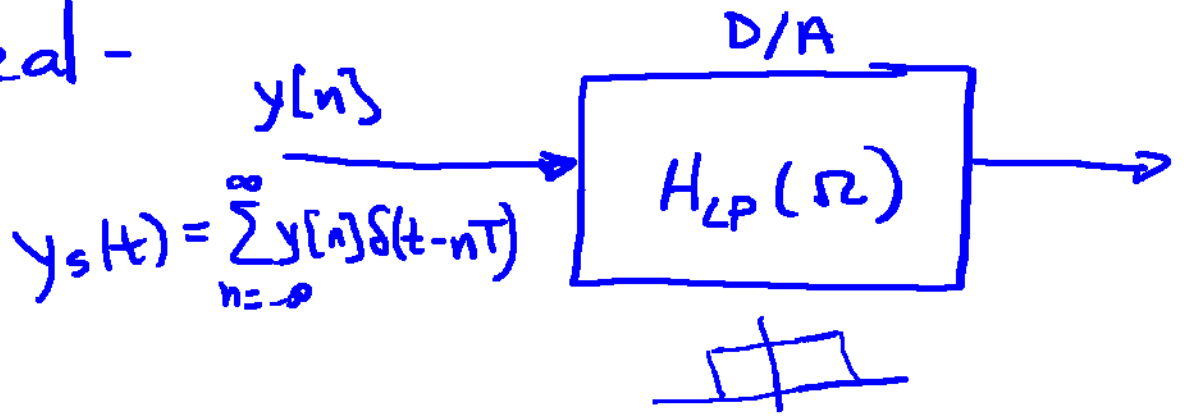
Practical Reconstruction

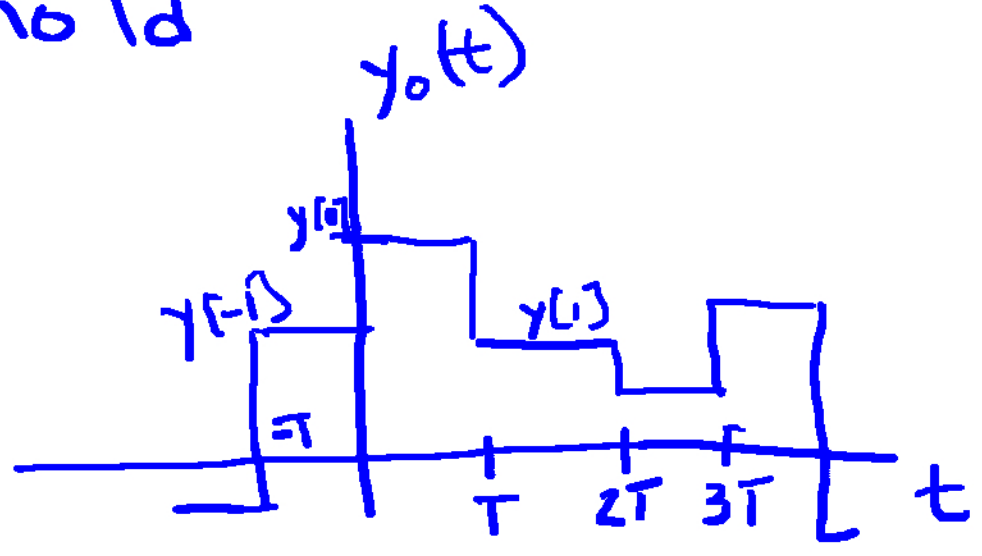
Ideal -



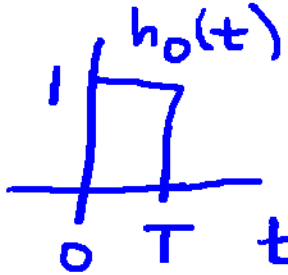
$$y(t) = T\sum_{n=-\infty}^{\infty} y[n]\,\frac{\sin\left(\frac{\Omega_s}{2}(t-nT)\right)}{\pi(t-nT)}$$

Practical – zero-order hold

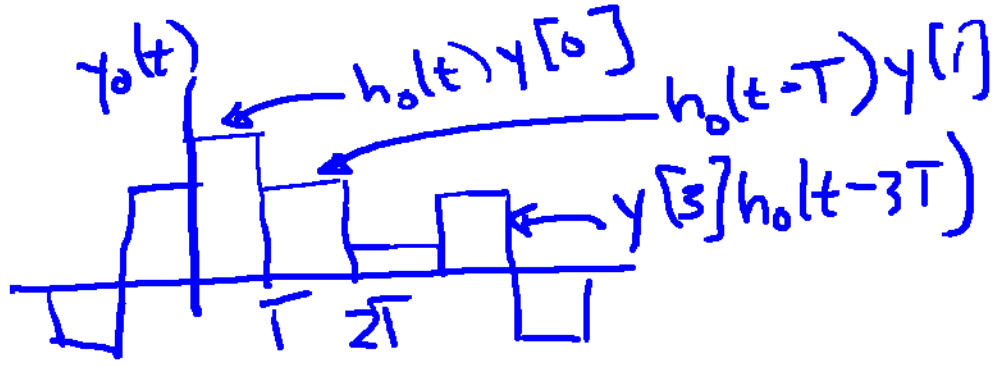
constant voltage of value y[n] is held for T sec



