Frequency Domain (FT) Interpretation of sampling

1) Relate Ω to ω

2) Relate FT of sampled signal to FT of cont. time signal

   a) Find FT of a discrete-time signal

   $$x[n] \xleftrightarrow{FT} X_s(\Omega)$$

   b) Find cont. time rep. for $x[n]$

   $$x_s(t) \xleftrightarrow{FT} X_s(\Omega)$$

   c) $$x_s(t) = f_1(x(t)) \xleftrightarrow{FT} X_s(\Omega) = f_2(X(\Omega))$$

1) Relate Ω to ω

Let $g(t) = A\cos(\Omega t + \phi)$, $g[n] = A\cos(\omega n + \phi)$

$$g[n] = g(nT) = A\cos(\Omega T n + \phi)$$

$$\boxed{\omega = \Omega T}$$

DT freq = CT freq · Sampling interval

$$\text{rads} = \frac{\text{rads}}{\text{sec}} \cdot \text{sec}$$

2a) FT of $x[n]$

$$x[n] \xleftrightarrow{DTFT} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$\omega = \Omega T$$

$$x[n] \xleftrightarrow{FT} X_s(\Omega) = X(e^{j\omega})\Big|_{\omega=\Omega T}$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega T n}$$

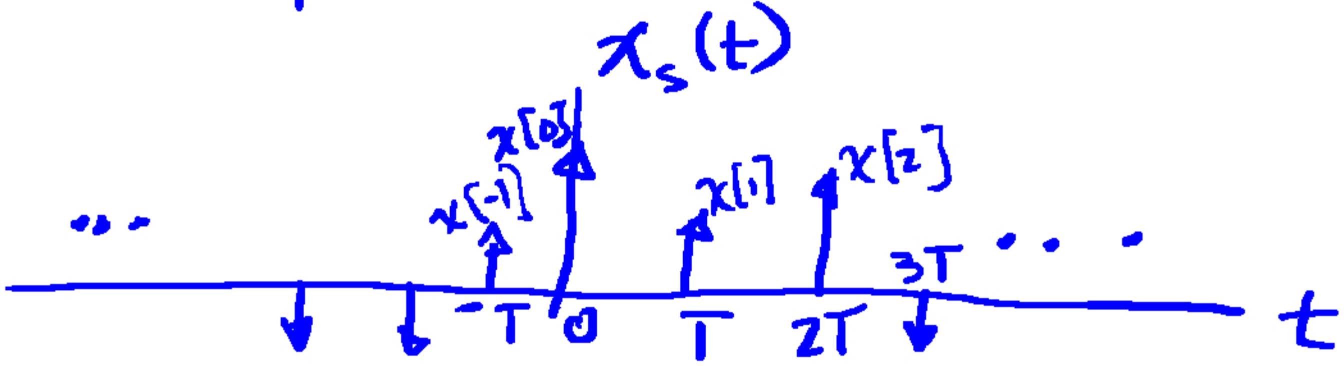
2b) Cont.-time Representation for $x[n]$

$$X_s(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega T n}, \quad \text{find } x_s(t)$$

$$\delta(t - nT) \xleftrightarrow{FT} e^{-j\Omega T n}$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\, \delta(t - nT)$$

CT rep for $x[n]$



2c) Express $X_s(\Omega)$ in terms of $X(\Omega)$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t-nT)$$

use $x[n] = x(nT)$

$$= \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

$x(t)\delta(t-nT) = x(nT)\delta(t-nT)$

$$= \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT)$$

$$= x(t)\,s(t)$$

where $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$

$\updownarrow$ FT

$$S(\Omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\Omega - k\frac{2\pi}{T}\right)$$

$\updownarrow$ FT

$$X_s(\Omega) = \frac{1}{2\pi} X(\Omega) * S(\Omega)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\Omega) * \delta\left(\Omega - k\frac{2\pi}{T}\right)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\Omega - k\frac{2\pi}{T}\right)$$

Summary

1) $\boxed{\omega = \Omega T}$

2) if $x(t) \xleftrightarrow{FT} X(\Omega)$ and $x[n] = x(nT)$, then

$$x[n] \xleftrightarrow{FT} X_s(\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\Omega - k\frac{2\pi}{T}\right)$$