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CALIBRATION OF A KELVIN VARLEY VOLTAGE DIVIDER

by

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CALIBRATION OF A KELVIN-VARLEY STANDARD DIVIDER

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Of
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SUMMARY

The linearity deviation of a multiple decade Kelvin-Varley voltage divider can be calibrated by comparing it decade by decade with a ten-step standard divider. The standard divider can be calibrated by precisely measuring its resistors and calculating its linearity. The basis for both of these techniques is derived mathematically. The procedure for measuring each decade and the method of combining the contributions of all the decades to find the linearity deviation of a given setting will be presented. The expected accuracy of the measurements will be analyzed. Contributions of resistor accuracy, resistor and contact stability, lead resistance, temperature variations, analytical simplifications, and power dissipation will be discussed.

INTRODUCTION

A multiple decade Kelvin-Varley voltage divider is often used as a standard for calibrating other voltage dividers. It is therefore necessary that we be able to calibrate this standard.

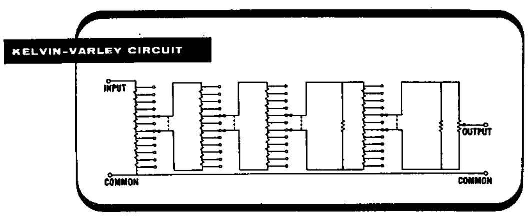


Figure 1 Kelvin-Varley Circuit

Figure 1 shows the circuit diagram for a Kelvin-Varley voltage divider. The first decade consists of eleven resistors. At any given setting two of these resistors are shunted by an interpolating divider with an input resistance equal to the value of the two resistors that it shunts. Thus, the input to the first decade looks like ten equal resistors in series so that the interpolating divider will work over a range of one tenth of the input. The second, third, etc. decades can also be Kelvin-Varley circuits. The last decade, however, consists of ten resistances in series with eleven taps brought out. It can be set to values of one, two, three, etc. up to ten. It is only through the use of this tenth setting that the divider can be set to full scale. The full scale dial reading will be 99···9TEN. This feature also means that there will be two ways of reaching any setting ending in zero. (Except zero and full scale)

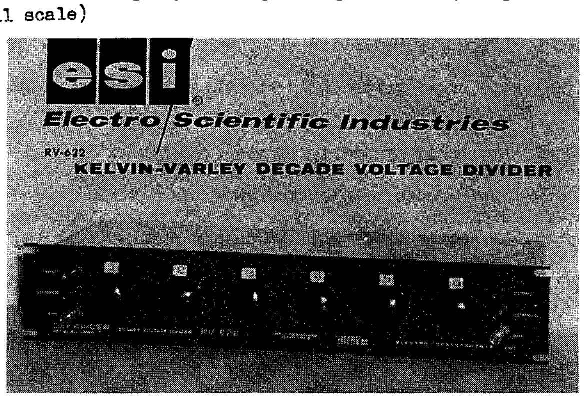


Figure 2 ESI Model RV 622 Dekavider® Decade Voltage Divider

At first it might seem desirable to have a calibration made at each possible setting. The divider shown in Figure 2, however, has one million one hundred thousand possible settings. If measurements could be made at the rate of one a second it would take about two months to complete the calibrations and the results would fill a volume of books comparable in size to an encyclopedia. There has to be another way. It would be preferable if each decade could be calibrated individually and the resulting deviations combined to give the deviation at any desired setting. We are going to tell one way that this can be accomplished.

CALIBRATING A TWO-STEP DIVIDER

We will start with a two decade divider. We are going to measure the linearity of the interpolating divider first; next the linearity of the first divider, and then we will show how to combine the results to give the linearity at any desired setting. Linearity measurements are made in terms of linearity deviation Γ. Linearity deviation is the difference between the ratio of output voltage to input voltage and the setting. The setting is the nominal ratio of output voltage to input voltage.

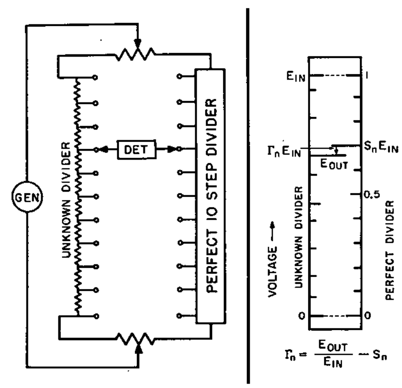


Figure 3 Measuring the Interpolating Divider

Figure 3 shows the circuit connections for measuring the linearity deviation of a ten-step unknown interpolating divider. We assume that a perfect ten-step divider is available for making the measurements. The potentiometers at top and bottom are provided for adjusting the zero and full scale settings of both dividers to agree. The result is shown in the voltage diagram at the right where the zero and full scale settings of both dividers are equal. The generator voltage is adjusted so that the high impedance detector will read linearity deviation directly in convenient scale divisions. The detector is moved from point to point and the linearity deviations are recorded.

