Fourier Representations

Joseph Fourier 1768–1830

Sums of sinusoids

Sinusoids are fundamental

- electromagnetic spectrum
- oscillatory motion

Frequency is pervasive

sound

Sinusoids are eigenfunctions of LTI Systems

$$e^{j\omega n} \longrightarrow \boxed{H} \longrightarrow \alpha e^{j\omega n}$$

Sum of sinusoids can rep. any signal

Four Fourier Representations

Time

		continuous (t)	discrete [n]		
Time	nonperiodic	Fourier Transform $x(t) \xleftrightarrow{FT} X(\Omega)$	Discrete-time Fourier Transform $x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$	continuous (Ω) or (ω)	Frequency
T or N	periodic	Fourier Series $x(t) \xleftrightarrow{FS;\ \Omega_0} X[k]$ $\Omega_0 = \frac{2\pi}{T}$ rads|sec	Discrete-time Fourier Series $x[n] \xleftrightarrow{DTFS;\ \omega_0} X[k]$ $\omega_0 = \frac{2\pi}{N}$ rads	Discrete [k]	
		nonperiodic	periodic 2π or N		

Frequency

Fourier Transform

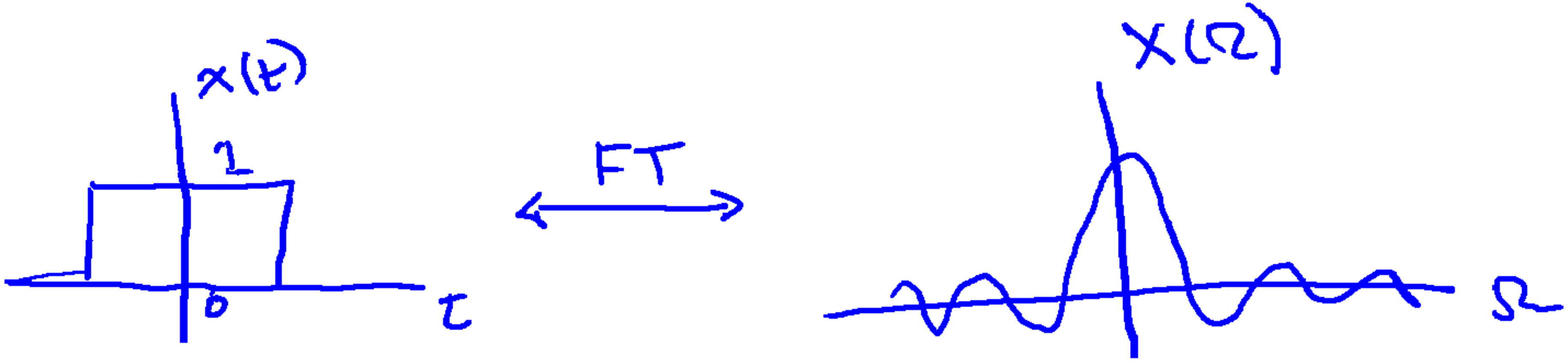
$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega$$

$$\approx \sum_{\Omega} \underbrace{\frac{X(\Omega)}{2\pi} d\Omega}_{\text{weight}} \cdot e^{j\Omega t}$$

cont in t, Ω

$-\infty$ to ∞



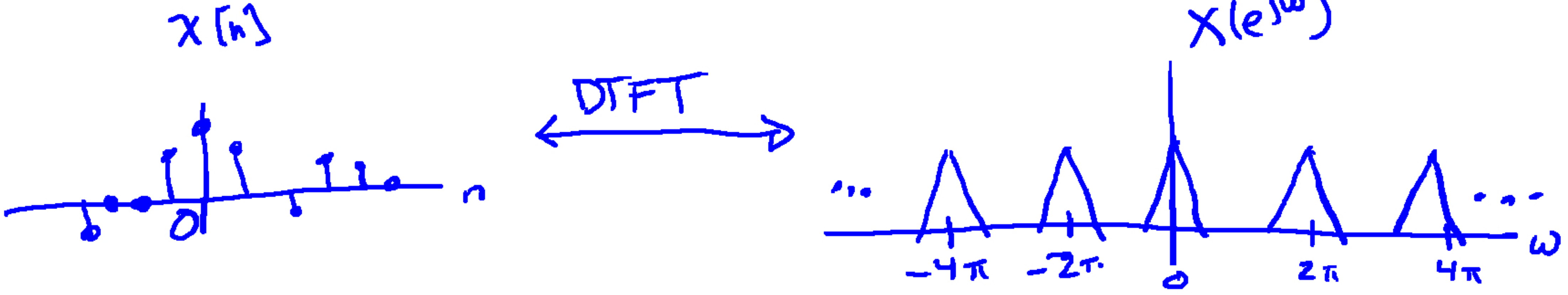
Discrete-Time Fourier Transform

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$x[n] \quad -\infty < n < \infty \qquad \approx \sum \left(\frac{X(e^{j\omega})}{2\pi}d\omega\right)\cdot e^{j\omega n}$$

