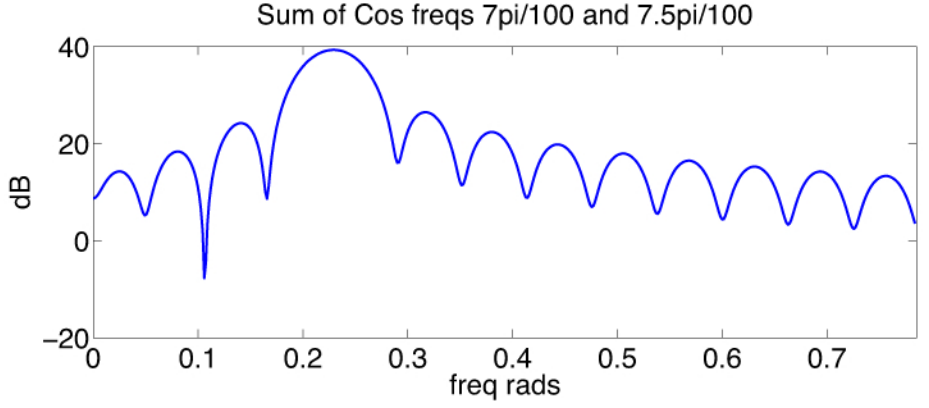








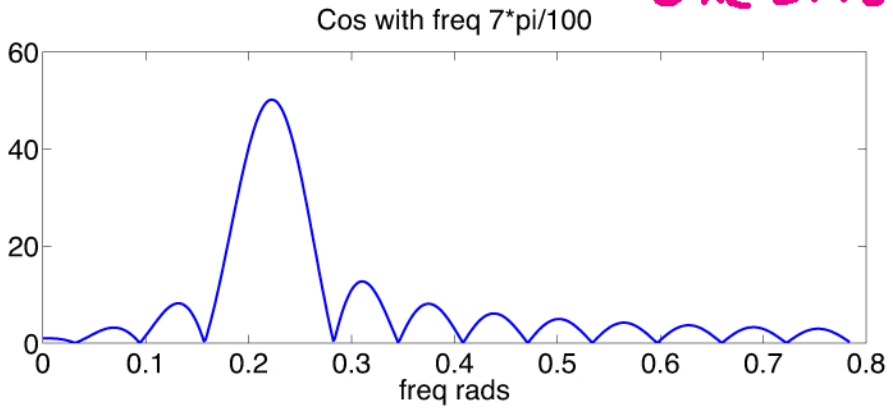


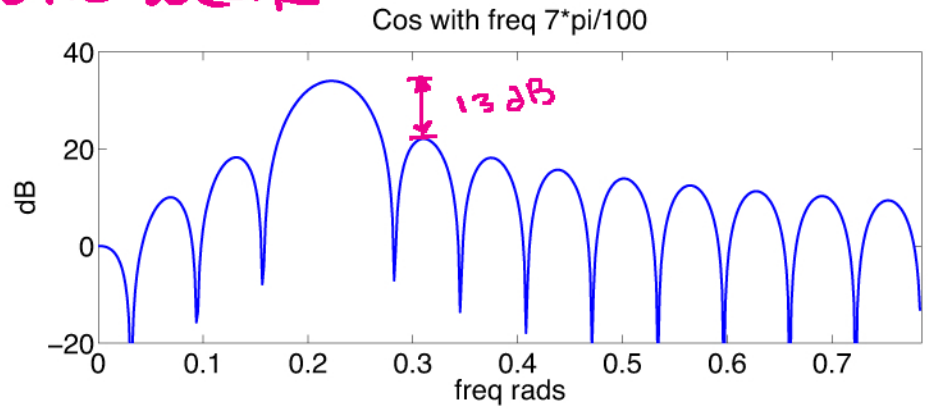


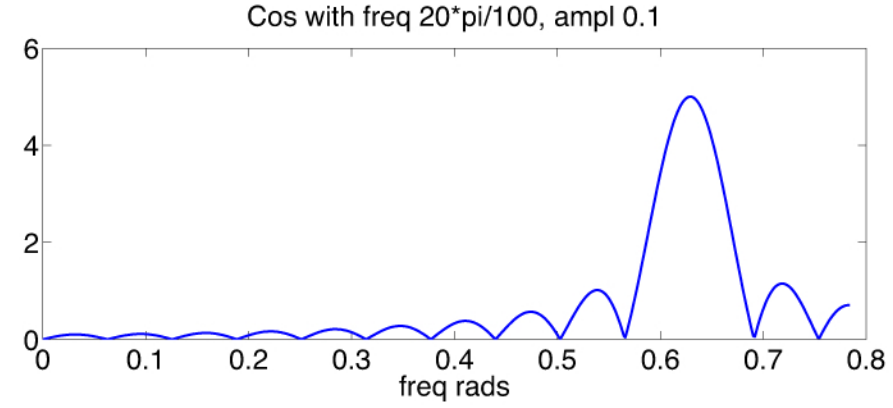
$|Z(e^{iw})|$