$$\Gamma = \frac{\text{ACTUAL OUTPUT VOLTAGE}}{\text{INPUT VOLTAGE}} - \text{SETTING}$$

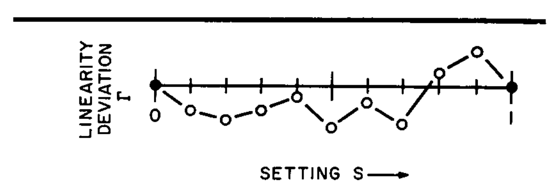


Figure 4 Linearity Deviation of the Interpolating Divider

In Figure 4 we have a graph showing typical results. Notice here that the "zero" and "one" settings show no linearity deviation. The end correcting potentiometers were adjusted to assure this.

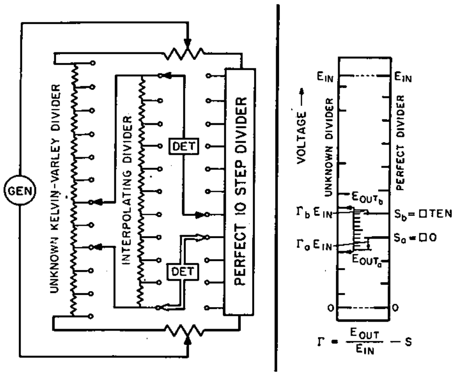


Figure 5 Measuring the Interpolated Divider

In Figure 5 the first decade is being calibrated. Here twenty measurements are necessary because each setting except zero and one can be reached in two different ways. The measurements are made by first setting the dividers to zero and adjusting the bottom potentiometer for agreement with the perfect ten-step divider then moving the settings to full scale and adjusting the top potentiometer. The voltage diagram at the right shows how the perfect ten-step divider is used for finding the linearity deviations at both ends of the interpolating divider for each setting of the first divider.

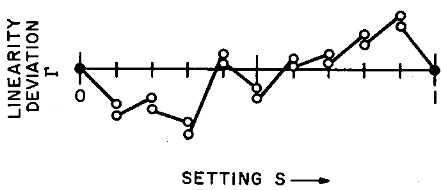
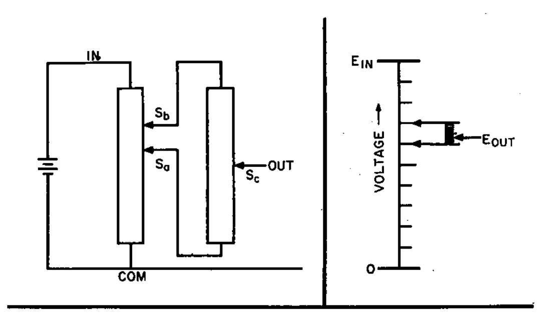


Figure 6 Contribution of the Interpolated Divider to Linearity Deviation

Figure 6 is a plot of the results of these measurements. Here a straight line is drawn between the readings obtained with the interpolating divider at zero and at full scale. This is done because a linear divider connected between these two voltages and set at a value somewhere between would have the linearity deviation shown by this straight line at the desired setting. We have now completed the linearity measurements of each decade separately and we need to know how these can be combined to find the linearity deviation at a given setting.



$$\Gamma = \frac{E_{OUT}}{E_{IN}} - S_a - \frac{S_c}{10}$$

Figure 7 Two Decade Voltage Divider

Figure 7 shows a two decade setting S where:

$$S = \frac{\text{Nominal } E_{Out}}{E_{In}} = S_a + \frac{S_c}{10}$$

The linearity deviation Γ is derived in relation to the setting. The voltage diagram shows how any desired output voltage may be obtained by moving the two decades.

