Advances in Cone Beam Tomography

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A Literature Survey

for partial fulfillment of the requirements of

Multidimensional Signal Processing

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Chapter 1

Introduction

1.1 Computerized Tomography

Computed tomography (CT) is a method of reconstructing a multidimensional signal from its projections in the lower dimensional space. The word tomography means 'reconstruction from projections' (from Greek *tomos* - slice)[Roe92]. One of the most common projection operations we come across is the formation of shadows. A shadow is a 2D projection of a 3D object formed due to visible light. Shadows are dependent on the internal properties of the 3D object (in this case, transparency or opacity). Hence, shadows can be used to study the internal properties of the object. Also, given a large number of shadows taken from different directions, we can reconstruct the given 3D object in a way such that the reconstructed object reflects the internal properties in question. In practical applications, x-rays, gamma rays, electrons or positrons are used instead of light to study the internal properties of the object. These physical agents (like x-rays, gamma rays) are called probes. When the probe is *outside* the object we call the technique, transmission tomography and if the probe is *inside* the object the term emission tomography is used.

There are a numerous applications of computed tomography in fields as diverse as medical imaging, seismology, non-destructive testing of industrial products, radio astronomy.

Perhaps, its use in medical imaging for purposes such as noninvasive diagnostics, surgical planning, etc., made it as famous as it is today. For a reader more inquisitive on the applications of tomography, we direct him/her to [KS88].

Typically, based on the geometry of the physical agents, three different types of tomographies are identified - parallel beam tomography, fan beam tomography and cone beam tomography. In this document, we explore the need for cone beam tomography, methods of achieving it and the problems that need to be solved that would make it more applicable to different fields that use tomography. But, for completeness and for later references during cone beam tomography discussion, we briefly elucidate parallel beam and fan beam tomographies.

1.1.1 Parallel Beam Tomography

In parallel beam tomography, a parallel beam of rays intersect the object of interest. The parallel beam is inclined at an angle θ to the x-axis, and each ray M can be characterized by its perpendicular distance, t to the origin (figure 1.1). Now a line integral is performed along the line $M_{\theta,t}$ and is denoted by

$$P_\theta(t) = \int_{M_{\theta,t}} f(x,y)ds \tag{1.1}$$

where s is along the direction of the ray. $P_\theta(t)$ is called the Radon transform of $f(x,y)$. For each θ, $\{P_\theta(t)|\theta \in [0,\pi)\}$ gives a complete collection of 1-D projections of the 2-D object $f(x,y)$. Using a parametric representation $t = xcos(\theta)+ysin(\theta)$, we can write the projections as

$$P_\theta(t) = \int\int_{\Re^2} f(x,y)\delta(xcos(\theta)+ysin(\theta)-t)dxdy \tag{1.2}$$

Computed tomography deals with the reconstruction of $f(x,y)$ from $P_\theta(t)$. Two basic methods are used, (1) Filtered Back-Projection and (2) Fourier Slice Method. We show the equations of Filtered Back-Projection as they are necessary for understanding the material of this survey. It can be shown that, [Dea83] in two dimensions, the inverse Radon transform is reduced to:

$$f(x,y) = \frac{1}{2\pi}\int_0^{\pi}\int_{-\infty}^{\infty}\frac{1}{\mathbf{x}.\mathbf{\Theta} - l}\frac{\partial P_\theta(l)}{\partial l}dld\theta \tag{1.3}$$

where $\mathbf{\Theta} = (cos(\theta), sin(\theta))$ and $\mathbf{x} = (x, y)$. Assuming $\partial P_\theta(l)/\partial l$ exists and is continuous, it

