FT and DTFT Properties

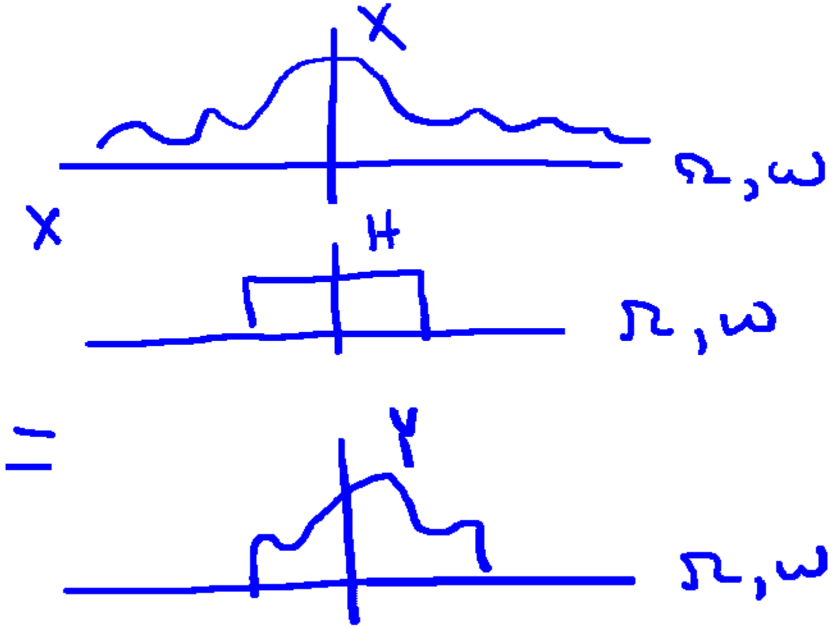
A) Convolution – multiplication

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \xleftrightarrow{FT} Y(\Omega) = X(\Omega) H(\Omega)$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \xleftrightarrow{DTFT} Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$$

$\xleftrightarrow{FT}$

$\xleftrightarrow{DTFT}$



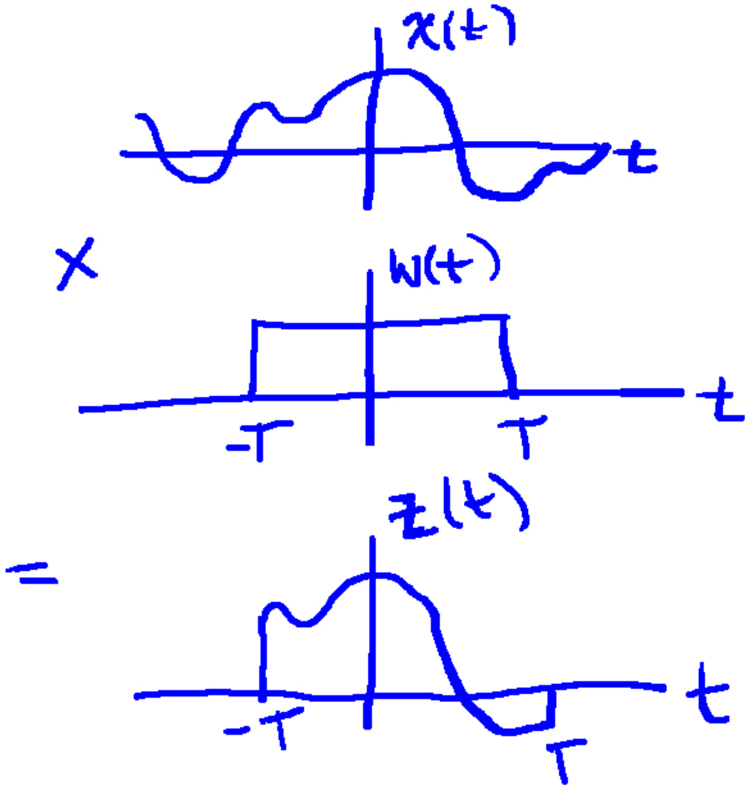
"Filtering"

B) Multiplication – Convolution (windowing)

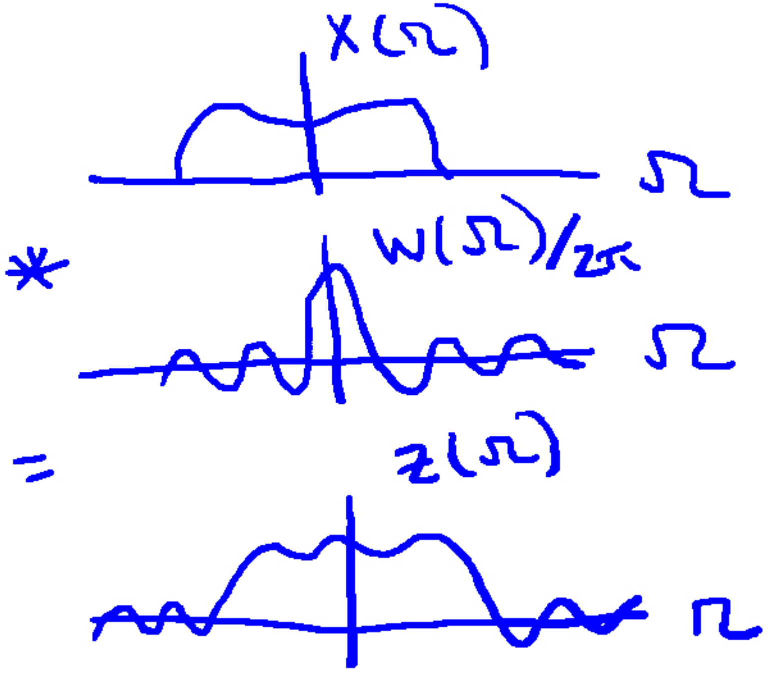
$$z(t) = x(t)\,w(t) \xleftrightarrow{FT} Z(\Omega) = \frac{1}{2\pi} X(\Omega) * W(\Omega)$$

