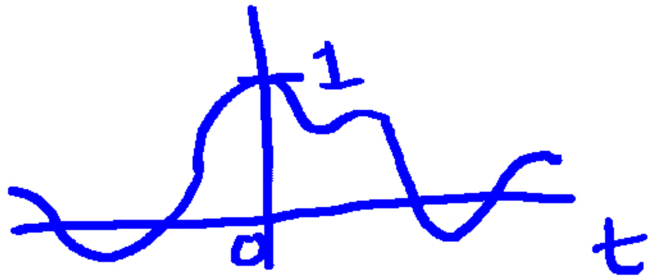
LTI Systems

system: maps an input signal to an output signal

$$x(t) \text{ or } x[n] \longrightarrow \boxed{H} \longrightarrow y(t) \text{ or } y[n]$$

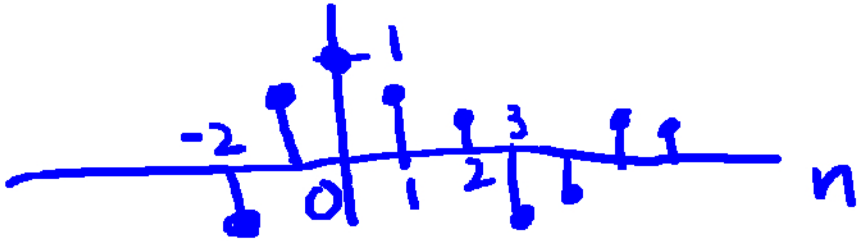
Continuous-time Signal

$x(t)$



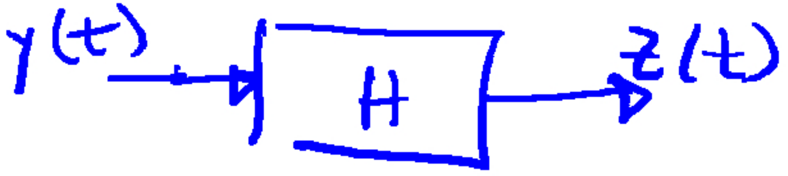
Discrete-time signal

$x[n]$



System 1) Model a physical phenomenon
2) implement desired characteristic

$y(t) = x(t) + bx(t-\tau)$



$|b| < 1$

$$
\begin{aligned}
z(t) = {} & y(t) \\
& - b\,y(t-\tau) \\
& + b^2 y(t-2\tau) \\
& - b^3 y(t-3\tau) \\
& + b^4 y(t-4\tau) \\
& \vdots
\end{aligned}
$$

$$
\begin{aligned}
& x(t) + \cancel{bx(t-\tau)} \\
& \cancel{-bx(t-\tau)} \cancel{-b^2 x(t-2\tau)} \\
& \cancel{+b^2 x(t-2\tau)} \cancel{+b^3 x(t-3\tau)} \\
& \cancel{-b^3 x(t-3\tau)} - b^4 x(t-4\tau) \\
& \vdots
\end{aligned}
$$

Linear System

Superposition holds: sum of inputs $\Rightarrow$ sum of outputs

$$x_1[n] \rightarrow \boxed{H} \rightarrow y_1[n]$$

$$x_2[n] \rightarrow \boxed{H} \rightarrow y_2[n]$$

$$a x_1[n] + b x_2[n] \rightarrow \boxed{H} \rightarrow a y_1[n] + b y_2[n]$$

Time Invariance

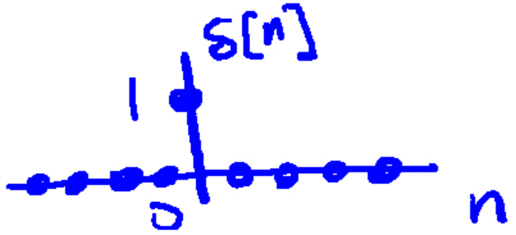
system responds the same now as it does later

$$x[n] \rightarrow \boxed{H} \rightarrow y[n] \quad \Rightarrow \quad x[n-n_0] \rightarrow \boxed{H} \rightarrow y[n-n_0]$$

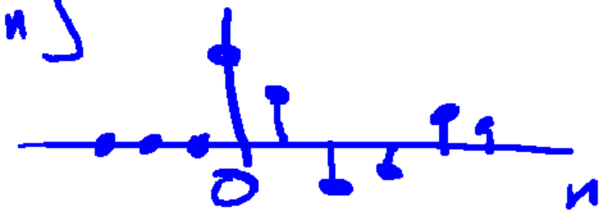
LTI satisfies both

I/O for LTI Systems

impulse response tells all



$$\delta[n] \longrightarrow \boxed{H} \longrightarrow h[n]$$



$$x[n] \text{ input} \Rightarrow y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$
$$= \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$
$$= h[n] * x[n]$$

causal

$h[n] < 0 \quad n<0$

stable

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Difference Equations

Important class of LTI systems

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

1) model physical systems
2) design filters
3) implement (compute) filters

$y[n] - \frac{1}{2} y[n-1] = x[n] \longrightarrow y[n] = \frac{1}{2} y[n-1] + x[n]$

imp resp $\quad x[n] = \delta[n]$

$y[-1] = 0$

$y[0] = \frac{1}{2} \cdot 0 + 1 = 1$

$y[1] = \frac{1}{2} \cdot 1 + 0 = \frac{1}{2}$

$y[2] = \frac{1}{2} \cdot \frac{1}{2} + 0 = \frac{1}{4}$

$\vdots$