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Application of the AC Josephson-Effect for Precise Measurement

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SUMMARY It is the purpose of this paper to review the generation of quantized voltage steps in Josephson-junctions, and also the recent practical application of these precise measurements. A 10-V Josephson-junction-array-voltage standard system has been established with a Josephson-junction-array, a phase-locked millimeter wave, and a precise null-detection system. Based on these technologies, the AC Josephson effect has been applied to other precise measurements such as DC error voltage of a multi-integrating analog-to-digital converter and for a pulse-width-modulation type precise voltage calibrator.

key words: *AC Josephson-effect, Josephson-junction-array, quantized voltage, phase-locked millimeter wave, null-detection, precise measurement*

1. Introduction

In 1962, Josephson [1] predicted that certain effects would occur when electron pairs (Cooper pair) tunnel from one superconductor through a thin dielectric barrier into another. At finite voltages, a quasi-particle current occurs, but there is also an AC super-current with a frequency f_J directly related to the DC voltage V appearing at the barrier $f_J = (2e/h)V$, where e is the electron charge and h is Plank's constant. This AC super-current can be frequency-modulated by an applied AC voltage of frequency f, then the current has Fourier components when $f_J = nf$, where n is the integer. This is called the AC Josephson effect. Constant voltage steps appearing at $V_n = n(h/2e)f$ were first observed by Shapiro [2] on $\mathrm{Al/Al_2O_3/S_n}$ junctions.

It is the purpose of this paper to describe the generation of quantized voltage steps in Josephson-junctions as a result of coherent-interference between the AC Josephson phenomena and an irradiated phase-locked millimeter wave, and its recent applications for precise measurements.

2. Quantized Voltage Steps in the Josephson-Junctions

All Cooper pairs in a superconductor occupy the same quantum state and are described by a single wave-function with a defined differential phase θ, where $\theta = \theta_1 - \theta_2$ is the difference between the phases θ_1 and θ_2 of the two superconductors at the junction.

$$I = I_c \sin\theta \tag{1}$$

$$d\theta/dt = (2e/\hbar)V \tag{2}$$

When a DC voltage V_0 is applied across the barrier, the phase difference θ continuously changes in time. Symbol $\hbar$ is defined as $h/2\pi$ The integration of (2) insertion into (1) yields:

$$I = I_c \sin[(2e/\hbar)V_0 t + \text{constant}] \tag{3}$$

The super current oscillates with a frequency of $\omega_0 = (2e/\hbar)V_0$. When a bias voltage is given as a sum of DC voltage V_0 and AC (applied millimeter waves) voltage $V_1 \cos\omega_1 t$, that is: $V = V_0 + V_1 \cos\omega_1 t$, substitute it and integrating (2) yields:

$$\theta = \int (2e/\hbar)V dt = (2e/\hbar)V_0 t + (2e/\hbar)(v_1/\omega_1)\sin\omega_1 t + \theta_0 \tag{4}$$

Substituting the result into (1), we obtain the following equation:

$$I = I_c \sin[(2e/\hbar)V_0 t + (2e/\hbar)(v_1/\omega_1)\sin\omega_1 t + \theta_0] \tag{5}$$

By Fourier-Bessel series extension, Eq. (5) becomes:

$$I = I_c \sum (-1)^n J_n[(2e/\hbar)(v_1/\omega_1)] \cdot \sin\{(\omega_0 - n\omega_1)t + \theta_0\} \tag{6}$$

where J_n expresses the Bessel function. When the super current oscillating frequency (ω_0) in the Josephson-junction is synchronized to the multiple applied millimeter frequency $(n\omega_1)$ like an "injection-locking," that is $\omega_0 = n\omega_1$, $\sin(\omega_0 - n\omega_1)t$ in Eq. (6) become to 0. So that (6) becomes:

$$I = I_c(-1)^n J_n[(2e/\hbar)(v_1/\omega_1)] \sin\theta_0 \quad (\omega_0 = n\omega_1) \tag{7}$$

Initial phase θ_0 is determined by the current through the junction. Kautz has investigated chaotic behavior in Josephson-junctions [3], and found out that the applied millimeter frequency f_1 $(= \omega_J/2\pi)$ that should be select at least three times the plasma frequency f_p which is determined by the critical current density J_c, length l, and width d at the junction, for stable locking.