$$z[n] = x[n]\,w[n] \xleftrightarrow{DTFT} Z(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) \circledast W(e^{j\omega})$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\gamma})\, W(e^{j(\omega-\gamma)})\, d\gamma$$



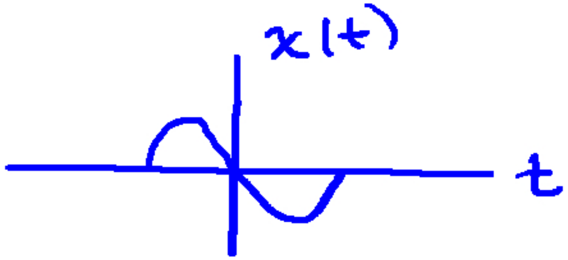
$\xleftrightarrow{FT}$



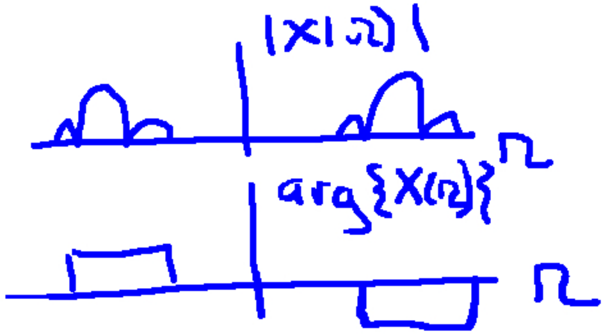
c) Time Shift

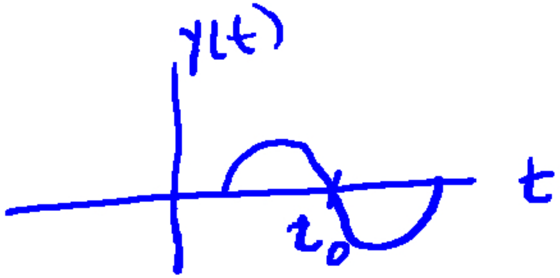
$$y(t) = x(t - t_0) \quad \xleftrightarrow{\ FT\ } \quad Y(\Omega) = e^{-j\Omega t_0} X(\Omega)$$

$$y[n] = x[n - n_0] \quad \xleftrightarrow{\ DTFT\ } \quad Y(e^{j\omega}) = e^{-j\omega n_0} X(e^{j\omega})$$

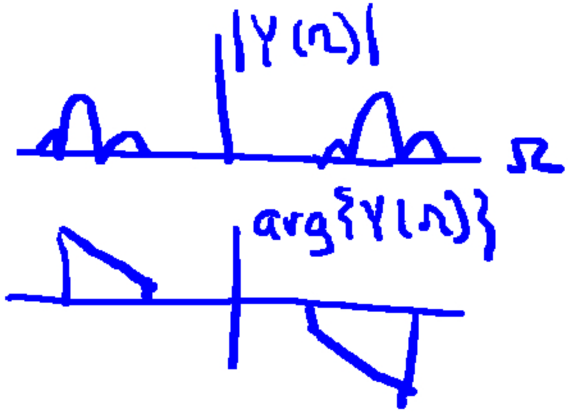


$\xleftrightarrow{\ FT\ }$





$\xleftrightarrow{\ FT\ }$

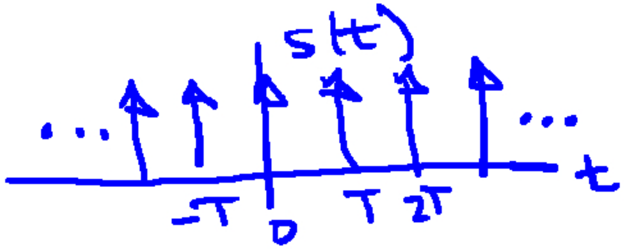


D) FT Representation for Periodic Signals

$x(t)$ has fund. period T: $x(t) \xleftrightarrow{FS;\, \Omega_0 = \frac{2\pi}{T}} X[k] = \frac{1}{T}\int_0^T x(t) e^{-jk\Omega_0 t}\, dt$

Then $x(t) \xleftrightarrow{FT} X(\Omega) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\, \delta(\Omega - k\Omega_0)$

Example:



$s(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT)$

① Find FS

$S[k] = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t) e^{-jk\frac{2\pi}{T}t}\, dt$

$= \frac{1}{T} \cdot 1 = \frac{1}{T}$

② $S(\Omega) = 2\pi \sum_{k=-\infty}^{\infty} \frac{1}{T}\, \delta(\Omega - k\frac{2\pi}{T})$