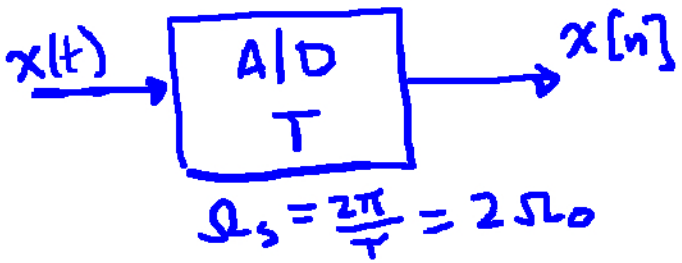
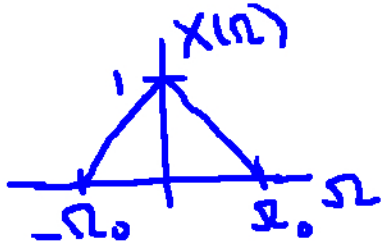
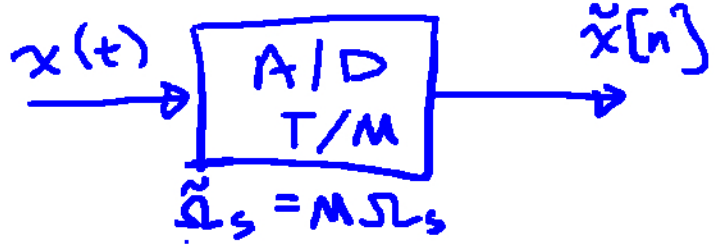
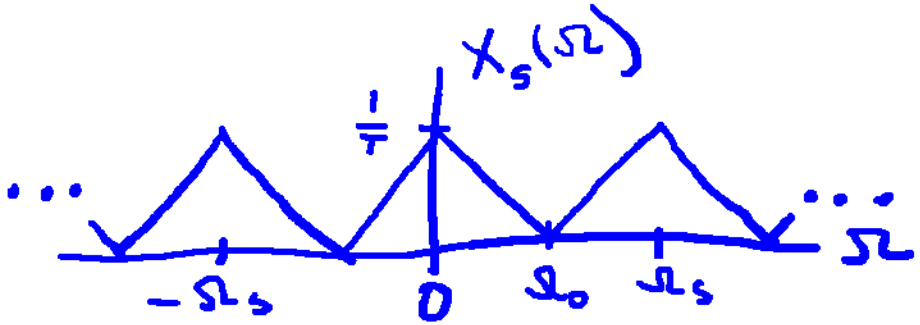
# Upsampling (Discrete-time Interpolation)

