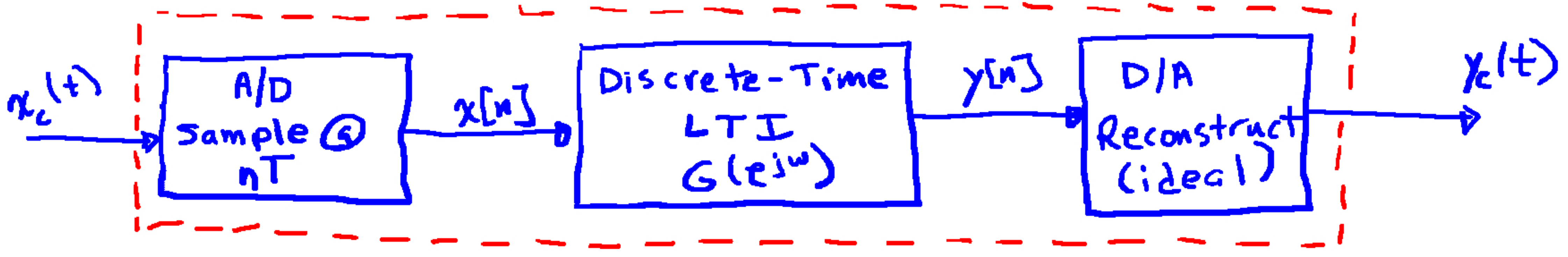
Equivalent Analog Filtering



$H(\Omega)$

Desired filter $H(\Omega) \Rightarrow G(e^{j\omega})$?

Example: Speech - remove low-frequency noise

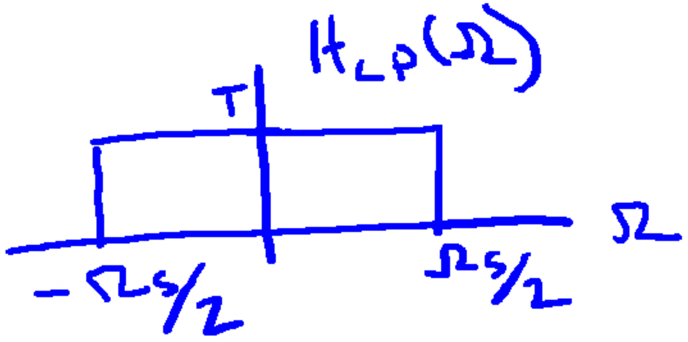
$|\Omega| < 100\,\text{Hz}$ Assume $X_c(\Omega) = 0$ $|\Omega| > 8000\pi \ \frac{\text{rads}}{\text{sec}}$

$\Omega_s = 20000\pi$ rads/sec

Analysis: $x_c(t) \xleftrightarrow{FT} X_c(\Omega)$; $x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$

$y_c(t) \xleftrightarrow{FT} Y_c(\Omega)$; $y[n] \xleftrightarrow{DTFT} Y(e^{j\omega})$

Recall - $\omega = \Omega T$; ideal reconst.