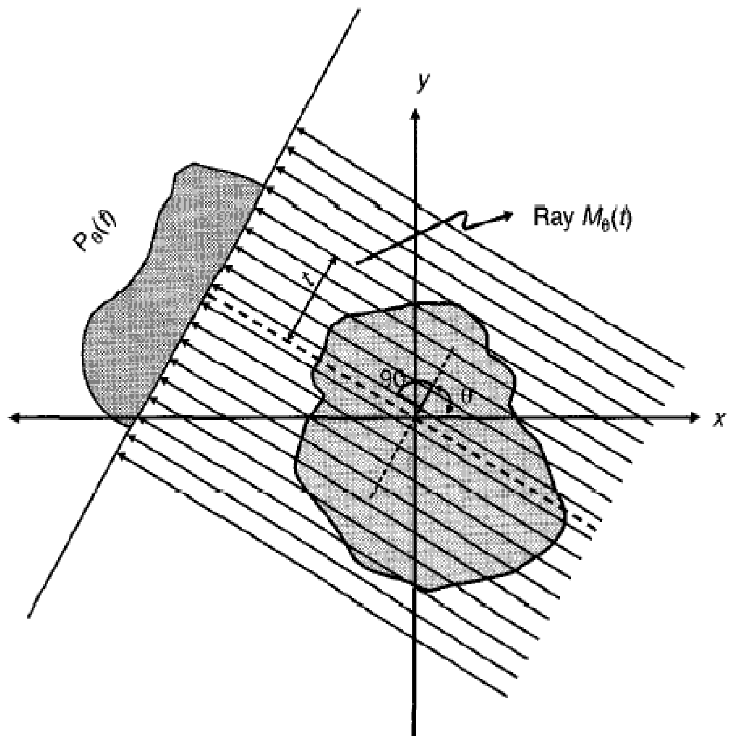

Figure 1.1: Parallel beam projection
Courtesy: Hiriyannaiah, [Hir97]

can be shown that

$$f(x,y) = \frac{1}{2\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty}P_\theta(l)H_\epsilon(x.\theta - l)dld\theta \tag{1.4}$$

where $H_\epsilon(l) = 1/\epsilon^2$ for $|l| < \epsilon$ and $H_\epsilon(l) = 1/l^2$ otherwise. Since equation 1.4 is in the form of convolution, this method is also called *convolution back projection.*

1.1.2 Fan Beam Tomography

In parallel beam tomography, the source and the detector are translated for each angle θ. This leads to large scanning times, which is particularly unwanted in medical diagnostics. Fan beam tomography offers solutions to some of these problems. In this configuration (figure 1.2), the source beam is collimated so that a thin, planar fan beam of rays originate from the source and pass through the object of interest before reaching the detector array. This is repeated for multiple angles by rotating the source around the object. Reconstructing

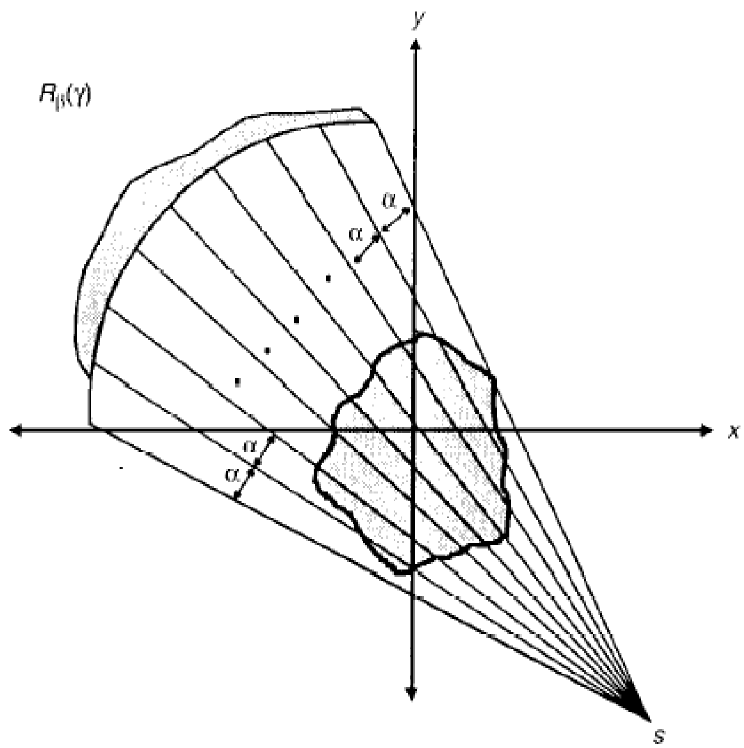

Figure 1.2: Fan beam projection
This figure shows a particular configuration of fan beam system called the equiangular fan beam system. Courtesy: Hiriyannaiah, [Hir97]