Let $h_0(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$



$$y_0(t) = \sum_{n=-\infty}^{\infty} y[n]\, h_0(t - nT)$$



$$y_0(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t - nT) * h_0(t)$$

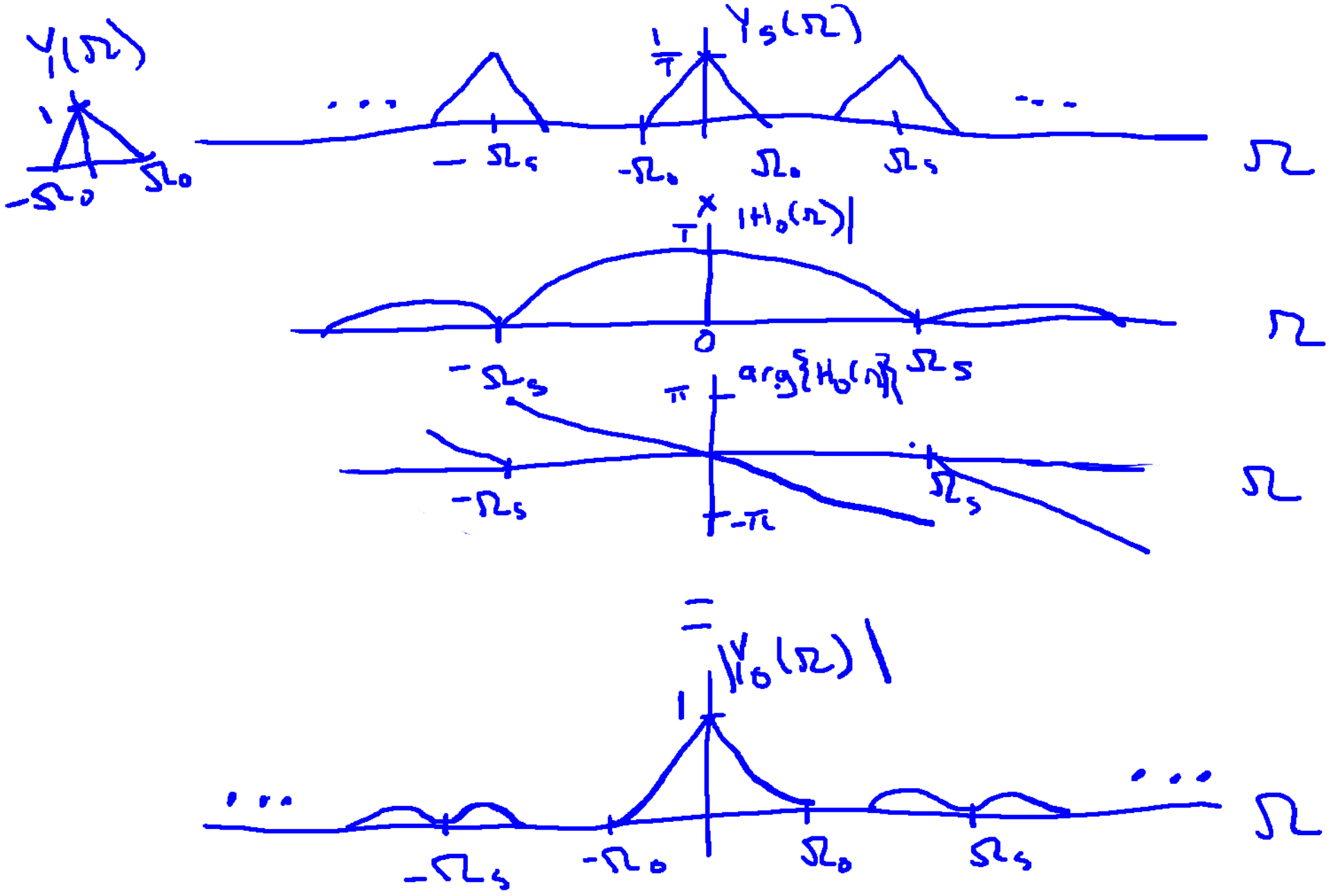
$$= y_s(t) * h_0(t)$$

$$\Downarrow$$

$$Y_0(\Omega) = Y_s(\Omega)\, H_0(\Omega)$$

$$Y_s(\Omega) = \frac{1}{T}\sum_{l=-\infty}^{\infty} Y(\Omega - l\Omega_s)$$

$$H_0(\Omega) = 2e^{-j\Omega T/2}\,\frac{\sin\left(\frac{\Omega}{2}T\right)}{\Omega}$$