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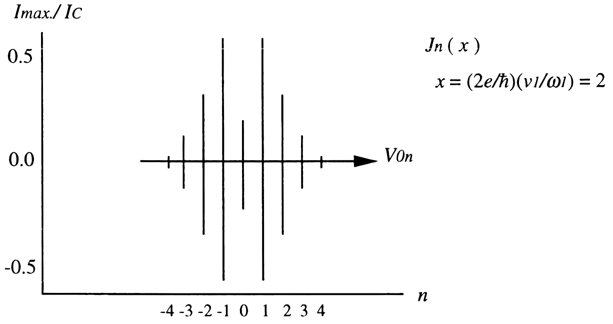


Fig. 1 Calculated quantized voltage steps in a single Josephson-junction by the AC Josephson effect.

Maximum current I_{max} in (7) is obtained in the case of $\sin\theta_0 = 1$,

$$I_{max} = I_c(-1)^n J_n[(2e/\hbar)(v_1/\omega_1)] \tag{8}$$

Therefore, DC super current expressed as Eq. (8) appears symmetrically at the quantized Josephson voltage. Quantized n-th Josephson voltage steps V_n (n: integer) are obtained on the synchronized condition, and are expressed as follows:

$$V_n = n(\hbar/2e)\omega_1 \tag{9a}$$

Equation (9a) is rewritten as follows by the relationship $\omega_1 = 2\pi f_1$:

$$V_n = n(h/2e)f_1 \tag{9b}$$

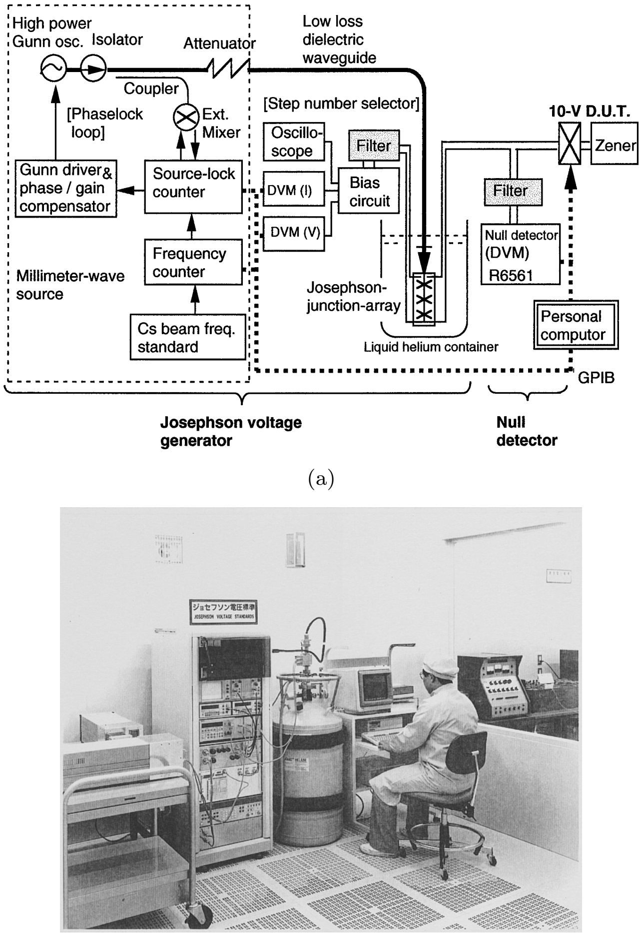
When a millimeter wave irradiates to single Josephson-junction, calculated Josephson voltage steps are shown as in Fig. 1. The current's width obeys the Bessel function. The voltage between steps is 194.376361 μV with an applied millimeter wave frequency of $f_1 = 94$ GHz.