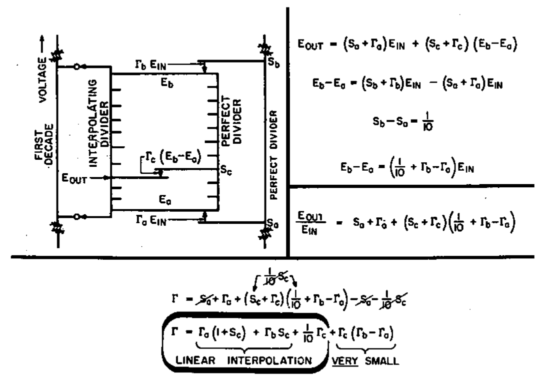


Figure 8 Combining Linearity Deviation Measurements

Part of this voltage diagram is enlarged in Figure 8 to show how the linearity deviation at any setting can be expressed in terms of the measurements previously made on each decade. Here, we restate the voltages in terms of settings and measured linearity deviations. Next we find the ratio of output voltage to input voltage and subtract from it the settings. The setting S is the nominal ratio of output voltage to input voltage. This difference is the linearity deviation for the divider. The linearity deviation is found to be a linear interpolation between the deviations found at the ends of the interpolating divider--plus 1/10 of the deviation of the interpolating divider. There is also a very small term which is the product of two linearity deviations. Both of these deviations will be in the order of ten parts per million typically so that their product will be only a few parts in 10^{10} and can therefore be ignored.

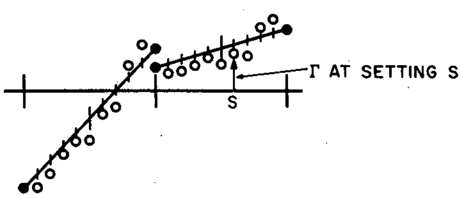


Figure 9 Linearity Deviation of a Two Decade Divider

Figure 9 shows graphically how this is done. Two steps of the first divider are shown. The linearity deviations of the end measurements are connected by a straight line. One tenth of the linearity deviation measurements of the interpolating divider are then added to the values along the straight line. These points then show the linearity deviation of the divider for each possible setting. There are 110 possible settings of this two decade divider. We can find the linearity deviation for any one of these settings by having made only 27 measurements. If we measure a three decade divider, the procedure is exactly the same except that both of the last two stages are the interpolating divider and they are set to 00 and to 9TEN at each setting of the first decade. The same procedure can be continued for as many decades as are desired.

CALIBRATING A "PERFECT" DIVIDER

How do we physically make these measurements? First we need the "perfect" ten-step divider. We calibrate a ten-step divider by resistance comparison. This gets us within ±0.1 ppm of knowing its linearity. Next we use this divider to set an adjustable ten-step divider within ± 0.2 ppm of "perfect"--we settle for this.

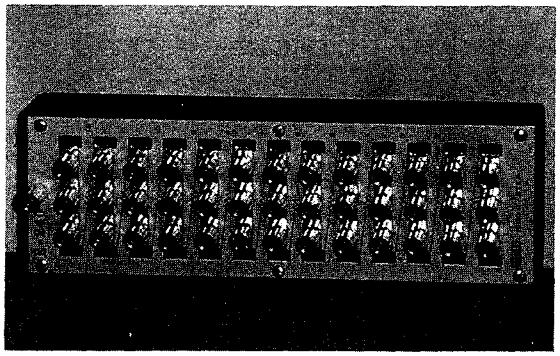


Figure 10 ESI Model SR 1010 Decade Resistance Standard

Figure 10 shows the ESI Model SR-1010 DECADE RESISTANCE STANDARD which is calibrated as a voltage divider.

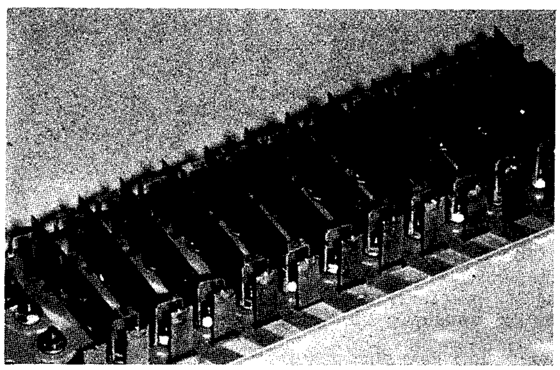


Figure 11 Resistor Configuration

This unit consists of twelve resistors permanently connected in series as shown in Figure 11.

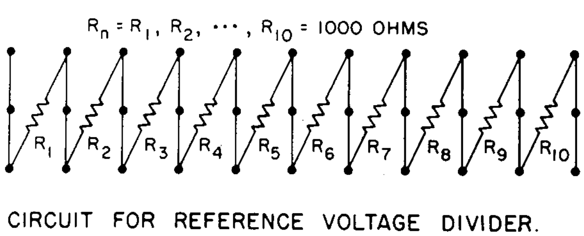


Figure 12 ESI Model SR 1010 Decade Resistance Standard Circuit Diagram

Extra terminals are provided as shown in Figure 12 so that four-terminal measurements of the individual resistors can be made.