$Y_1(\Omega)$
1
$-\Omega_0$
Ω_0
$Y_s(\Omega)$
$\frac{1}{T}$
$-\Omega_s$
$-\Omega_0$
Ω_0
Ω_s
Ω
$|H_0(\Omega)|$
T
0
Ω
$-\Omega_s$
$\arg\{H_0(\Omega)\}$
π
Ω_s
$-\Omega_s$
$-\pi$
Ω
$|V_0(\Omega)|$
1
$-\Omega_s$
$-\Omega_0$
Ω_0
Ω_s
Ω

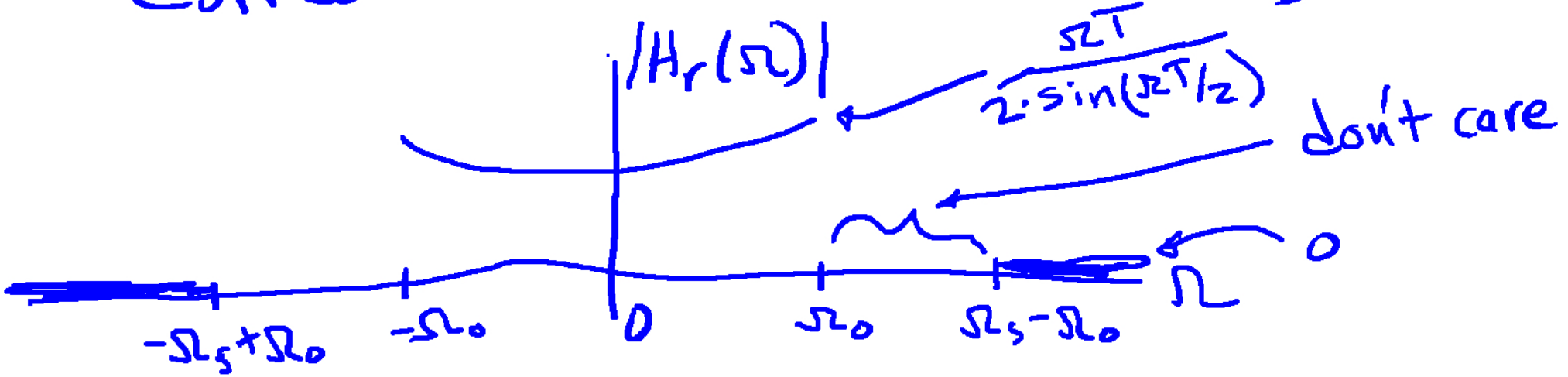
Effects of Zero-Order Hold

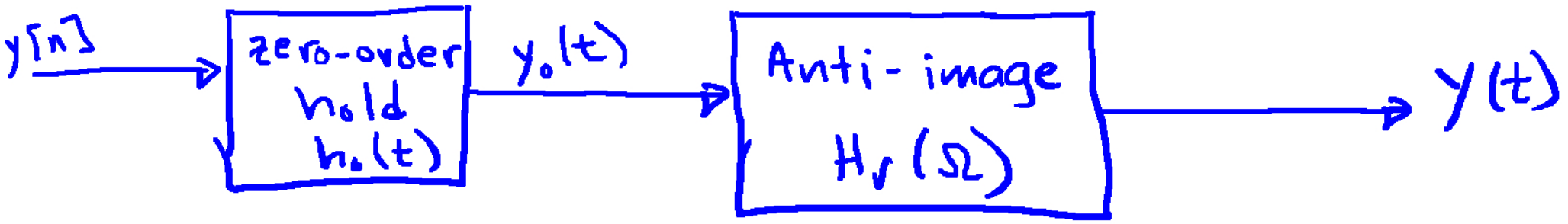
1) Linear phase shift $\Leftrightarrow$ time delay of $T/2$

2) Distortion between $-\Omega_0$ and Ω_0
(mainlobe of $H_0(\Omega)$)

3) Distorted/attenuated replicates of $Y(\Omega)$ at $k\Omega_s$ $(k \neq 0)$

Correct 2+3 with antiimaging filter





Goal: $|H_r(\Omega) H_0(\Omega)| = \begin{cases} T & |\Omega| < \Omega_0 \\ 0 & |\Omega| > \Omega_s - \Omega_0 \end{cases}$

$H_r(\Omega)$ is easier to build for $\Omega_s >> \Omega_0$