from fan beam projections is an extension of the concept of reconstructing from parallel beam projections. If we replace (t, θ) by generalized sampling coordinates, (ζ, η), we get the equation for reconstruction. It is given by the formula [Hor78]:

$$f(r,\phi) = \frac{1}{4\pi^2} \lim_{\epsilon \to 0} \int \int P_\theta(l-t) H_\epsilon(t) J(l,\theta,\zeta,\eta) d\zeta d\eta \tag{1.5}$$

where J is the Jacobian of the transformation from (ζ, η) space to (l, θ) space. It is given by:

$$J = \frac{\partial t}{\partial \zeta} \frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \zeta} \frac{\partial l}{\partial \eta} \tag{1.6}$$

The above equation for reconstruction can be simplified to the convolution form.

1.2 Motivation for Cone Beam Tomography

Cone beam tomography is a 3D extension of the 2D fan beam tomography.. For generating cone beam projections, we essentially use the same setup as that of the fan beam, with the exception of the collimator. In this case, we remove the collimator. Cone beam tomography helps in generating 3D reconstructed images easily. Usually, in computed tomography, 3D images are formed by stacking 2D slices. Since, these slices, in practice are of finite width, there is a huge problem of spatial resolution in reconstructing 3D images using these slices. Smith [Smi90b] observes that though it is possible to improve spatial resolution by overlapping cross-sections, it, unfortunately, results in increasing the dose to the patient. Also, it is extremely difficult to obtain data for reconstruction of objects that change their state rapidly. When scanning patients, one has to make sure that they do not move, which is highly unlikely. If they move, motion artifacts are formed. In monetary terms, this would result in higher costs as fewer patients can be scanned in a given amount of time. These problems can be avoided by using cone beam tomography.

Another major reason for using cone beam tomography is its effectiveness in SPECT (Single Photon Emission Computed Tomography). A small photon count in SPECT results in data with large statistical uncertainty, which can be reduced by employing cone beam tomography. SPECT is very attractive, especially in medical diagnostics, because, in some instances, “SPECT can reveal diseases before structural damage has been done”[Smi90a].

1.3 Organization of the Report

The report is organized as follows. Chapter 2 describes the problem of reconstruction from cone beam projections and introduces a major problem that plagues cone beam tomography, namely, the long object problem. In chapter 3, we explore the existing approaches and their limitations for both reconstruction and the long object problem. Unsolved problems are discussed in chapter 4. The report concludes with chapter 5.

Chapter 2

Problem Description and Background

2.1 Reconstruction from Cone-beam Projections

We pointed out earlier that cone beam tomography is a 3D extension of 2D fan beam tomography. This might lead to the conclusion that cone beam data could be reconstructed by using the well-understood, well-documented theories of fan beam and parallel beam reconstruction. However, Smith [Smi87] argues that fan beam theory cannot be used, as, data from vertices of one circle can be reconstructed. It was also shown that parallel beam theory cannot be applied for cone beam reconstruction owing to the fact that "a complete one-dimensional family of integrals" is not known [Smi85]. Hence it was generally accepted that newer functions for reconstruction have to be developed.

The problem of reconstruction from cone beam data is defined and a "convolution type" equation for reconstruction is given below [Smi87]. If, $f = f(\mathbf{x})$ is the three dimensional object to be reconstructed, and if the support of f is a sphere of radius R, then, a function F can be defined as

$$F(\beta, l) = \frac{1}{\pi} \lim_{\epsilon \to 0} \int_{-\infty}^{\infty} H_\epsilon(l - t)\breve{f}(\beta, t)dt \tag{2.1}$$

where, $H_\epsilon(t) = 1/\epsilon^2$ for $|t| < \epsilon$ and $-1/t^2$ otherwise. In the above equation, $\beta \in S^2/2$ and $\breve{f}$

is the three dimensional Radon transform of the object given by

$$\breve{f}(\beta, l) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(l\beta + s\beta_{\perp\mathbf{1}} + t\beta_{\perp\mathbf{2}}) ds dt \tag{2.2}$$

where β, $\beta_{\perp\mathbf{1}}$ and $\beta_{\perp\mathbf{2}}$ are three orthonormal vectors. Now f can be obtained from the following proposition:

$$f(\mathbf{x}) = \frac{1}{4\pi^3} \int_{S^2/2} \lim_{\epsilon \to 0} \int_{-R}^{R} F(\beta, l) H_\epsilon(\mathbf{x}.\beta - l) dl d\beta \tag{2.3}$$