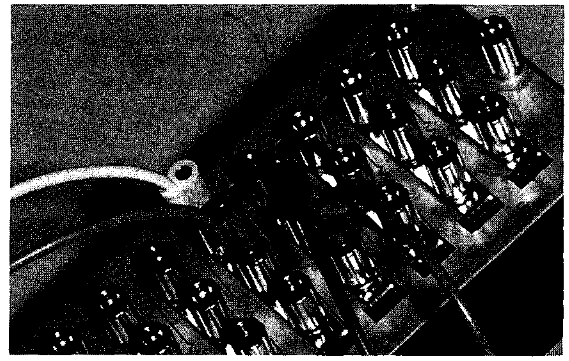


Figure 13 Four-Terminal Connections

Figure 13 shows the way to make these four-terminal connections so the resistance between center terminals can be measured accurately. These center terminals are the ones used for voltage divider taps.

$$\Gamma = \frac{E_{OUT}}{E_{IN}} - S \qquad \Gamma = \frac{\sum_{n=1}^{10S} R_n}{\sum_{n=1}^{10} R_n} - S$$

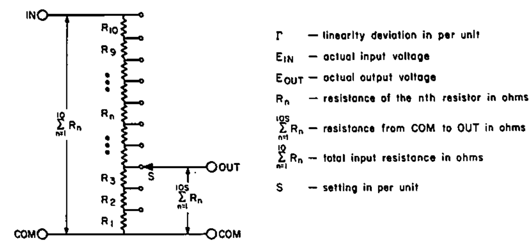


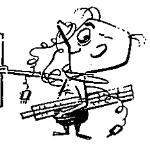
Figure 14 Linearity Deviation as a Function of Resistance

The linearity deviation equation is rewritten in terms of resistance values as shown in Figure 14.

Figure 15 ESI Model 242 Resistance Measuring System

Absolute resistance measurements of the accuracy needed are considered impractical--but resistance comparisons to better than one ppm are easy with the ESI Model 242 Resistance bridge shown in Figure 15.

$$R_n \equiv R_s \, (1+\Delta_n)$$



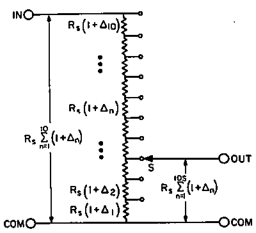
R_n — resistance of the nth resistor in ohms

R_s — resistance of the standard resistor in ohms

Δ_n — per unit deviation of the unknown from the standard

Figure 16 Defining Resistance Deviation

All of the divider resistors are compared to a single standard resistor. The resulting values of the deviation from the standard (defined in Figure 16) are recorded.



$$\Gamma = \frac{\cancel{R_s}\sum_{n=1}^{10S}(1+\Delta_n)}{\cancel{R_s}\sum_{n=1}^{10}(1+\Delta_n)} - S = \frac{\sum_{n=1}^{10S}\Delta_n - S\sum_{n=1}^{10}\Delta_n}{10 + \sum_{n=1}^{10}\Delta_n}$$

$$\Delta_{AV} = \frac{1}{10}\sum_{n=1}^{10}\Delta_n$$

$$\Gamma = \frac{\sum_{n=1}^{10S}(\Delta_n - \Delta_{AV})}{10\,(1+\Delta_{AV})}$$

Figure 17 Linearity Deviation in Terms of Resistance Deviation

In Figure 17 the equation for the linearity deviation is converted into an equation in terms of the resistance deviations. Note that the resistance of the standard resistor conveniently cancels out of the equation for linearity deviation. The value of Δ_{AV} is so much smaller than one that it can be removed from the denominator to give the formula actually used for calculation.

$$\Gamma \approx \overbrace{\frac{1}{10}\underbrace{\sum_{n=1}^{10S}\left(\underbrace{\Delta_n - \Delta_{AV}}_{\text{STEP 1}}\right)}_{\text{STEP 2}}}^{\text{STEP 3}}$$

Step 1. Calculate the average deviation.

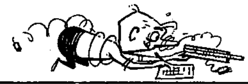
$$\Delta_{AV} \equiv \frac{1}{10}\sum_{n=1}^{10}\Delta_n$$

Step 2. Subtract the average from each deviation to get the difference from the average.

Step 3. Sum these differences from the average and divide by 10 to get the linearity deviations at each tap.

Figure 18 Calculation of Linearity Deviation from Resistance Deviation Measurements

Figure 18 shows the process for actually calculating the linearity deviation at each step on our ten-step divider. First the resistance deviation values are summed to find the average. Then the average is subtracted from the individual readings and the linearity deviation is found by taking one tenth of the sum of these deviations from the average.