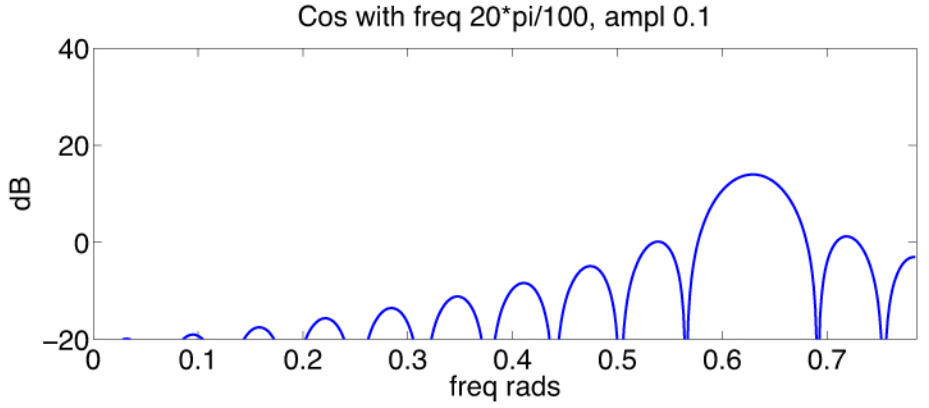
N=100

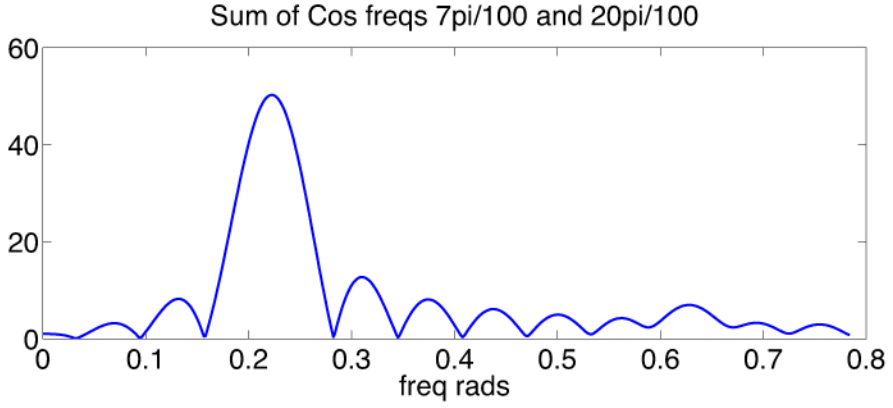
One strong, one weak

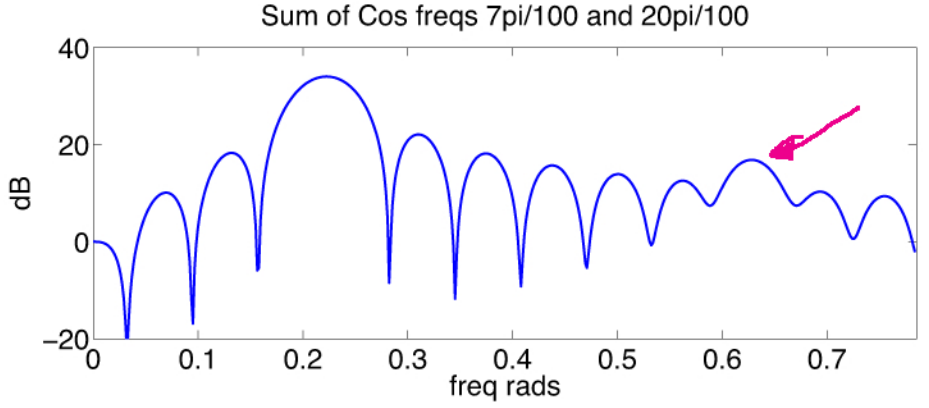












$|z(e^{j\omega})|$

$N=100$

one strong, one weak