3. Applications of the AC Josephson-Effect for Precise Measurements

10-V Josephson-junction-array voltage standard (JJAVS) and other two practical applications are introduced in this section. 10-V JJAVS is the most precise instrument being composed of key technologies: a series junction-array, a stabilized millimeter-wave, and a precise null-detection. Two other practical applications are based on these technologies developed for 10-V JJAVS.

3.1 Josephson-Junction-Array Voltage Standard

After the discovery and experimental verification of the AC Josephson effect, several experiments were performed to check the Josephson frequency voltage relationship. The value $2e/h$ is defined as the Josephson constant K_{J-90}, because the Comité Consultatif d'Électricité (CCE) suggested that all standard laboratories should adopt the value $2e/h = 483597.9$ GHz/V since the beginning of 1990 [4].



(a)

(b)

Fig. 2 (a) A basic configuration of the modern JJAVS which can calibrate directly for 10-V output of the Zener reference standard. (b) Picture of the modern JJAVS.

By using the Josephson constant, Eq. (9b) is expressed as follows:

$$V_n = nf_1/K_{J-90} \tag{10}$$

Based on this Eq. (10), three key technologies are required to establish the modern JJAVS. The first one is the Josephson-junction-array (JJA) which can extend the direct calibration range; second is the frequency stabilization sub-system for millimeter-wave source; and third is the precise null-detection sub-system. Figure 2(a) shows a basic configuration of the modern JJAVS, and Fig. 2(b) shows its picture composed of these elements [4] for calibrating the secondary standards, that is, 10-V output voltage of the Zener reference standards.

(1) Josephson-junction-array

As the quantized voltage of each junction is sum uped in series, JJA has been developed with the aim of extending the direct calibration range of JJAVS. In 1987, Lloyd et al. (NIST) [6] reported the first 10-V JJA

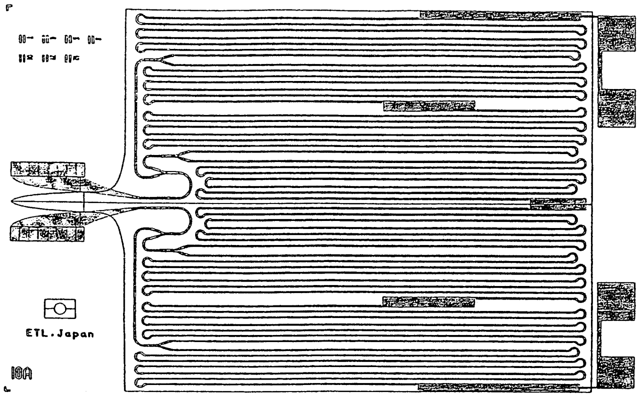


Fig. 3 Top view of the new JJA pattern of 25,944 of Nb/Al-Al_2O_3/Nb junctions with effective short-double fin-line antenna. Wafer size = 17.0 mm×10.5 mm.

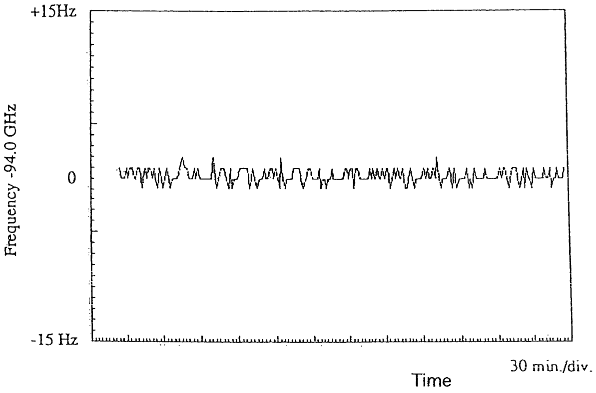


Fig. 4 Frequency stability of Gunn-diode oscillator at 94 GHz after improvement of the PLL circuit.