RESISTOR NUMBER	MEASURED DEVIATION PPM	DIFFERENCE FROM THE AVERAGE PPM	LINEARITY DEVIATION Γ PPM
10S	Δ_n	$\Delta_n - \Delta_{AV}$	$\frac{1}{10}\sum_{n=1}^{10S}(\Delta_n - \Delta_{AV})$
1	1.4	+0.36	+0.04
2	+0.8	−0.24	+0.01
3	+5.0	+3.96	+0.41
4	+2.2	+1.16	+0.52
5	+2.4	+1.36	+0.66
6	−0.4	−1.44	+0.52
7	+0.4	−0.64	+0.45
8	−2.4	−3.44	+0.11
9	+1.4	+0.36	+0.14
10	−0.4	−1.44	0
TOTAL	$\sum_{n=1}^{10}\Delta_n = +10.4$	0	

LINEARITY CALCULATED FROM RESISTANCE MEASUREMENTS OF AN ESI MODEL SR-1010 DECADE RESISTANCE STANDARD

Figure 19 Sample Calculations of Linearity Deviation

Figure 19 is a table of measurements and calculations which were made for calibrating a divider.

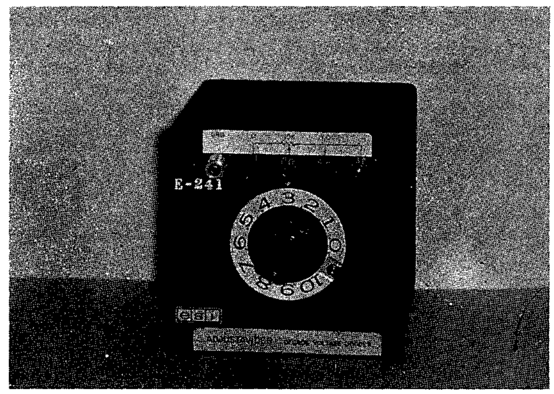


Figure 20 Adjustable Decade Voltage Divider

In Figure 20 we see the adjustable voltage divider which can be set to within 2/10 of a part per million by using our previously calibrated divider and a calibrated meter for measuring linearity deviation.

CALIBRATING A MULTIPLE DECADE DIVIDER

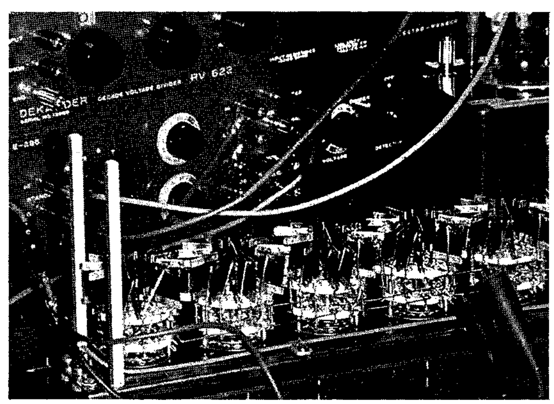


Figure 21 Calibrating a Voltage Divider Decade

Figure 21 is a production set-up for measuring the deviation of one decade of a six decade divider. In the foreground is the divider being tested. In the background we see the adjustable divider which is assumed to be perfect. The panel directly behind contains the two end correcting potentiometers for setting the ends equal, and a voltage adjusting control for setting the sensitivity so that the linearity deviation can be read directly from a meter.

RANGE OF DECADE	END DEVIATION	FIRST DECADE	SECOND DECADE	THIRD DECADE	FOURTH DECADE	FIFTH DECADE	SIXTH DECADE
0 TO 0X*	+0.3	0.0 TO +0.6	0.00 TO −0.10	0.000 TO −0.050	0.000,0 TO −0.001,0	0.000,00 TO +0.000,20	0.000,000
1 TO 1X	—	−0.3 TO 0.0	+0.05 TO −0.14	+0.003 TO −0.050	−0.007,0 TO −0.006,0	+0.000,50 TO +0.000,90	−0.000,140
2 TO 2X	—	−0.7 TO +0.4	−0.03 TO −0.10	+0.030 TO −0.020	+0.003,0 TO +0.005,0	+0.000,70 TO +0.000,90	−0.000,011
3 TO 3X	—	−0.5 TO +0.5	−0.07 TO −0.11	+0.060 TO +0.010	+0.004,0 TO +0.008,0	+0.000,60 TO +0.000,60	−0.000,040
4 TO 4X	—	+0.1 TO +1.1	+0.06 TO +0.05	+0.080 TO +0.030	+0.013,0 TO +0.012,0	+0.000,10 TO +0.000,00	−0.000,020
5 TO 5X	—	+0.5 TO +1.4	+0.21 TO +0.14	+0.100 TO +0.035	+0.019,0 TO +0.019,0	+0.000,30 TO +0.000,90	−0.000,060
6 TO 6X	—	+0.5 TO +1.0	+0.21 TO +0.09	+0.090 TO +0.010	+0.021,0 TO +0.023,0	−0.000,20 TO +0.000,40	−0.000,050
7 TO 7X	—	+0.4 TO +0.8	+0.20 TO +0.02	+0.080 TO 0.000	+0.020,0 TO +0.020,0	+0.000,70 TO +0.000,80	−0.000,100
8 TO 8X	—	0.0 TO +0.3	+0.14 TO +0.01	+0.060 TO −0.010	+0.023,0 TO +0.021,0	+0.000,10 TO +0.000,20	−0.000,080
9 TO 9X	−0.3	−0.4 TO 0.0	+0.14 TO 0.00	+0.050 TO 0.000	+0.015,0 TO 0.000,0	−0.000,03 TO 0.000,00	−0.000,120 0.000,000