$$-\pi < \omega < \pi$$

$$X(e^{j(\omega+\ell 2\pi)}) = X(e^{j\omega}) \quad \therefore \; X(e^{j\omega}) \text{ has period } 2\pi$$



Fourier Series: $x(t)$ period T

$$X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\Omega_0 t}dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\Omega_0 t}$$

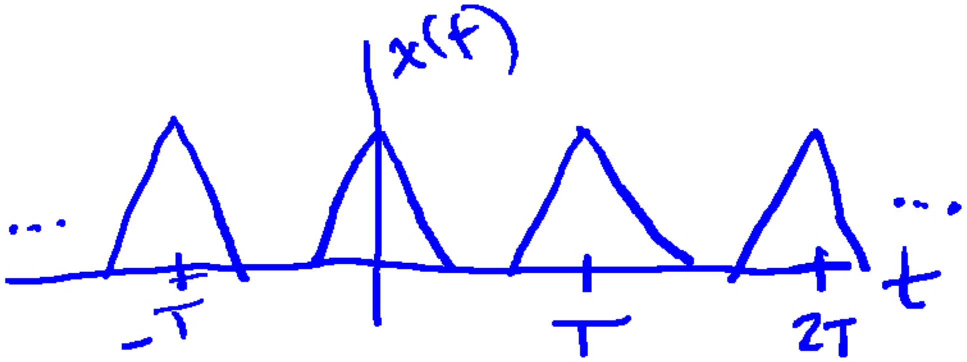
$$\Omega_0 = \frac{2\pi}{T}$$

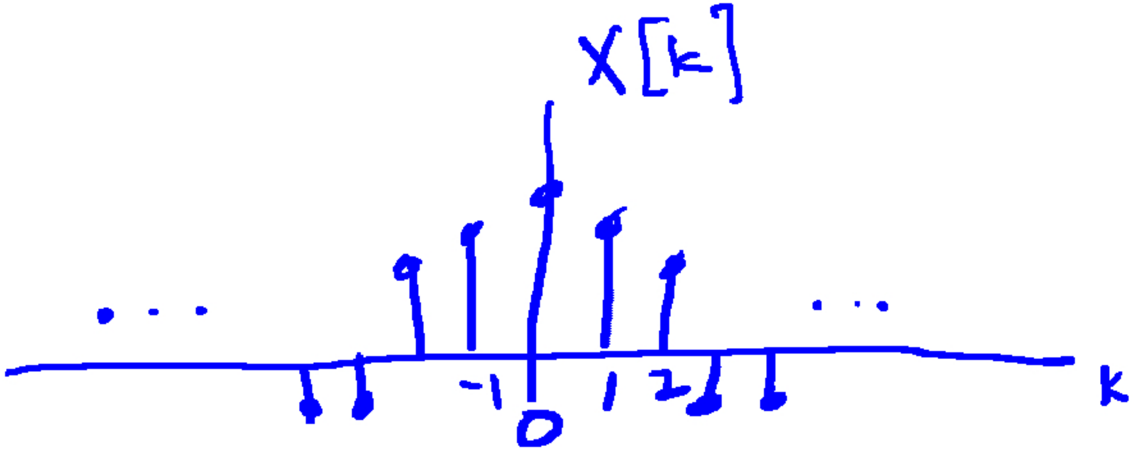
$X[k]$ weight for $e^{jk\Omega_0 t}$ freq $k\Omega_0$

$x(t)$ periodic T

$x(t)$ $0 < t \leq T$

$k = \ldots -2, -1, 0, 1, 2 \ldots$





Discrete-Time Fourier Series: $x[n]$ period N

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n} \qquad x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\omega_0 n}$$

$$\omega_0 = \frac{2\pi}{N}$$

$X[k]$ weight applied to $e^{jk\omega_0 n}$

freq $k\omega_0$ rads

$x[n]$ $n=0,1,2,\dots N-1$

$X[k]$ $k=0,1,2,\dots N-1$

Discrete Fourier Transform

DFT