containing 14,184 Nb/Nb_2O_5/PbInAu junctions. This JJA made the direct calibration of 10-V Zener reference standards possible. Besides the Nb/Nb_2O_5/PbInAu technology, $Nb/Al_2O_3/Nb$ technology has the advantage of long-time storage and multiple temperature-cycling between liquid helium and room temperature. In addition to this, the shallow penetration depth and the dielectric constant of Al_2O_3 ($\varepsilon_r \sim 10$) allow constant voltage steps to be obtained with current widths larger than any other technology presently in use. This technology is in use at three developing laboratories, the NIST, the PTB [7], and the ETL [8]. By using this technology, the latest JJA of 25,944 of Nb/Al-Al_2O_3/Nb junctions was developed in collaboration with the ETL and the Advantest Laboratories as shown its pattern configuration in Fig. 3 [9]. The newest one is able to generate zero-crossing step voltage between −18.5 V and +18.5 V.

(2) Stabilization for millimeter-wave source

Equation (10) means that the accuracy of the quantized step voltages are mainly determined by the accuracy of the irradiated millimeter-wave frequency f_1, because n is an integer, and K_{J-90} is a constant. As a millimeter-wave source for modern JJAVS, a solid-state Gunn-diode oscillator has been widely used instead of a klystron oscillator. The frequency of a Gunn oscillator has to be stabilized in a phase-locked loop (PLL) in order to meet the specifications for use in the JJAVS system. Instead of a former electronic feedback circuit, a new circuit with phase and gain compensated feedback loop for the Gunn oscillator was proposed by the authors for stabilization of the Gunn oscillator in 1991 [10]. Then, frequency stability of the phase-locked Gunn oscillator at 94 GHz of 3×10^{-11} (peak to peak) was established as shown its result in Fig. 4 [10]. Since 1991, twenty-five of improved PLL circuits for JJAVS have been used world-widely to ensure that the national standards are compatible.

Aldo Godone and Dominico Andreone reported that the spectrum of short-term fluctuation of the voltage across the Josephson-junction-array is equal to the fractional frequency fluctuation of irradiating millimeter-wave. A value of $\sigma_V/V_n \approx 6\times10^{-12}$ is calculated for a fully phase-locked Gunn oscillator, while the value $\sigma_V/V_n \approx 3 \times 10^{-10}$ is the approximate estimate for the case of a frequency controlled Gunn oscillator, suggesting that the phase-locked method of stabilization is best suited for the JJAVS [11].

(3) Precise null-detection

In order to calibrate 10 V directly by the AC Josephson effect with an irradiating millimeter-wave frequency at 94 GHz, the step number n should be selected either 51,446 or 51,447. Unfortunately, such a high order step voltage may easily be caused to "step-out" by noise coming in the instruments and/or from outside sources, because the current width of the Josephson steps decreases by about 10 μA at step numbers above 50,000. Inaccurate capturing of the desired step numbers may be caused by a few milli-volts difference between the voltage to be calibrated and the Josephson step as a reference. If a digital voltmeter (DVM) is used as a null detector, then the voltage reading includes errors from the DVM, such as those due to offset-error, linearity-error, gain-error and stability. In order to improve accuracy in null-detection, a precise null-balancing technique has been developed for the 10-V JJAVS [12].

As a precise null-balancing technique, a mercury reed-relay switch in which chattering is eliminated by a single-action separation due to mercury viscosity, is used to remove the bias circuit in order to select the desired Josephson step. The accurate capturing at 10 V by the mercury reed-relay switch from a large number of Josephson steps around 10 V is shown in Fig. 5. Once the desired step voltage is caught, the difference in voltage between the reference and voltage to be calibrated should be minimal. This adjustment is achieved by reading the voltage difference ΔV of a DVM as a null-detector, then calculating the adjustment frequency Δf by using the basic Josephson voltage-frequency rela-

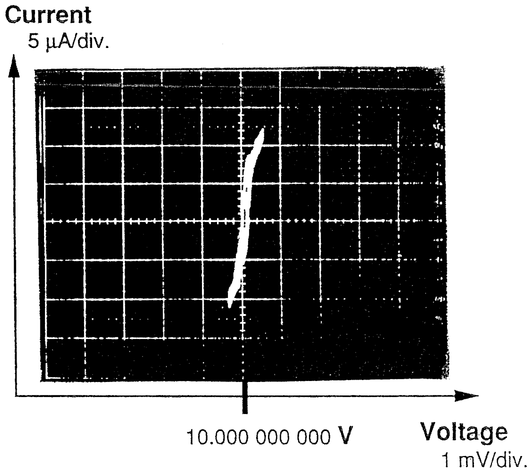


Fig. 5 Capture of the desired Josephson voltage step at 10 V from a large number of Josephson steps around 10 V.

