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Measurement Uncertainty

A Practical Guide for Secondary Standards Dosimetry Laboratories

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FOREWORD

The need for international traceability for radiation dose measurements has been understood since the early nineteen-sixties. The benefits of high dosimetric accuracy were recognized, particularly in radiotherapy, where the outcome of treatments is dependent on the radiation dose delivered to patients. When considering radiation protection dosimetry, the uncertainty may be greater than for therapy, but proper traceability of the measurements is no less important.

To ensure harmonization and consistency in radiation measurements, the International Atomic Energy Agency (IAEA) and the World Health Organization (WHO) created a Network of Secondary Standards Dosimetry Laboratories (SSDLs) in 1976. An SSDL is a laboratory that has been designated by the competent national authorities to undertake the duty of providing the necessary link in the traceability chain of radiation dosimetry to the international measurement system (SI, for Système International) for radiation metrology users. The role of the SSDLs is crucial in providing traceable calibrations; they disseminate calibrations at specific radiation qualities appropriate for the use of radiation measuring instruments. Historically, although the first SSDLs were established mainly to provide radiotherapy level calibrations, the scope of their work has expanded over the years. Today, many SSDLs provide traceability for radiation protection measurements and diagnostic radiology in addition to radiotherapy. Some SSDLs, with the appropriate facilities and expertise, also conduct quality audits of the clinical use of the calibrated dosimeters — for example, by providing postal dosimeters for dose comparisons for medical institutions or on-site dosimetry audits with an ion chamber and other appropriate equipment.

The requirements for traceable and reliable calibrations are becoming more important. For example, for international trade where radiation products are manufactured within strict quality control systems, it is necessary that they conform to given safety and performance criteria. The demonstration of the competence of calibration laboratories is achieved through comparisons and the establishment of a quality system following the International Organization for Standardization (ISO) Guide 17025. One of the requirements of the quality system of a calibration laboratory is the assessment of the measurement uncertainty for all its calibration services. General guidance on the estimation of measurement uncertainty was published by ISO in 1995. However, that document addresses all calibration and testing laboratories and not specifically dosimetry calibration laboratories. To provide SSDLs of the IAEA/WHO Network with a practical guide on the assessment of the measurement uncertainty, two consultants meetings were held at IAEA Headquarters on 26–30 April, 2004 and 19–23 September 2005. The present publication was prepared during these meetings. It is addressed to scientists working in calibration laboratories and to physicists involved in radiation dosimetry measurements.

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CONTENTS

1. INTRODUCTION

The IAEA/WHO Network, through Secondary Standards Dosimetry Laboratories (SSDLs) designated by the Member States, provides a direct linkage of national dosimetry standards to the international measurement system (SI, for Système International). Through the proper calibration of field instruments by the SSDLs, these measurements are traceable to the Primary Standards Dosimetry Laboratories (PSDLs) and the Bureau International des Poids et Mesures (BIPM). The Network has proven to be of value in improving national capabilities for instrument calibration and the level of awareness of the need for better accuracy and traceability [1]. To ensure that the services provided by SSDL members to end users follow internationally accepted standards, the IAEA has set up a comparison programme using ionization chambers. In this way, the laboratories verify the integrity of their national standards and the procedures used for the transfer of the standards to the end users [1]. During the implementation of the comparison programme, the IAEA identified important discrepancies in the way SSDLs report their calibration uncertainty. Although the general Guidance on the expression of Uncertainty in Measurement (GUM) was published by ISO in 1995 [3], no specific publication deals with the implementation of the ISO guidance in the calibration of dosimeters. The need to prepare a guidance document for SSDL members on this topic was recognized by the IAEA and supported by the IAEA Standing Advisory Group, the Scientific Committee of the IAEA/WHO SSDL Network organized in 2004 [4].