The above equation holds only if F is known in the set $\{(\beta, l) : \beta \in \frac{S^2}{2}, |l| < R\}$. The above equation is of the form of convolution and forms the basis for many efficient approaches. These approaches will be discussed in chapter 3.

2.1.1 Completeness

Completeness is determining whether the information that is contained within some geometry of vertices is enough to perform reconstruction without artifacts. From the above set of equations, we know that if for any geometry, it is possible to obtain F, then we can perform artifact-free reconstruction. This is specified in the completeness condition, which states [Smi90b]

> *If on every plane that intersects the object there lies a vertex, then one has complete information about the object.*

Some of the examples of complete geometries are *sine on the cylinder*, *Baseball* etc. Examples of incomplete geometries are *circle*, *Straight line* etc. This shows that only some of these geometries can be used for scanning. This forms the basis for designing the systems for projecting the rays.

2.2 Long Object Problem

One of the most desirable properties of data collection is scanning only the region of interest (ROI) without having to scan the whole object. This is particularly desirable in medical applications as this would avoid unnecessary radiation to the patient. This problem is called the long object problem. The intent in the long object problem is to reconstruct a central ROI of a long object when the helical path extends only in object's immediate vicinity. Traditionally, the reconstructed problems with truncated cone beam data related to helical scan are divided into short object problem and long object problem. Reconstructing a whole object having a bounded support in the axial direction when the helical path extends a little bit above and below the object support forms the short object problem where as in the long-object problem one aims at reconstructing a central region of interest (ROI) of a long object having an unbounded support [KND00]. The short object problem and the long object problem are illustrated in figure 2.1. It is clear to see that the long object problem is more difficult to solve than the short object problem. Even as of today, there is no satisfactory method to solve the long object problem in terms of quality and computational complexity.

Formally, the long object problem can be formulated as follows [KND98]. Let $f(\mathbf{x})$ be an object that contains the ROI, where $\mathbf{x} = (x, y, z)^T$. The object is supported inside an infinite cylinder $x^2 + y^2 = Q^2$, the central axis of which coincides with the z-axis. The aim of the long object problem is to reconstruct $f(\mathbf{x})$ over a central ROI given by

$$\Omega = \{\mathbf{x} | x^2 + y^2 \leq Q^2, z_{min} \leq z \leq z_{max}\} \tag{2.4}$$

Recent approaches to solve the long object problem and their limitations are given in the next chapter.