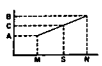
*X — means that the decades to the right of the one being measured are set to maximum. For example 0.4X for the second decade indicates a dial setting of 0.049,99TEN.

LINEARITY DEVIATION MEASUREMENTS OF AN ESI MODEL RV-622 DEKAVIDER® DECADE VOLTAGE DIVIDER

Figure 22 Measurements on a Typical Six Decade Voltage Divider

Figure 22 shows the 101 linearity deviation measurements on a specific divider. These measurements have already been multiplied by the appropriate powers of ten so that they represent the contribution to the linearity deviation at any given setting. The values shown are in ppm of input voltage.

$$C = A + \frac{(B-A)(S-M)}{(N-M)}$$



D — decade for which calculation is being made
S — setting of decade D and decades following
M — setting of decade D with following decades at zero
N — setting of decade D with following decades at 99••• TEN
A — linearity deviation at M
B — linearity deviation at N
C — linearity deviation at S (calculated)

D	END DEVIATION	FIRST DECADE	SECOND DECADE	THIRD DECADE	FOURTH DECADE	FIFTH DECADE	SIXTH DECADE	TOTAL
M	0.000,000	0.500,000	0.040,000	0.003,000	0.000,200	0.000,010	—	
N	0.999,99TEN	0.599,99TEN	0.049,99TEN	0.003,99TEN	0.000,29TEN	0.000,01 TEN	—	
S	0.543,212	0.543,212	0.043,212	0.003,212	0.000,212	0.000,012	0.000,002	
Appm	+0.3	+0.5	+0.06	+0.06	+0.003	+0.000,5	—	
Bppm	−0.3	+1.4	+0.05	+0.01	+0.005	+0.000,9	—	
Cppm	−0.025,930	+0.888,908	+0.056,788	+0.049,400	+0.003,240	+0.000,580	−0.000,011	+0.972,975

CALCULATING THE LINEARITY DEVIATION AT A SAMPLE SETTING OF 0.543,212

Figure 23 Interpolation Calculation From a Set of Typical Linearity Deviation Measurements

Figure 23 shows the interpolation equation and the calculations necessary for finding the linearity deviation at a sample setting from the measurements on the individual decades.

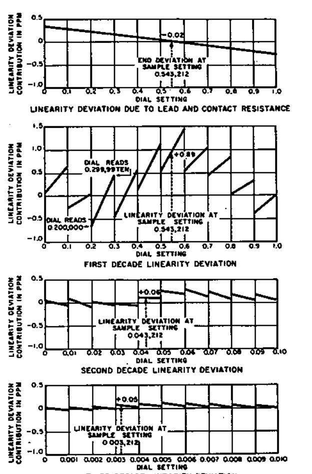


Figure 24 Graphical Representation of Linearity Distribution Contributions

Figure 24 shows a graphical representation of the linearity deviation of the first three decades of a six decade divider. A sample setting is given to show how to find and combine the contributions of each decade. Figure 24 also shows the end linearity deviation due to lead and contact resistance at the input. These end linearity deviation values are found by first setting the divider to zero and measuring the voltage between the common and output taps. Then the divider is set to full scale and a measurement is made of the voltage difference between the input and output taps. This end linearity contribution is also linearly interpolated to find its contribution at any setting. External circuit lead and contact voltage drops must also be added for accurate measurement. End correcting potentiometers can be used with the divider to eliminate all lead and contact contributions from the measurements.