A guidance document on the assessment of measurement uncertainty will also benefit SSDLs that signed the Mutual Recognition Arrangement (MRA) of National Measurement Standards and of the Calibration and Measurement Certificates Issued by the National Metrology Institutes. The IAEA signed the MRA in October 1999 and has, since then, contributed to strengthening the SSDL capabilities in this field. The signing by the IAEA of the MRA imposes stricter demands on its dosimetry comparisons and may require a modification of the criteria of acceptability of the level of performance achieved by SSDLs in these comparisons. A common procedure for uncertainty assessment among all SSDL Network members would greatly facilitate the review of comparison results by the IAEA and other regional or international institutions.

The purpose of this publication is to provide guidance to SSDLs on how to assess and report measurement uncertainty related to their calibration services in a way that is consistent with the GUM. It includes a review of the calibration procedures and model equations used by SSDLs in Section 2, a tutorial part on how to assess the various uncertainty components in Section 3, and two practical examples of typical calibrations in ^{60}Co gamma ray beams. The practical examples, linked to the concepts described in Sections 2 and 3, are given to help the user in preparing the uncertainty budget. Although the scope of this publication is limited to a model equation used in ^{60}Co gamma beam for radiotherapy dosimetry, the concept can be extended to cover X ray dosimetry for radiotherapy and radiation protection dosimetry. For the calibration of instruments used in diagnostic radiology, the guidance given in IAEA technical Reports Series No. 457 [5] should be followed.

2. REALIZATION AND DISSEMINATION OF STANDARDS AT SSDLs

An SSDL is equipped with secondary standards which are traceable to the PSDLs or the BIPM. The standards of about 40% of the SSDL members are traceable to the IAEA, 45% to a PSDL and 15% directly to the BIPM. While the scope of the work of an SSDL may be quite broad in some countries and restricted in others, depending on national needs, there are certain duties which an SSDL is required to fulfil to retain full membership in the IAEA/WHO SSDL Network [1]. These duties include the provision of calibration services to end users in the country for radiotherapy, diagnostic radiology and/or radiation protection level dosimetry.

The training of SSDL staff, type of equipment used and calibration procedures have a direct impact on the uncertainty level related to the calibration services. For example, the use of a fine focus telemeter or a precision rod to position an ion chamber in a radiation field will lead to a lower uncertainty on the distance than the use of a simple scale meter. This section provides a methodology to derive a general model equation used in calibration as well as an overview of the various influence quantities that have a significant impact on the calibration results [6].

2.1. Determination of a calibration coefficient: The model equation

Two procedures can be used to calibrate instruments in a radiation field: tip-to-tip or substitution. In the tip-to-tip method, two dosimeters (the reference dosimeter and the dosimeter to be calibrated) are placed in the radiation beam and irradiated simultaneously. In calibration by substitution, first the reference dosimeter is placed at the calibration point to determine the reference output rate of the beam through a set of readings. It is then replaced by the dosimeter to be calibrated, and a similar set of readings is taken. Each procedure has advantages and disadvantages [6] and an SSDL may well choose one procedure on some occasions and another on other occasions. Most SSDLs use the substitution method, and it was therefore decided to use it as a basis for the guidance provided in this publication. The practical examples given in the appendices are also based on the substitution method. It is believed that readers who use the tip-to-tip method can easily derive the corresponding model equation.

Using the substitution method, the calibration coefficient of an instrument is determined in two steps:

Step 1: The output rate[1] $\dot{K}_{a,Q}$ of a radiation beam of quality Q is determined with the SSDL reference standard, calibrated at a PSDL or the IAEA at the beam quality Q_0.

$$\dot{K}_{a,Q} = N_{K,Q_0}^{\text{ref}} M_{corr}^{\text{ref}} k_{Q,Q_0} \tag{1}$$

where

N_{K,Q_0}^{ref} is the calibration coefficient of the SSDL reference standard (calibrated at a PSDL or IAEA) for the beam quality Q_0,

M_{corr}^{ref} is the reading of the reference dosimeter corrected for the influence quantities, and k_{Q,Q_0} (denoted k_Q in the rest of this publication) is a factor to account for the difference in beam qualities of the IAEA (PSDL) and the SSDL [7].

