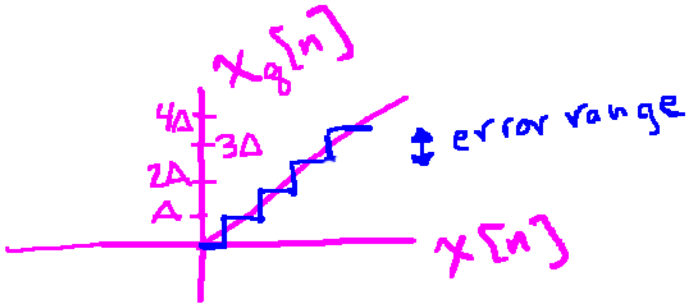
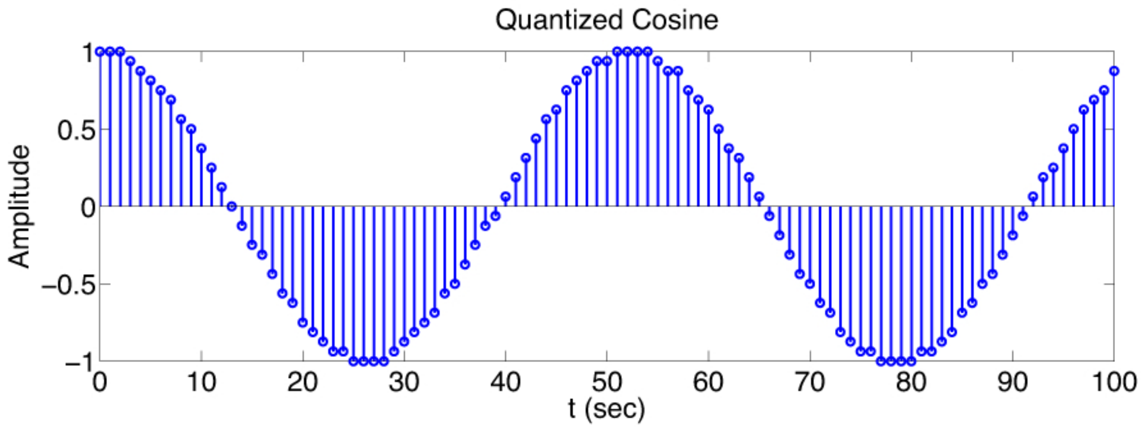
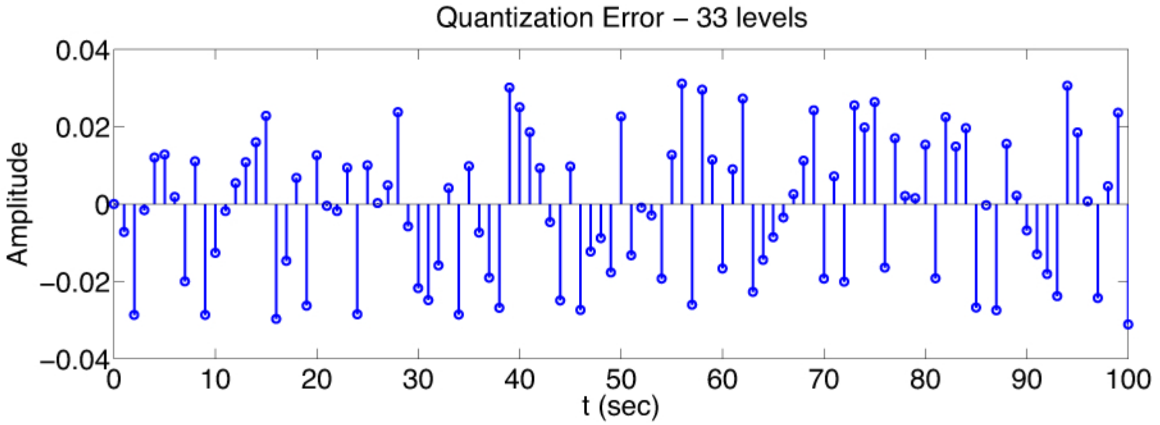
Quantization Error Analysis