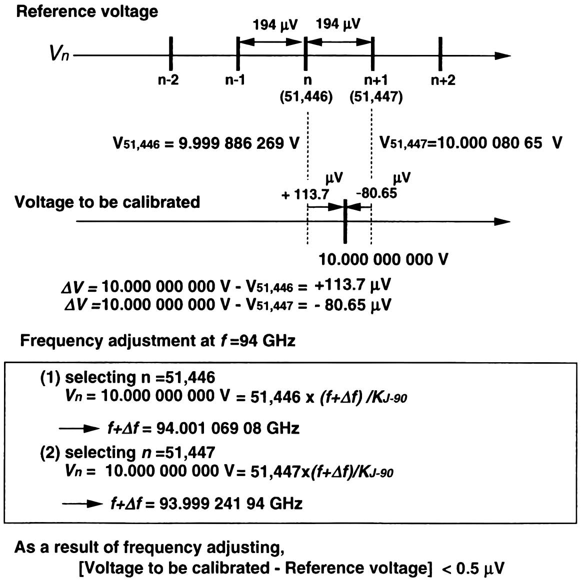


Fig. 6 Null-balance by adjusting the millimeter-wave frequency. Once the desired Josephson step voltage is caught, the difference voltage between the nearest reference and voltage to be calibrated is balanced better than 0.5 μV by using frequency adjustment.

tionship $\varDelta f = \varDelta V K_{\mathrm{J-90}}$. The calculated frequency is fed back to the source-lock counter as shown in Fig. 6. Thus, the null-balance voltage is reduced to within $\pm 0.5\,\mu$V. When the voltage difference becomes small, effects of linearity-error and gain-error of the DVM to the measurement uncertainty become small. Although the linear drifting offset-error can be eliminated by the polarity reversing technique, but it is not easy to eliminate residual offset error that contains nonlinear drifting term and data scattering. Therefore, DC stability (= residual offset error which can not eliminate by polarity reversing) is a significant part of the error in null-detection. Compared with a direct-type DVM and a chopper-type DVM as a null detector, the chopper-type is more suitable, because the chopper mechanism reduces drift and data scattering in the analog-to-digital conversion circuit of the DVM by synchronous detection [12].

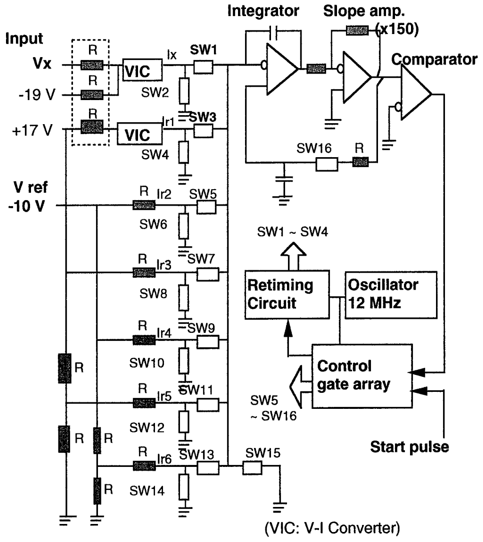


Fig. 7 Configuration of the multi-integrating ADC.

Combined with these techniques, the modern 10-V JJAVS has been established. The total rss uncertainty of the direct calibration for 10-V output of the Zener reference standard (Fluke 732A) is 6×10^{-9} [13]. The main contribution of this uncertainty comes from short-term and long-term scattering of the 10-V output of the Zener voltage standard.