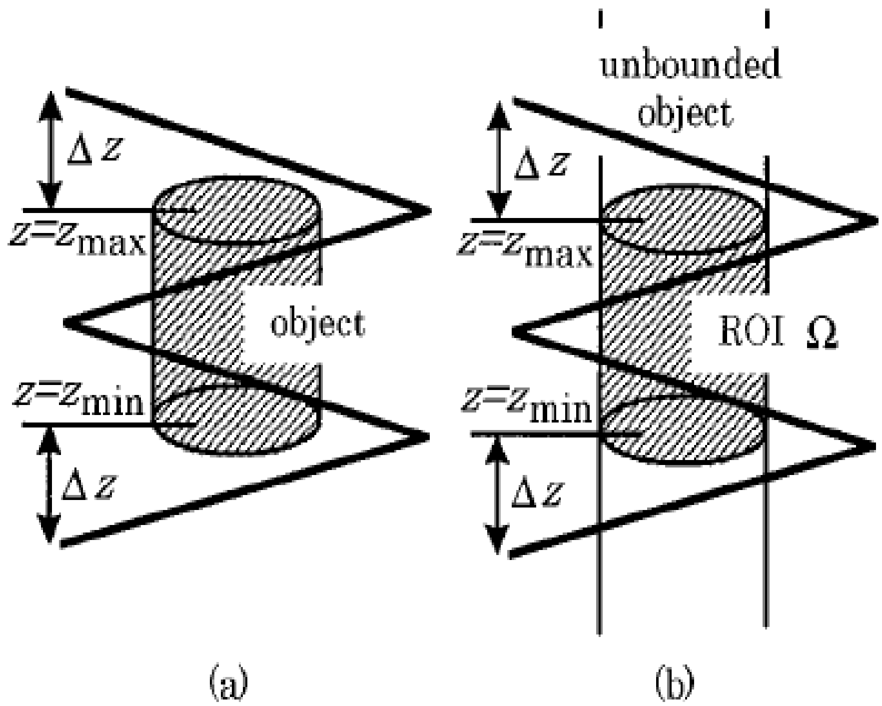

Figure 2.1: Long object problem
(a) Short object problem (b) Long object problem. Courtesy: Kudo et al, [KND00]

Chapter 3

Existing Approaches and their Limitations

3.1 Reconstruction from Cone-beam Projections

In chapter 2, we showed the equation for inversion of 3D Radon transform. This is the most direct method that is similar to the standard filtered back projection we perform in two dimensions. However, in three dimensions, this algorithm is computationally very expensive, which requires $O(N^5)$ operations for recovering N^3 volume from N^3 samples of Radon transform. Marr et. al [MCL81] factorized the 3D Radon transform into 2D Radon transforms. This results in a computational complexity of $O(N^4)$. In [ZG92], the authors propose a backprojection algorithm by inverting the Radon transform in the frequency domain. However, the computational complexity of this algorithm is very high. In [Dus96], Dusaussoy describes a volumetric image reconstruction algorithm that has a computational complexity of $O(N^3 \log N)$. The technique uses Fourier transform for the inversion of the Radon transform we saw in chapter 2. The method is a direct implementation of the slice theorem for plane integrals. In this algorithm the spectral data that lie on concentric circles are bilinearly interpolated on to the sides of the concentric cubes. This is a generalization of 2D Fourier image reconstruction. Though this method reduces the computational complexity to a great extent, the reconstruction quality is not great. Also, there have been no tests to analyze the noise properties of the reconstructed images and the effects of scatter and beam hardening in experimental cone beam data. Also, this method takes a lot more time for rebinning than the standard Grangeat algorithm[Gra91].

There are many algorithms with complexity $O(N^3 \log N)$ [AD94],[SFS97]. However most of these algorithms are based on the factorization of the 3D Radon transform in to a pair of 2D Radon transforms. This factorization relies on angularly separability of sampling pattern of the parameter space, which is an unrealistic assumption. A native 3D algorithm that does not rely on factorization was proposed by Basu et. al [BB02]. The algorithm uses a hierarchical decomposition of the 3D Radon transform to recursively decompose the backprojection operation. Though the method reduces the computational complexity and provides a reconstruction with a high quality, it still has to be combined with a 2D reprojection algorithm to compute the 2D Radon transforms that arise from Smith transformation from cone beam to 3D Radon data. Another $O(N^3 \log N)$ algorithm was proposed by Kudo and Saito [KS96]. They reduce the computational complexity by using the subsampled Radon space for reconstruction. They argue that this process will not decrease the quality of reconstruction as the Radon space was oversampled to start with. They tested their method against [AD94] and they claim that their method is 2.3 times faster than the latter. This amount of savings is specially significant when reconstructing volumes of dimensions 512^3.

In [BKS$^+$00], Bluder et. al., compare multirow Fourier reconstruction (MFR) [SFS97] algorithm and advanced single slice rebinning (ASSR) [KSK00] method to identify a practical and efficient approximate cone beam method to extend its potential for medical use. The parameters that were used for comparison are image artifacts, spatial resolution, contrast resolution, and image noise. They found that the ASSR method is more practical and efficient and was providing image quality comparable to that of a single-row scanning system even with a 46 row detector. They also observed that both the algorithms tolerate any table feed below the maximum value associated to the detector height.