$$Y_c(\Omega) = H_{LP}(\Omega)\, Y(e^{j\omega})\Big|_{\omega = \Omega T}$$

$$= H_{LP}(\Omega)\, Y(e^{j\Omega T})$$

$$Y(e^{j\omega}) = G(e^{j\omega})\, X(e^{j\omega})$$

$$Y_c(\Omega) = H_{LP}(\Omega)\, G(e^{j\Omega T})\, X(e^{j\Omega T})$$

Since $$X_{cs}(\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s)$$

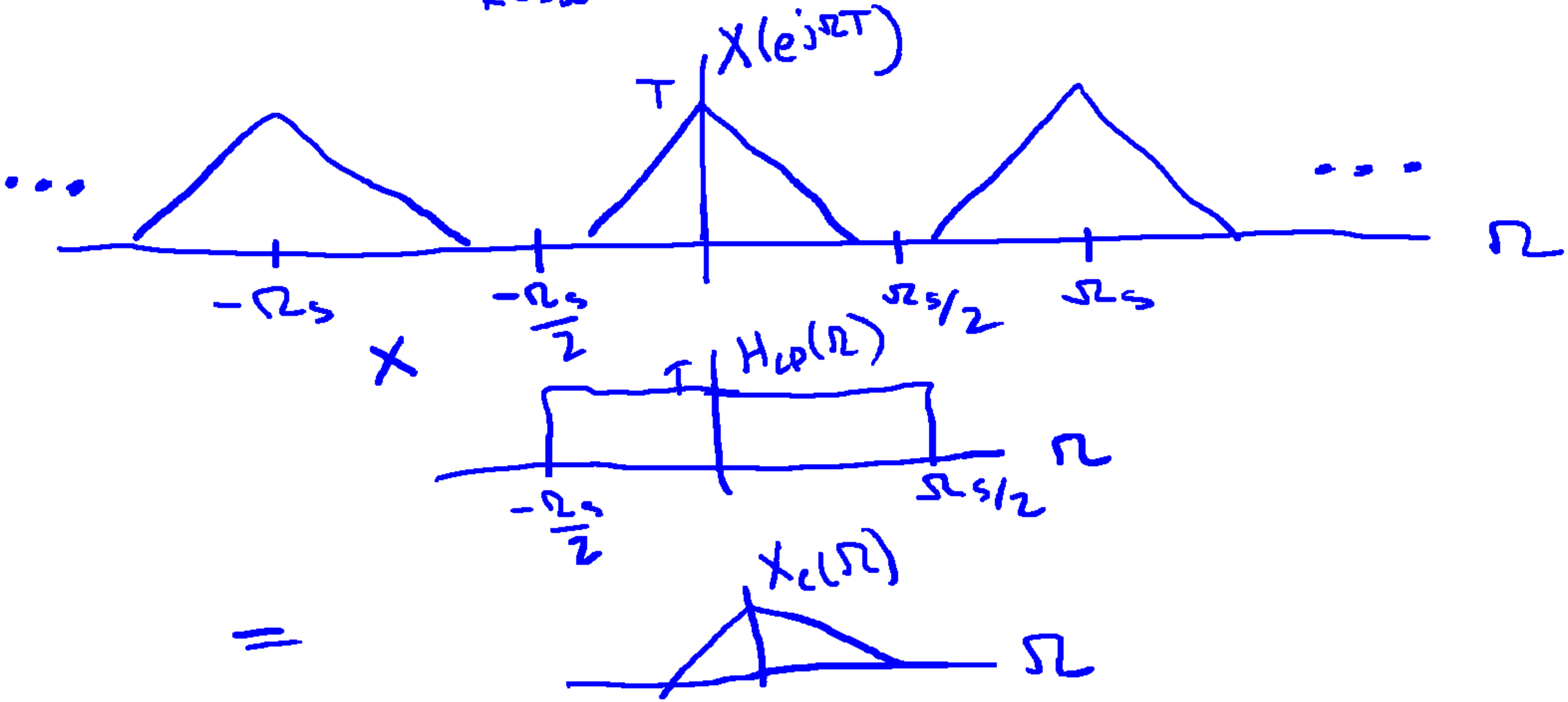
$$X(e^{j\omega}) = X_{cs}(\Omega)\Big|_{\Omega=\frac{\omega}{T}} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega}{T} - k\Omega_s\right)$$

$$X(e^{j\Omega T}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s)$$

$$\underbrace{Y_c(\Omega)}_{\text{output}} = H_{LP}(\Omega)\, G(e^{j\Omega T})\, \frac{1}{T}\sum_{k=-\infty}^{\infty} \underbrace{X_c(\Omega - k\Omega_s)}_{\text{input}}$$

Assuming $|X_c(\Omega)| = 0, \ |\Omega| > \frac{\Omega_s}{2}$ (Sampling thm)