3.2 Gain- and Linearity-Error Check of an Analog-to-Digital Converter (ADC)

Gain- and linearity-errors are key points to check for a high-class DVM. Those errors come from analog-to-digital converter (ADC) of the DVM. A multi-slope, integrating type 24-bit ADC has been developed for an 8-digit DVM by Advantest Ltd. The ADC is an advanced type of dual-slope ADC, designed for compatibility with high-resolution and high-speed measurement of the DVM. Resolution of the DVM is 0.01 ppm at 10 of power line cycle (PLC) (200 msec.), 0.1 ppm at 1 PLC (20 msec.) In general, it should be taken long time in input charging time and discharging time to obtain high resolution in dual slope ADC. Multi-slope circuit consist of reference voltage, switches, integrator, slope amplifier, comparator and re-timing circuit as shown in Fig. 7 reduces discharging time with selecting reference voltage in order, then the measurement time become short with keeping high resolution.

A precise dc voltage generator (calibrator) is required to verify the DC error voltage of the ADC. Regarding the output voltage of the DC voltage generator as a reference, the reading value of the ADC is

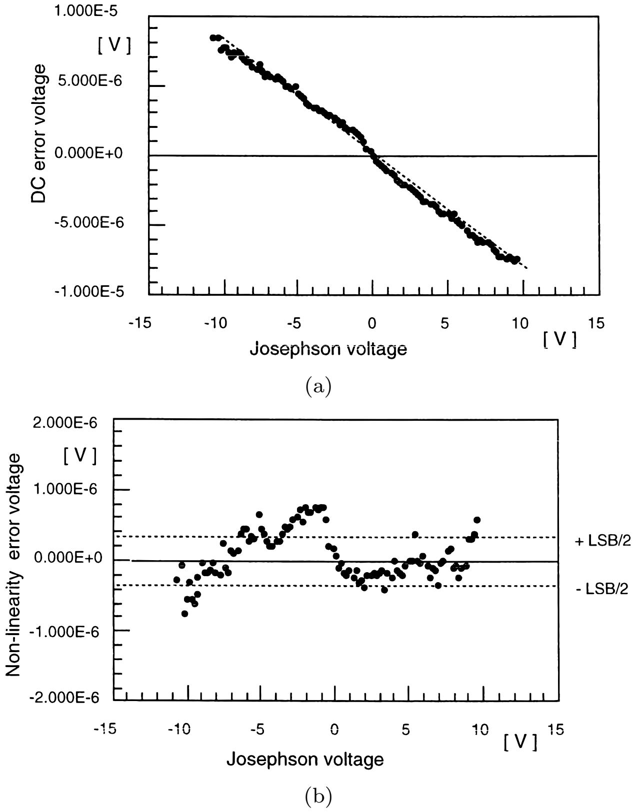


Fig. 8 (a) DC error voltage of the multi-integrating ADC in the 10 V range. (b) Non-linearity error of the multi-integrating ADC in the 10 V range. It is calculated from Fig. 8(a) as the difference between DC error voltage and its linear-fitting regression line.

checked. The best accuracy of any commercially available DC voltage calibrator (a kind of precise DC voltage generator) based on a Zener diode, is 1 to 2×10^{-6} (ppm) at present. This is insufficient to check the latest high-resolution ADC. Based on recently developed techniques in the 10-V JJAVS, we can supply a precise reference voltage in the range from -10 V to $+$ 10 V with selecting step one after another [13] and with accuracy of 10^{-10} at least as described in the previous section.

The DC error voltage of the ADC is measured as a difference between the reading value of the ADC (averaged 10 data obtained with each 50 PLC integrating time) and the selected Josephson step voltage. The measured result of the DC error voltage curve including gain-error, linearity-error, and offset-error in the 10 V range is shown in Fig. 8(a). Gain-error and offset-error can be adjusted, but the residual-error (total DC error curve minus linear-fitting regression line of the total DC error curve) can not be eliminated by adjusting. I defined the residual-error as a non-linearity error. The calculated non-linearity error from Fig. 8(a) is shown in Fig. 8(b). It is allowable if the non-linearity error falls within the least significant bit (LSB) of the ADC, but Fig. 8(b) shows that the ADC has the non-linearity error in excess of one LSB. It means that the ADC electronics circuit should be improved.