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[1] The model equation is also valid for integral kerma.

Step 2: The instrument to be calibrated is placed at the same position as the SSDL reference standard in the beam of quality Q. The calibration coefficient, N_Q^{user}, for the beam quality Q of the instrument to be calibrated is determined as the ratio of the output rate, $\dot{K}_{a,Q}$, determined in step 1, to the mean reading obtained from the instrument to be calibrated, corrected for the influence quantities.

$$N_Q^{\mathrm{user}} = \frac{\dot{K}_{a,Q}}{M_{corr}^{\mathrm{user}}} k_{source} \tag{2}$$

where

k_{source} is a correction for the effect of a change in source position,

M_{corr}^{user} is the reading obtained with the user chamber (already corrected for influence quantities).

The following calibration conditions are assumed to be valid throughout the calibration process:

- the calibration source (^{60}Co unit or ^{137}Cs) is of the type where the source is moved to the fully shielded position between measurements with different chambers, and so an allowance is made in the analysis for variations in the source position from one chamber measurement to another.
- the collimator setting is fixed, and
- corrections to the readings with the reference and user instruments are based on measurements with the same barometer and thermometer.

In Equation (3) given below, M_{corr}^{ref} and M_{corr}^{user} are denoted as M_{corr}

$$M_{corr} = M_{raw}\, k_{TP}\, k_{dist}\, k_{stab}\, k_{others} \tag{3}$$

where

M_{raw} is the mean value of the readings taken after the instrument is settled,

k_{TP} is a factor to correct for departure of air density from reference conditions,

k_{dist} is a factor to correct for the deviation of chamber position from the reference position,

k_{stab} is a factor to correct for the stability of the SSDL reference standard,

k_{others} is a factor including all the corrections whose uncertainties are too small to consider individually in the uncertainty budget, because they are estimated to be much less than 0.1%. Nevertheless, SSDLs are advised to review and assess these factors independently to ensure that their overall contribution is indeed negligible (less than 0.1%).

k_{others} is given by

$$k_{others} = k_{elec} k_{lin} k_s k_{leak} k_h k_{pol} k_{rot} k_{fs} k_{\mathrm{hom}} \tag{4}$$

where:

k_{elec}	is the calibration coefficient of the measuring assembly, in case the chamber and measuring assembly are calibrated separately,
k_{lin}	is a factor to correct for non-linearity of the measuring assembly sensitivity,
k_s	is a factor to correct for the lack of saturation due to recombination,
k_{leak}	is a factor to correct for leakage current (possibly converted from an additive correction),
k_h	is a factor to correct for any departure of humidity from the reference condition: 50% relative humidity (RH),
k_{pol}	is a factor to correct for any change in the reading due to changing the polarizing voltage from its value at calibration,
k_{rot}	is a factor to correct for any misalignment (rotation, tilt) of the chamber in use,
k_{fs}	is a factor to correct for departure of the field size from the reference condition, and
k_{hom}	is a factor to correct for radial non-homogeneity of the beam.

Combining Equations (1) and (4) gives:

$$N_Q^{\mathrm{user}} = k_{source} k_{stab} N_{K,Q_0}^{ref} \frac{M_{raw}^{ref}}{M_{raw}^{user}} \frac{k_{TP}^{ref}}{k_{TP}^{user}} \frac{k_{dist}^{ref}}{k_{dist}^{user}} \frac{k_{other}^{ref}}{k_{other}^{user}} k_Q \tag{5}$$

By replacing k_{TP} by the explicit expression in terms of temperature in degrees Celsius, T, and pressure, p, Equation (5) becomes:

$$N_Q^{\mathrm{user}} = k_{source} k_{stab} N_{K,Q_0}^{ref} \frac{M_{raw}^{ref}}{M_{raw}^{user}} \left(\frac{273.15 + T^{ref}}{273.15 + T^{user}} \right) \frac{p^{user}}{p^{ref}} \frac{k_{dist}^{ref}}{k_{dist}^{user}} \frac{k_{other}^{ref}}{k_{other}^{user}} \tag{6}$$

The overall uncertainty of N_Q^{user} is obtained from the component uncertainties that arise from the influence quantities given in the right hand side of Equation (6).