Now we have accomplished what we set out to do. We have found a technique for calibrating a voltage divider. We have also found how to use the calibration to correct any setting of the divider. As usual there are still a few weeds in the garden to be pulled. We had to drop a few small terms to simplify the calculations that we have used but we were careful to see that they are well beyond the measurement accuracy with which we are dealing. There are a few other problems which are not quite so easily dismissed, however. The first of these is resistor stability with time. This can only be shown by repeated measurements over a prolonged period. Typically we have found stabilities of a few ppm per year. Units normally become more stable with age. The next worry is temperature stability. By careful wire selection, 100% inspection of the temperature coefficient of individual resistors and such precautions as winding all of the resistors in each of the first few decades from the same spool of wire, the effects of ambient temperature can be minimized. Linearity temperature coefficients of less than two ppm per degree centigrade are typical.

EFFECT OF FULL POWER

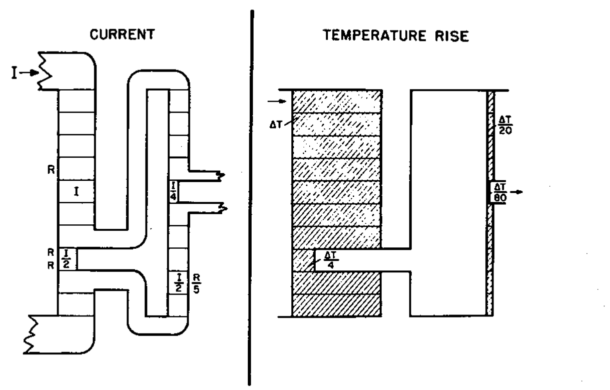


Figure 25 Temperature Distribution in a Kelvin-Varley Voltage Divider

Maximum power, however, gives us a different temperature problem. Here, the individual resistors are not heated equally. Figure 25 shows the current distribution through the first two decades of a Kelvin-Varley voltage divider. The resistance values are also shown. The chart on the right shows the temperature distribution among the resistors. By taking the temperature rise of the individual resistors of the first decade as Δt we find that the two resistors which are bridged by the interpolating divider only reach a temperature of $\Delta t/4$. The individual resistors of the second decade reach a value of $\Delta t/20$ except for the two that are bridged which only reach a temperature difference of $\Delta t/80$. The temperature rise on the third and later decks is so small it can be ignored. But on the first decade the difference in temperature rise between the bridged resistors and the rest of the resistors is significant. Therefore, the resistor temperature coefficients must be low in addition to being matched.

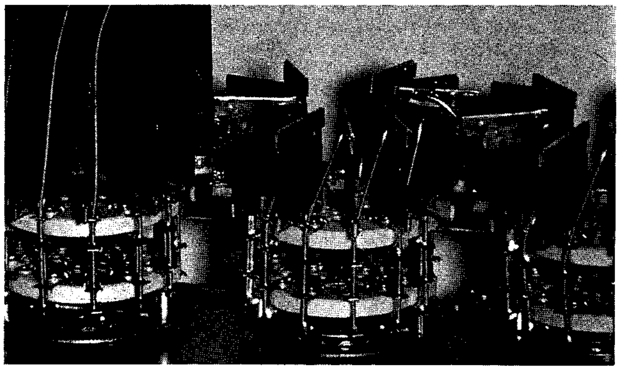


Figure 26 Voltage Divider Resistance Decades

Figure 26 shows that the resistors in the first decade have been made much larger than those in the following decades to reduce the effects of temperature rise with applied power. The temperature rise on these resistors for a five watt divider input is about 10° C. but remember that two of these resistors have a temperature rise of only about 2½° C. There are many variables involved in the effects of power on linearity. We take all of the precautions which we can and then see statistically what happens.

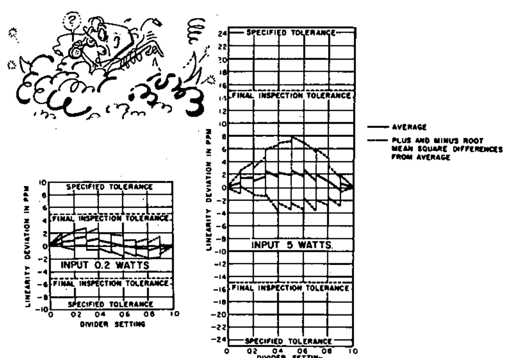


Figure 27 What Happens to Linearity Deviation When Full Power is Applied

Since linearity deviations of the first decade are the only ones materially affected by power changes we can make measurements on this first decade to find what actually happens. Here we have average linearity deviations of ten dividers for each setting, and the RMS differences (one sample deviation) from this average. In Figure 27 these values have been plotted for the negligible temperature change which results from 2/10 watt input and for the results when five watts are supplied to the input terminals.