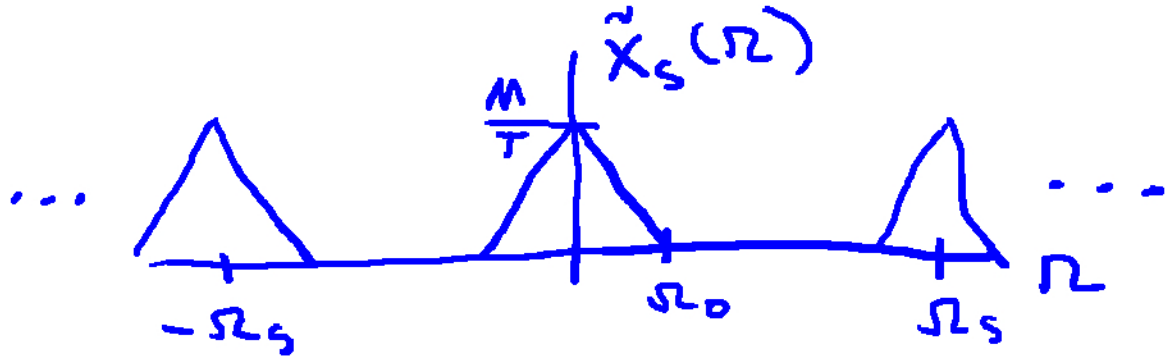




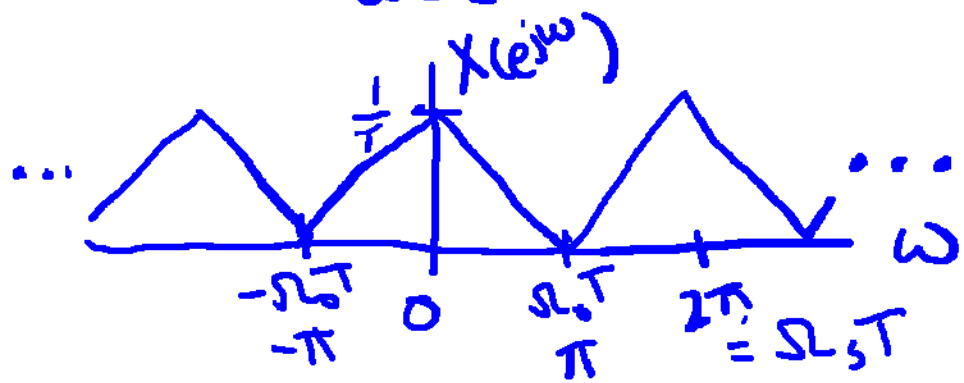



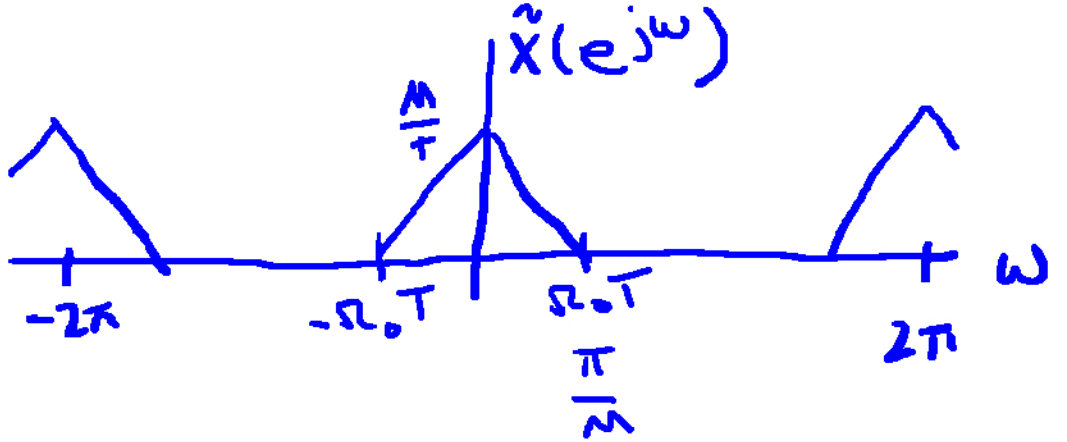



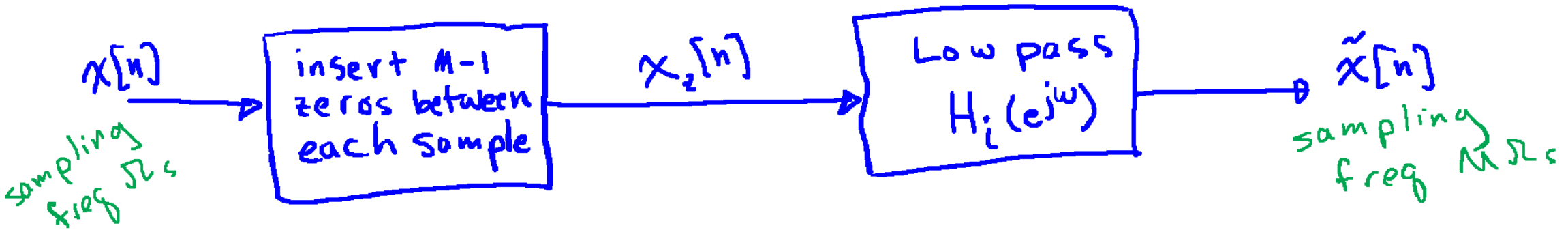



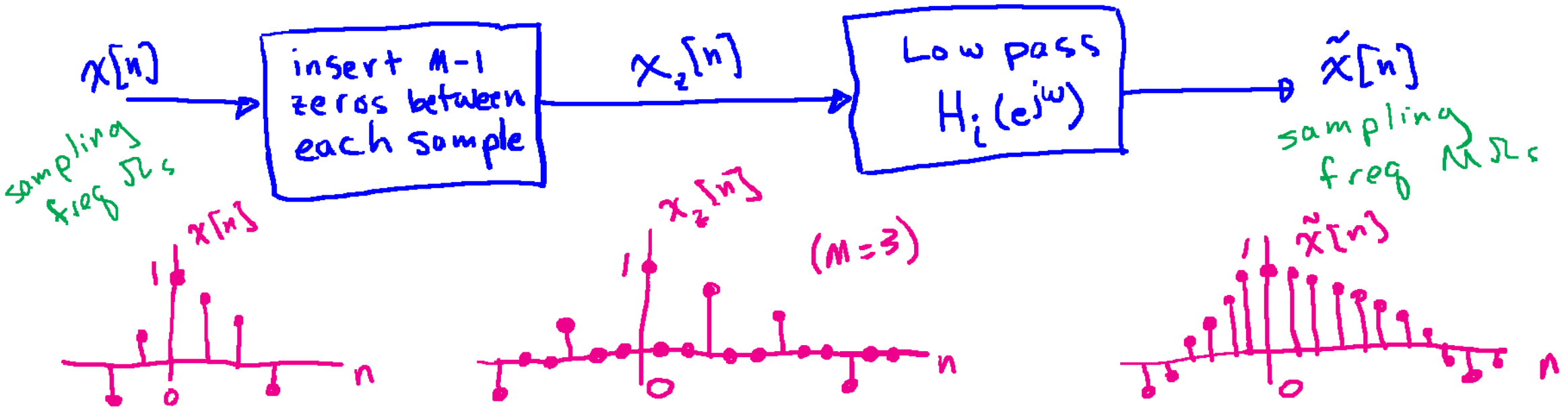



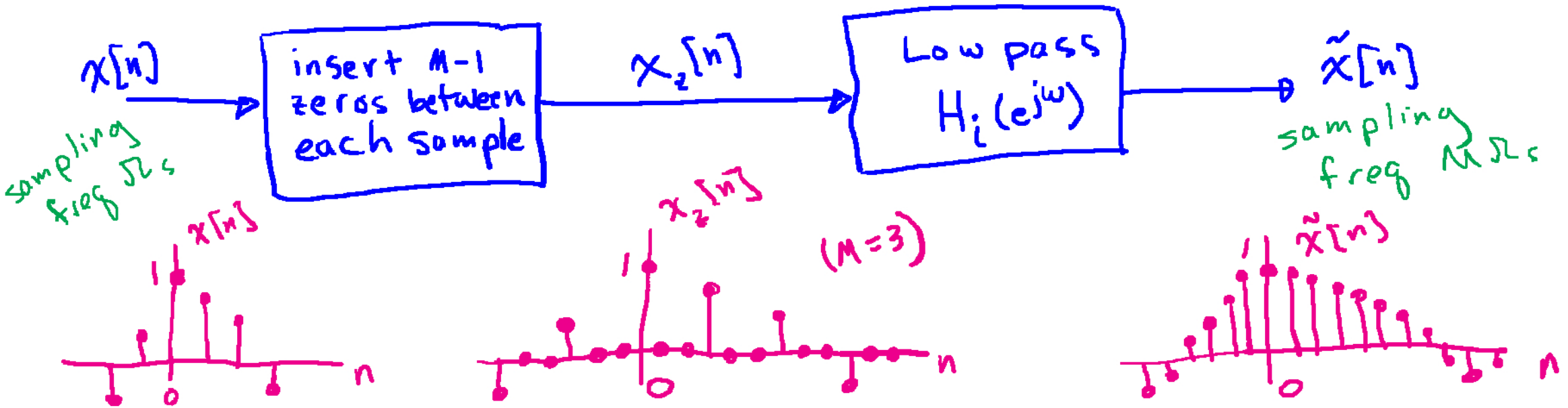

$\omega = \Omega T$



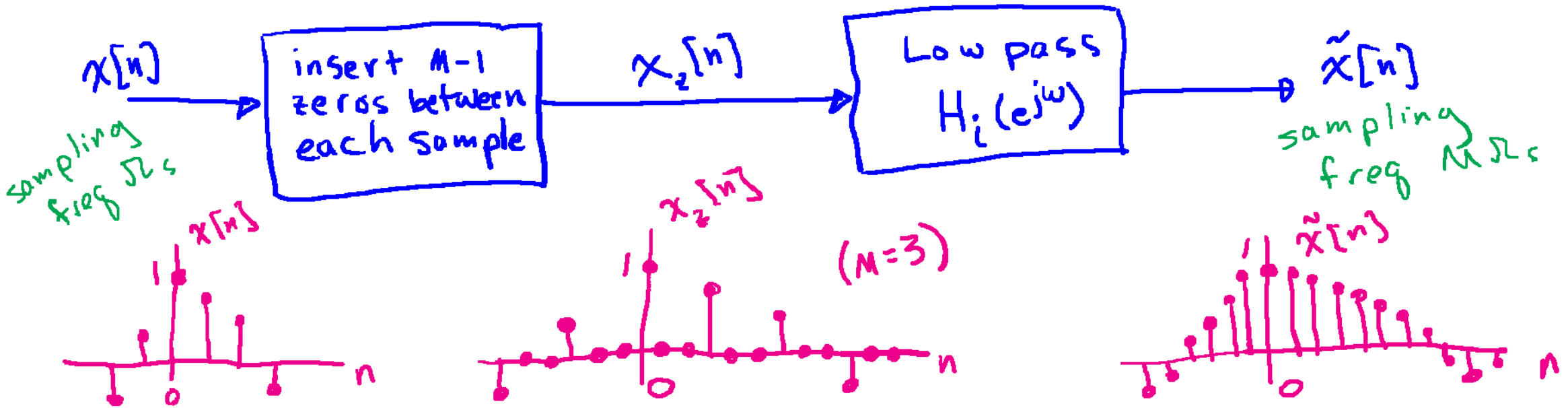




x[n]
sampling freq Ωs
insert M-1 zeros between each sample
x_z[n]
Low pass
H_i(e^jω)
x̃[n]
sampling freq MΩs


x[n]
insert M-1 zeros between each sample
$x_z[n]$
Low pass
$H_i(e^{j\omega})$
$\tilde{x}[n]$
sampling freq $\Omega_s$
sampling freq $\Omega_s$
x[n]
1
0
n
$x_z[n]$
(M=3)
1
0
n
$\tilde{x}[n]$
1
0
n




$$x_z[n] = \begin{cases} x[n/M] & , \ n/M \text{ integer} \\ 0 & , \ \text{other} \end{cases} \quad \text{or} \quad x_z[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kM]$$




$$x_z[n] = \begin{cases} x[n/M], & n/M \text{ integer} \\ 0, & \text{other} \end{cases} \quad \text{or} \quad x_z[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kM]$$