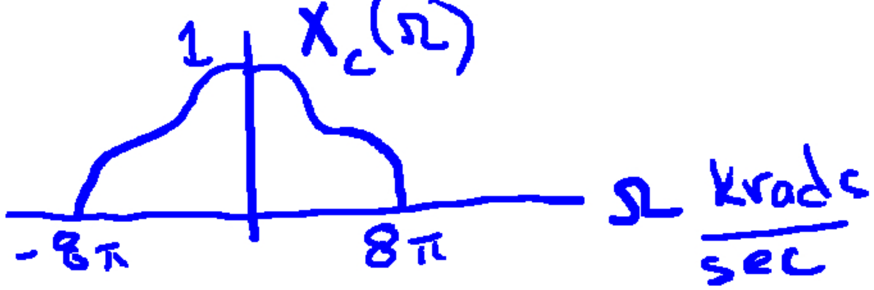
$$H_{LP}(\Omega)\, \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(\Omega - k\Omega_s) = X_c(\Omega)$$



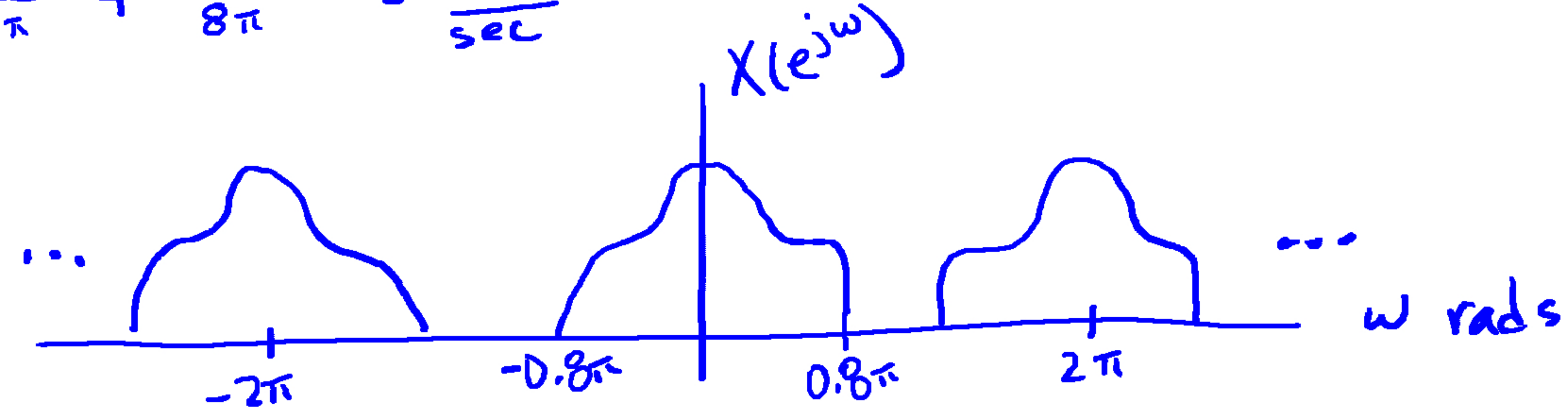
Thus $\underline{\underline{Y_c(\Omega) = G(e^{j\Omega T})\, X_c(\Omega)}}$ $\quad |\Omega| < \Omega_s/2$

Specs on $H(\Omega)$ transfer to $G(e^{j\Omega T})$ for $|\Omega| < \Omega_s/2$

Speech Example: $f_s = 10^4$ Hz $\Rightarrow T = 10^{-4}$ sec, $\Omega_s = 20\pi \frac{\text{Krads}}{\text{sec}}$