Wang and Ning [WN99] propose a reconstruction algorithm for circle-plus-arc data acquisition geometry. We know from the completeness condition that only certain types of data acquisition geometries would guarantee us complete reconstruction. The circle-plus-arc geometry is shown in figure 3.1. In this geometry, the object is bounded within the sphere of radius R, which is concentric with the circle-plus-arc orbit. γ_{min} is the minimum cone angle and δ_{min} is the minimum spanning angle of the arc orbit. This data acquisition scheme can

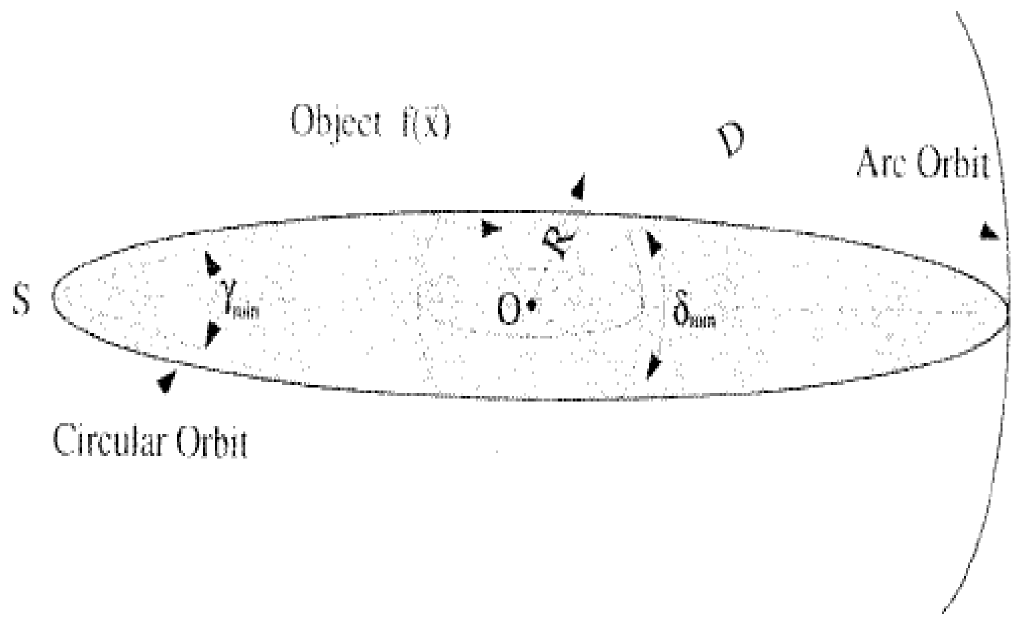

Figure 3.1: Circle plus arc geometry
Courtesy: Wang et. al, [WN99]

provide a complete set of projection data. The authors are of the opinion that this system can be designed in practice, but a little mechanical modification was required.

In [WVC99], Wang et. al. propose iterative tomography for reconstruction from incomplete data. They use Expectation-Maximization (EM) and Algebraic Reconstruction Technique (ART) in their algorithm. This algorithm also reduces the artifacts. The novelty of their approach is the introduction of a projection mask and computation of a 3D spatially varying relaxation factor. This allows for beam divergence and data truncation. Another important work in this field was proposed by Hiriyanniah et. al [HSR96]. They propose a projection onto convex sets (POCS) for handling incomplete data. They demonstrate the effectiveness of the algorithm for circular trajectories, but claim that the method is basically geometry independent. ART was used by Mueller et. al. [MYW99] for reconstruction of low contrast objects. They experiment with cone angles greater than 20^0 and use ART and the concept of depth dependent interpolation kernel to reduce the aliasing artifacts. In fact, ART was used to calculated the weights of the depth dependent interpolation kernel. But there still needs to be a lot of research done in the area of low-contrast object reconstruction for practical applicability.

Several authors have explored the possibility of reconstruction through MIMD computers. Most notable among those are [LPCA98] and [RFCS96]. The former experiment the possibilities of parallelization if basic operators (local vs global) and parallelization of reconstruction algorithms. The latter partitions the volume into variable width slabs and sends the each slab to the workstation. While the methods are different, they agree on the benefits of parallel implementation, namely,

- Computation of realistic-sized images
- Reduction in computation times with respect to the number of processors.

Other interesting implementations can be found in [MMW96] and [SP96].