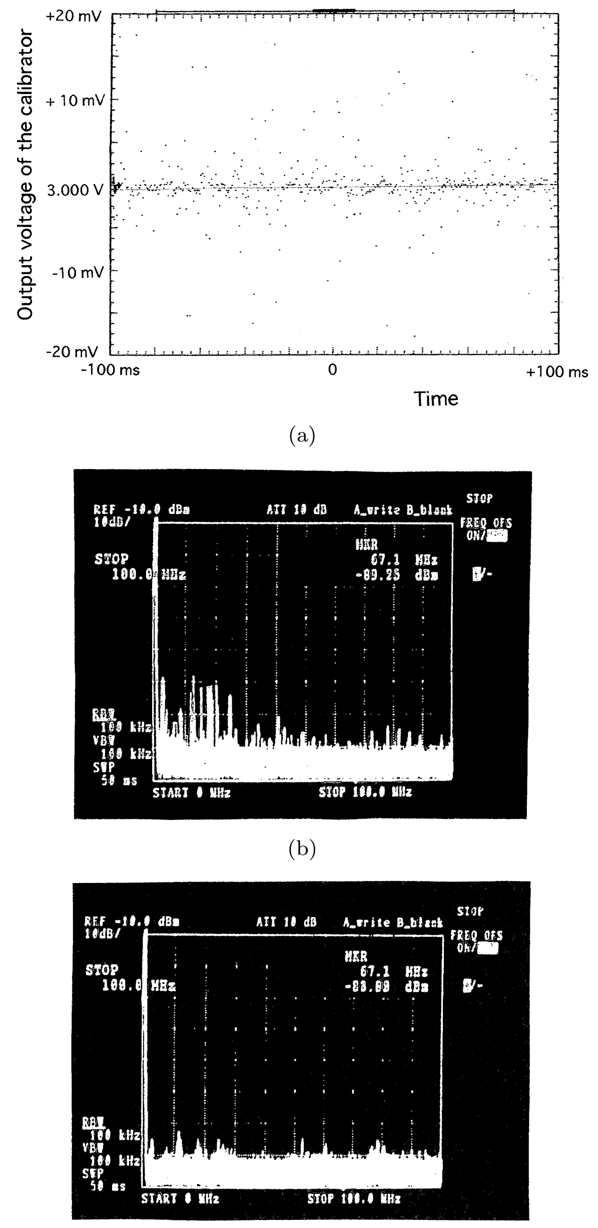
On the basis of 10-V JJAVS technologies, we can achieve such precise and accurate measurement for industrial products.

3.3 Offset-Error Check of the Precise Digital-to-Analog Converter (DAC)

In order to reflect the standard accuracy of the production line, a pulse width modulation (PWM) type precise voltage generator (calibrator) has been widely used. PWM is a convenient method to generate an accurate and precise voltage with integrating pulse-width synchronized to clock, so that it works as a digital-to-analog convertor (DAC), but it irradiates high-frequency noise produced by the clock pulse. It has been difficult to measure the noisy output voltage of the calibrator directly by the 10-V JJAVS, because high-frequency noise on a DC output voltage cause a “step out” phenomenon from the captured Josephson step. Model 5700 A (Fluke) is still one of the most widely used PWM-type calibrators used in production lines. Though its long time averaged DC output voltage is very precise and accurate, it is not so quiet due to the PWM method. Observed 3.000 V output of the 5700 A in the time domain is shown in Fig. 9(a) and its spectrum in the frequency domain is shown in Fig. 9(b). If the calibrator is able to calibrate by the JJAVS directly, the accuracy of the JJAVS reflects to industrial products, that is, the accuracy of industrial products will be improved.

