DFT Properties

1) Periodicity. Let $x[n] \xleftrightarrow{DFT;N} X[k]$

a) $X[k]$ is N periodic, i.e., $X[k+N] = X[k]$

- follows from 2π-periodicity of $X(e^{j\omega})$

b) $x[n]$ is N periodic, i.e., $x[n+N] = x[n]$

- sampling in freq $\Rightarrow$ periodicity (replicates) in time

2) Linearity

$$\underline{ax[n] + by[n]} \xleftrightarrow{DFT;N} aX[k] + bY[k]$$

3) Time Shift.

$$x[n-m] \xleftrightarrow{DFT;N} X[k]e^{-j\frac{2\pi}{N}km}$$

4) Modulation

$$x[n]e^{j\frac{2\pi}{N}mn} \xleftrightarrow{DFT;N} X[k-m]$$

5) Conjugate Symmetry. For $x[n]$ real,

$$X^*[k] = X[-k] = X[N-k]$$

- $\text{Real}\{X[k]\}$: even
- $\text{Imag}\{X[k]\}$: odd
- $|X[k]|$: even
- $\arg\{X[k]\}$: odd

6) Inversion.

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi}{N}kn} \quad ; \quad X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1}\left(\sum_{m=0}^{N-1} x[m]\, e^{-j\frac{2\pi}{N}km}\right) e^{j\frac{2\pi}{N}kn}$$

$$= \sum_{m=0}^{N-1} x[m]\, \frac{1}{N}\sum_{k=0}^{N-1}\left[\underline{e^{j\frac{2\pi}{N}(n-m)}}\right]^{k}$$

$$= \sum_{m=0}^{N-1} x[m] \begin{cases} 1, & n=m \\ 0, & n\neq m \end{cases}$$

$$= \underline{x[n]}$$