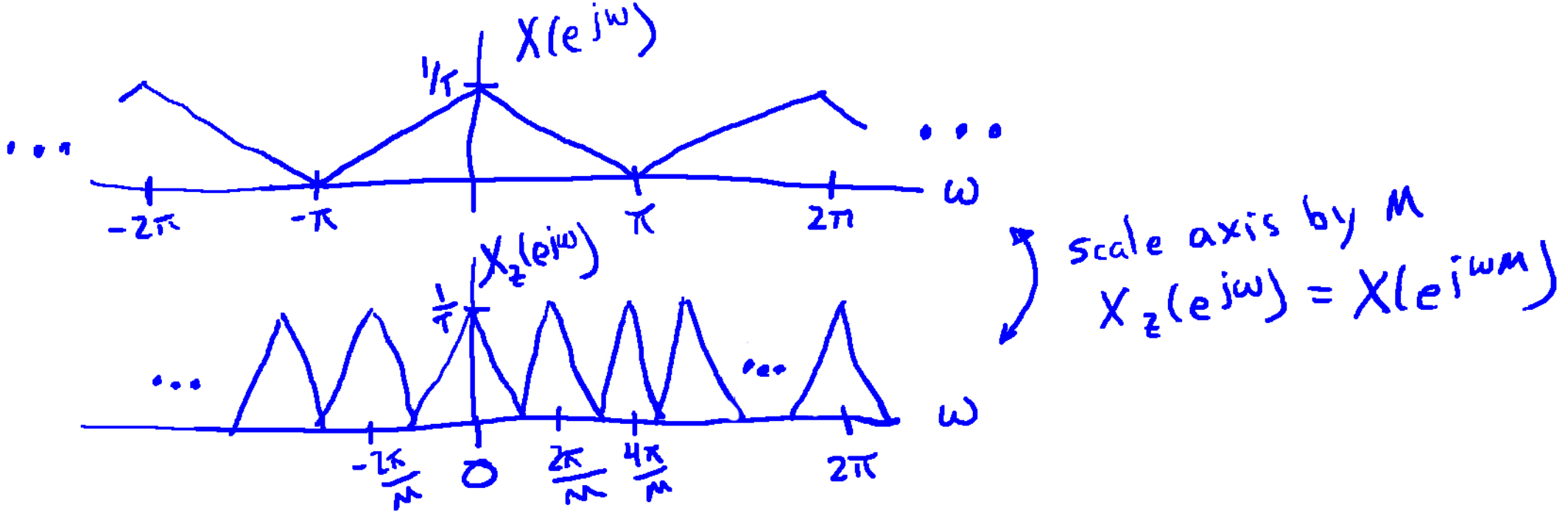
So

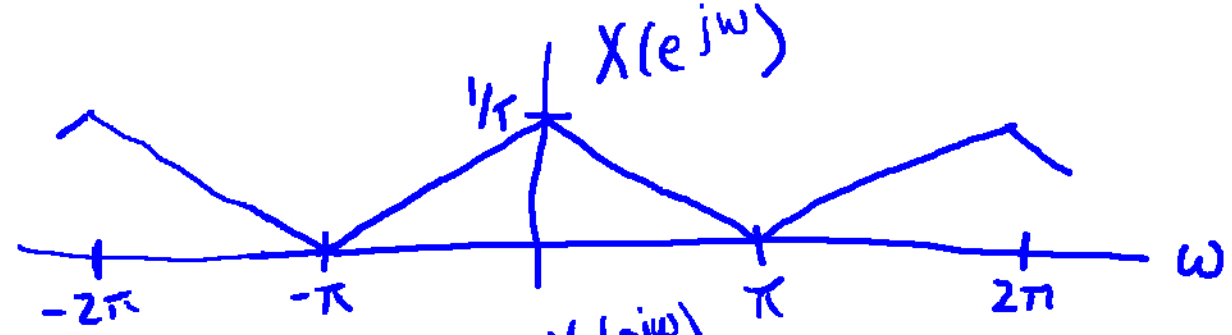
$$X_z(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left( \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kM] \right) e^{-j\omega n} = \sum_{k=-\infty}^{\infty} x[k] \left( \sum_{n=-\infty}^{\infty} \delta[n-kM] e^{-j\omega n} \right)$$

$e^{-j\omega Mk}$

$$= \sum_{k=-\infty}^{\infty} x[k]\, e^{-j\underbrace{\omega M}_{\omega'} k} = X(e^{j\omega M})$$

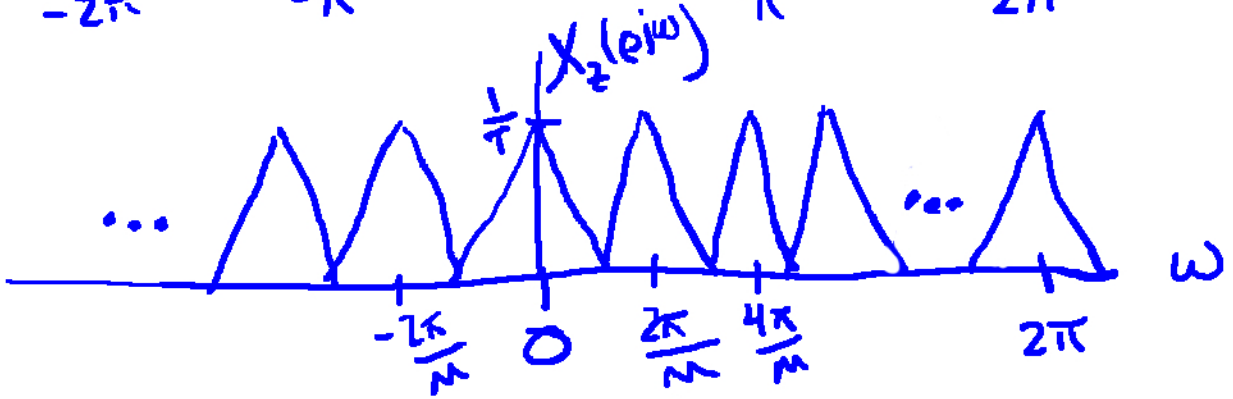
scale $\omega$ by $M$


X(e^jω)
1/T
...
...
ω
-2π
-π
π
2π
X_z(e^jω)
1/T
...
...
ω
-2π/M
0
2π/M
4π/M
2π
scale axis by M
X_z(e^jω) = X(e^iωM)




scale axis by $M$

$X_z(e^{j\omega}) = X(e^{j\omega M})$




$\times$ filter by $H_i(e^{j\omega})$