To reduce high-frequency noise coming from the output of the calibrator, we developed a low-pass filter consisting of a normal-mode filter for power-line noise and common-mode filter for noise between signal source and ground of instruments based on the following consideration. The allowable maximum DC resistance of the filter is determined from the voltage drop (less than 0.1 nV, i.e., 1/100 of one digit resolution of the DVM) by an input leakage current (5 pA) of the null-detecting DVM flowing through the filter. Then, the DC resistance should be less than 20 Ω. Since the output resistance of the calibrator is about 1 kΩ, it is necessary to keep the leakage resistance of the overall JJAVS at least 10^{11} Ω to obtain an uncertainty of less than 10^{-8}. A circuit diagram of the realized low-pass filter is shown in Fig. 10, its DC resistance is 0.5 Ω, wire-to-wire leakage resistance is 1×10^{12} Ω, and wire-to-guard leakage resistance is 2×10^{14} Ω. The filtered spectrum for Fig. 9(b) is shown in Fig. 9(c).

As a result of noise filtering, the output voltage of the calibrator has been measured every 1 V steps in the range -10 V to $+10$ V by using the 10-V JJAVS as shown in Fig. 11 [14]. This means that the accuracy of the in-house primary standard reflects directly to the



(a)

(b)

(c)

Fig. 9 (a) Time-domain output voltage of the PWM type calibrator 5700 A. (Output voltage: +3 V, Vertical scale: 5 mV/div., Horizontal scale: 20 ms/div.) (b) Frequency-domain spectrum of Fig. 9(a). (Vertical scale: 10 dB/div., Horizontal scale: 10 MHz/div. Frequency range: 10 kHz to 100 MHz) (c) Filtered spectrum for Fig. 9(b)

calibrator, so that the accuracy of industrial products in the production line expected to be improved by reducing uncertainties of the traceability chain.

4. Conclusion

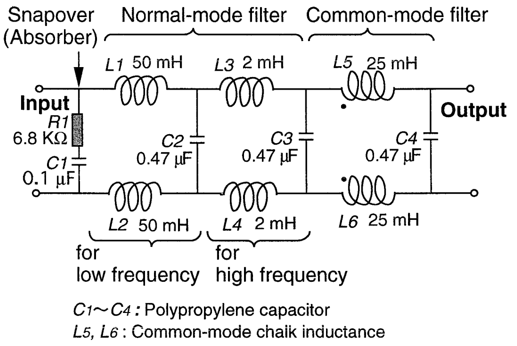


Fig. 10 Low-pass filter circuit consisting of normal- and common-mode filter. Filtering effect is shown in Fig. 9(b).

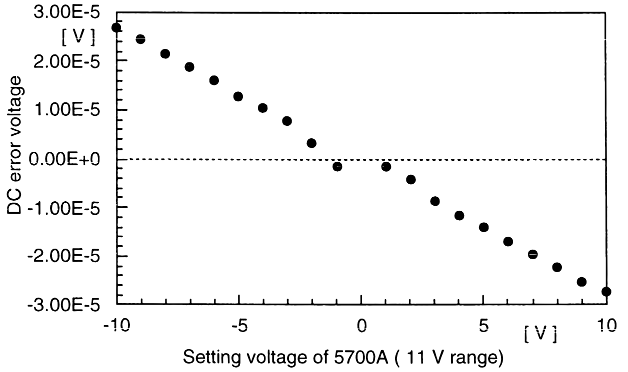


Fig. 11 Measured DC error voltage of the PWM type calibrator 5700 A in the 11 V range.

Since Josephson's prediction in 1962, so many people have been trying to put his theory into practical use. As a result of long and many developing histories, JJAVSs are used to calibrate the output voltage of Zener reference standards in daily works, not only for national standards but also for industrial use. Based on the 10-V JJAVS's technologies, the quantum effect has been applied to other practical precise measurements such as the DC error voltage of a DVM, and a PWM-type precise voltage calibrator. It is obvious that the new DC measurement technique based on the Josephson-effect make a breakthrough in accuracy.

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