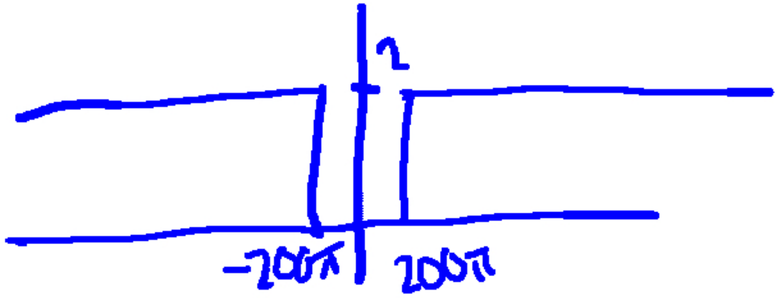


$\omega = \Omega(10^{-4})$

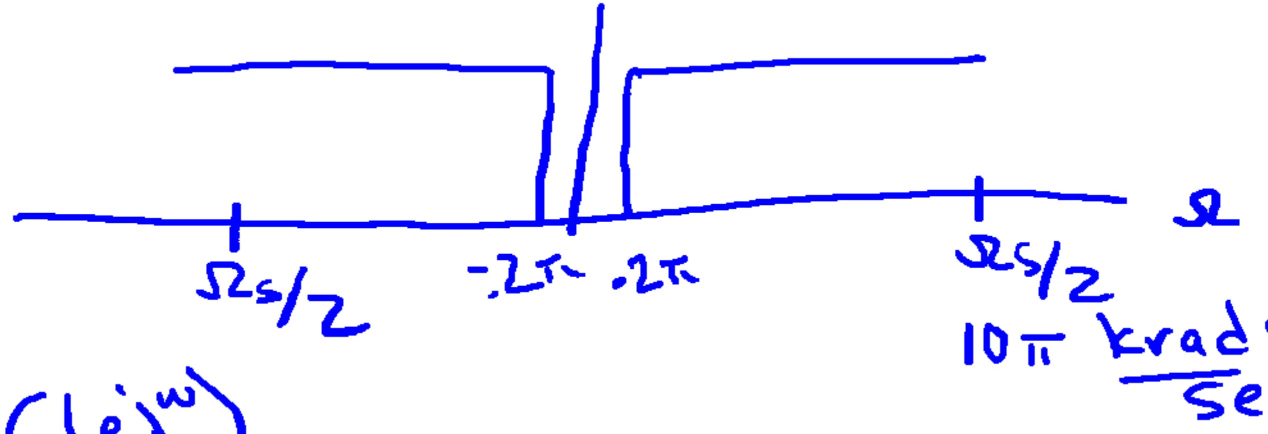


Highpass filter w/cutoff 100Hz $\Rightarrow$ $200\pi \ \frac{\text{rads}}{\text{sec}}$

$H(\Omega)$

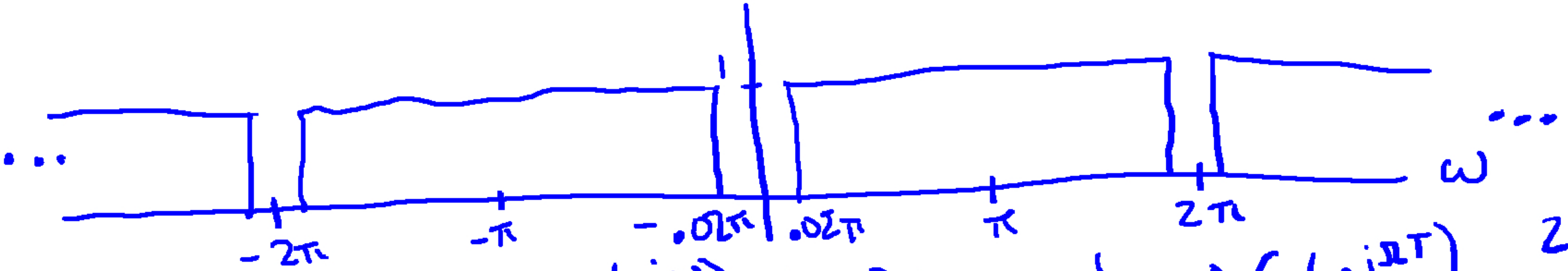


$G(e^{j\Omega T})$



$\omega = \Omega T$

$G(e^{j\omega})$



$G(e^{j\omega})$ is 2π periodic $\Rightarrow G(e^{j\Omega T})$

$\frac{2\pi}{T} = \Omega_s$