$$e_q[n] = x[n] - x_q[n]$$

$$-\Delta/2 \le e_q[n] \le \Delta/2$$





$e_q[n]$ is complicated or unpredictable

⇒ statistical characterization

Terminology

1) Stationary - stat. properties do not change with time

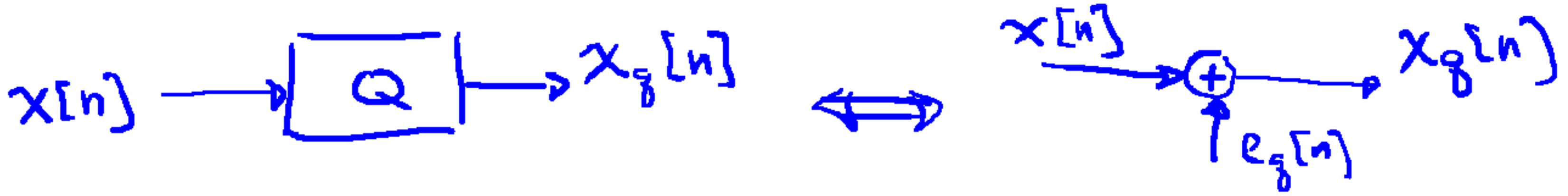
2) Random process - model for a signal with uncertain characteristics

3) Probability distribution - describes how likely it is that we observe the different possible values of a random quantity

4) Uncorrelated - values of signal at any two times appear unrelated

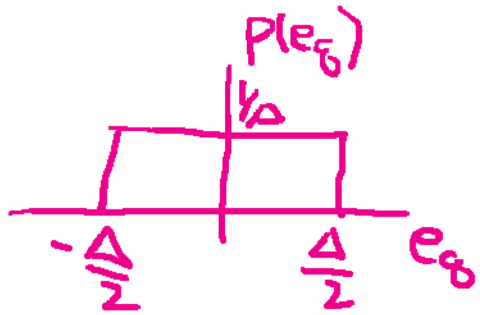
5) Independent - no relationship or pattern of any sort

Quantization Error Model



1) $e_q[n]$ is an observation of a stationary random process

2) $e_q[n]$ is uniformly distributed on $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$



3) $e_q[n]$ is uncorrelated

4) $e_q[n]$ is independent of $x[n]$

Reasonable if: Δ is small enough so $x[n]$ traverses several quant levels between samples

$x[n]$ appears random

Signal to Quantization Noise Ratio

$$SQNR = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) \text{ dB}$$

σ_x^2 : "power" in $x[n] = E\{x^2[n]\}$

σ_e^2 : "power" in $e_g[n] = E\{e_g^2[n]\}$

$$\sigma_e^2 = \int_{-\frac{\Delta}{2}}^{\Delta/2} e^2 p(e)\,de = \int_{-\frac{\Delta}{2}}^{\Delta/2} e^2\left(\frac{1}{\Delta}\right)de = \left.\frac{e^3}{3\Delta}\right|_{-\Delta/2}^{\Delta/2} = \frac{\Delta^2}{12}$$

$$SQNR = 10\log_{10}\left(\frac{12\sigma_x^2}{\Delta^2}\right)$$

but $\Delta = R/2^{b+1}$ (since $R = \max - \min$)

$$SQNR = 10\log_{10}\left(12\sigma_x^2\frac{(2^{b+1})^2}{R^2}\right)$$

$$= 20\log_{10}2^b + 20\log_{10}2 + 10\log_{10}12 - 20\log_{10}\left(\frac{R}{\sigma_x}\right)$$

40 dB ⇒ b = 7 or 8 bits

if $R = 4\sigma_x - (-4\sigma_x) = 8\sigma_x$ ⇒ ± 4 standard deviations

$$SQNR \approx 6.02b - 1.25 \text{ dB}$$

6 dB per bit!

Quantization Noise and Filtering