3.2 Long Object Problem

As we said before, long object problem is of great interest in the computed tomography research community for the advantages it offers. Some of the important publications in this field have been reviewed here. Schaller et. al. [SNS+00] propose a technique of virtual object to solve the long object problem. In this method, they introduce a virtual object $f_\phi(x)$ for each azimuth angle ϕ in the image space, such that the virtual object has the property of being equal to the true object $f(x)$ in some ROI Ω_m. They show that the Radon transformed data can be calculated for the parallel projection of $f_\phi(x)$ onto the meridian plane of angle ϕ for each ϕ. In this manner, they can perform "exact" reconstruction of $f(x)$. Though the computational complexity of this algorithm is less, the quality of reconstruction is outside the bounds of satisfaction.

In [KND00], Kudo et. al., propose an Quasi-exact algorithm whose overall structure is in the form of filtered backprojection. This is derived by extending the triangulation of Grangeat's formula [Gra91]. One advantage of this approach is that it does not require two circular scans at the ends of the helical paths, which were inseparable from the other approaches. While the approach shows some promise in terms of reconstruction quality, the computational complexity is a discouraging. The complexity is far greater than that for the Feldkamp [FDK84] algorithms.

Zhao and Wang [ZW00] proposed a Feldkamp-type reconstruction algorithm from the wavelet perspective. The advantage is that the algorithms could be used for either global or local reconstruction. The novelty of the work lies in identifying the fundamental relationship between the wavelet transform and Feldkamp algorithm. The authors also report that a 3D ROI can be reconstructed without severe artifacts. This could help in solving the long object problem. However, the research is still in the preliminary stage and refinements are necessary for it to *solve* the long object problem.

In [LTS00] Lauritsch et. al. hypothesize that using local ROI's would help in increasing the quality of the reconstruction in the long object problem. They design an algorithm based on this hypothesis. As a part of their algorithm they take the derivative of the Radon data for different local ROIs that are adapted to the scan path. They make sure that the contributing cone beams are not contaminated by object information outside the local ROI. This method though it made a headway in improving the reconstruction quality, the computational complexity is extremely high. The authors propose to perform 1D convolutions in order to decrease the computations, but it still has to have lesser computations for any practical applications. We discuss more about long object problem in the chapter *Unsolved Problems*.

Chapter 4

Unsolved Problems

4.1 Long Object Problem

Long object problem is a problem that has been plaguing researchers in the computed tomography community for the past several years. Many algorithms have been proposed, but none of them have been able to solve this problem, although, it must be said that research in the past couple of years has been very fruitful. In chapter 3, we saw some important contributions to the solution of this problem. Kudo et. al observe [KND00] that long object problem "is a challenging problem for which solutions are currently under investigation by researchers". Sourbelle et. al [SLT^{+}02] compare different algorithms and come to conclusion that all these techniques lead to reconstructions with very poor spatial resolution. One reason for the degradation in spatial resolution is probably due to interpolations in computing radon derivatives and inversion.

4.2 Low Contrast Object Reconstruction

It has been a general notion in many fields that most researchers always try to solve problems solved by other people, maybe, by other techniques, but largely ignore the unsolved problems for various reasons. Low contrast object reconstruction would form a fine example of that. Most researchers in computed tomography always design their systems and test them for high-contrast objects [MYW99]. High contrast may not always be satisfied, wherein the technique might fail or give substandard results. One of the major advantages of solving this problem would be in the field of automatic diagnosis. Presently, an expert is needed to interpret the results of a reconstruction. This is partly because, the reconstructions are poor in case of low contrast and needs a human expert to classify them. Solving the problem of low contrast reconstruction would make it much easier to develop algorithms to classify the reconstructed data. It could also help the medical expert make better decisions.

Chapter 5

Conclusions

In this report, we presented a brief survey on the advances in cone beam tomography. Cone beam tomography is advantageous for many reasons, few of which are i)convenience, ii)better 3D reconstruction iii)lesser radiation dose. However, many more innovations have to take place before cone beam tomography may become truly useful. Recent approaches for solving various problems and some of their limitations are discussed. In this report we have also identified some unsolved problems and gave reasons as to why they have to be solved.

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