EFFECT OF SWITCHING LATER DECADES

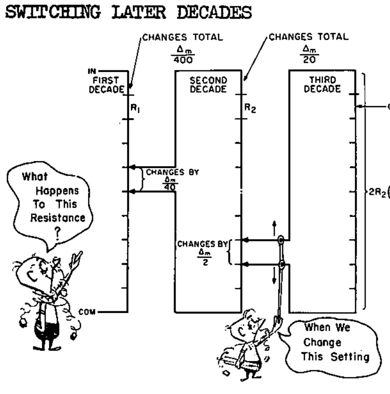


Figure 28 Effect of Resistance Variations Due to Switching Later Decades

Figure 28 illustrates a different problem. As the second decade setting is changed its interpolating divider is connected across slightly different resistance values. As a result the resistance presented to the first decade will change slightly. This will result in changing the voltage available at the tap points on the first decade. The effect on the first decade is the same as though the resistance of the third decade had been changed by an amount equal to the variations in the resistances of the second decade. In Figure 28 we approached the problem from this view point. A variation of Δ_m in the resistance of the third decade will result in half this variation in the resistance seen by the second decade. It will make a change of $\Delta_m/20$ in the resistance of the second decade. This will make a change of $\Delta_m/40$ in the resistance seen by the first decade and will change the total resistance of the first decade by $\Delta_m/400$.

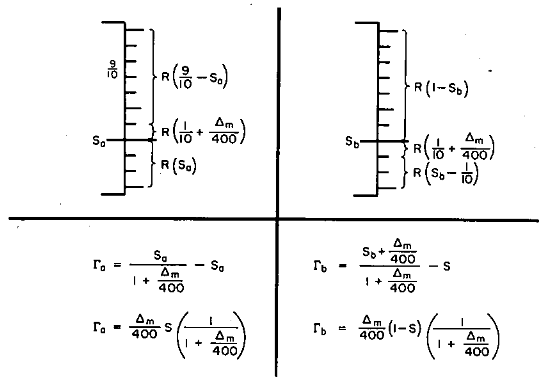


Figure 29 Effect of One Resistor in a Ten-Step Voltage Divider

Figure 29 shows the calculation for the effects of this resistance on the accuracy of the settings of the first decade. In the expressions for the linearity deviations the term $(1+\Delta_m/400)$ can be set equal to one without any appreciable effect in the accuracy of the result.

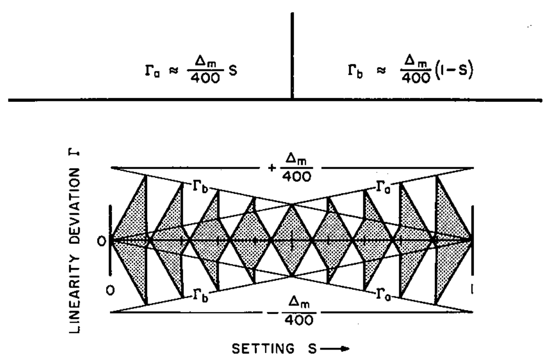


Figure 30 How the Linearity Changes When the Following Decade is Switched

In Figure 30 we have a graphical representation of the amount of linearity deviation experienced by the first decade because of a Δ_m variation in the second decade resistors. We see that the linearity deviation in the first decade is always less than 1/400 of the resistance deviation of the second decade. This would be 1/160,000 of the resistance variation of the third decade and so on.

EFFECT OF CONTACT RESISTANCE VARIATION

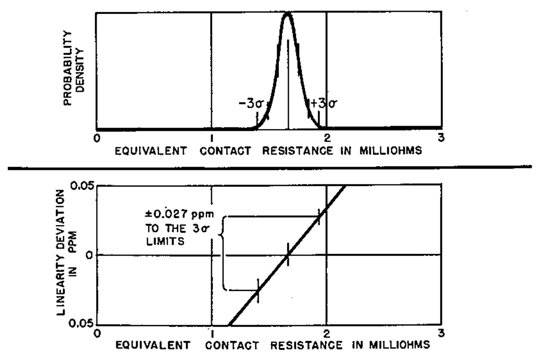


Figure 31 Contribution of Contact Resistance Variations To Linearity Deviation For a 10 Kilohm Divider

Another possible source of trouble is contact resistance variation. To minimize this problem all switch contacts of the Model RV 622 are doubled. Measurements of large numbers of contacts and a statistical analysis of the results have revealed the results shown in Figure 31. A linearity deviation of less than ±0.025 ppm can be expected from contact resistance variations on a 10 kilohm ESI Model RV 622 Six Decade Voltage divider.

Now the voltage divider is calibrated and we know what precautions are necessary to use it for part-per-million measurements.