Equation (6) represents the model of the measurement process and serves as 'the model equation' for the 'output quantity' (N_Q^{user}), which is a function of the input quantities (right hand sides).

2.2. Influence quantities

Influence quantities are those quantities that are not the subject of measurement, but yet influence the reading of the dosimeter or the quantity under measurement (e.g. air pressure, ageing and zero drift of dosimeter, beam quality, dose rate, field size, etc.).

When calibrating a dosimeter as many influence quantities as practicable should be kept under control. However, many influence quantities cannot be controlled (e.g. air pressure or dose rate in a ^{60}Co gamma beam). The corresponding effects should be corrected by applying appropriate factors. In either case the incomplete knowledge about the values of the influence quantities and their impact on the final result must be considered in the uncertainty analysis.

For practical reasons it is advisable to list all potential effects and their corresponding uncertainty value. Based on experience or documented reference, one should set a reasonable limit for the smallest uncertainty one wants to consider in the uncertainty analysis. All effects whose uncertainty is smaller than this selected limit would be ignored in the uncertainty budget. It should however be checked if their uncertainty is really below the limit. Nevertheless, one should mention such neglected effects in the uncertainty budget in order to document that they were not forgotten.

It is recommended that SSDLs consider adopting 0.1% as the lower limit for relative standard uncertainties in their uncertainty budget [8]. However, in the case of temperature and pressure measurements and positioning an ion chamber for measurements in air, relative standard uncertainties of less than 0.1% were found. To maintain clarity, the corresponding relative standard uncertainty values (although less than 0.1%) were retained in the tables given in the appendices. The relatively small value of uncertainty for temperature is based on the assumption that the same thermometer is used during the measurements with the reference and the user chambers, and the high resolution assumed for the thermometer. Similarly, the relatively small value of uncertainty for ion chamber positioning in air is due to the use of a fine focus telescope and the assumed ability of the operator to position the SSDL reference and the user chambers within 0.2 mm. Such practical details and capabilities will vary from one SSDL to another, and the uncertainty analysis must take the local conditions into account.

2.2.1. Air pressure and temperature

The air mass in the cavity of open (vented) ionization chambers is affected by variations of temperature and pressure. This is accounted for by applying a correction factor k_{TP} which converts the cavity air mass to the mass under reference conditions (usually 20ºC and 101.325 kPa).

For measuring the air pressure, a calibrated barometer should be used. The barometer should be located in the same room as the ionization chamber (or a room permanently connected with that of the chamber), preferably at about the same height as the chamber. The chamber should be properly ventilated and the pressure measured at (nearly) the same time as the chamber reading is obtained. Under these circumstances, the uncertainty stated in the calibration certificate of the barometer can be used as an estimate for the uncertainty of air pressure. As a general rule it should be possible to determine air pressure with a relative standard uncertainty of 0.1%.

For measuring the temperature, a calibrated thermometer should be used. It should be positioned near the ionization chamber (without disturbing the radiation field). If the ionization chamber is located in a phantom, the temperature in the phantom should be measured. There should be enough time to let the temperature of the chamber (and the phantom) equilibrate with the temperature of the environment. The temperature should be measured at (nearly) the same time as the chamber reading is obtained. Under these circumstances the uncertainty stated in the calibration certificate of the thermometer can be used as an estimate of the uncertainty of air temperature. If not all of these conditions are fulfilled one should think about how the temperature in the chamber cavity differs from the measured temperature. As a general rule it should be possible to determine the temperature of the air in the cavity with a standard uncertainty of 0.5 K.

The barometer and thermometer used should be calibrated at regular intervals, or at least checked against another barometer/thermometer for stability. If from these regular measurements effects of ‘ageing’ are observed (a drift) then this effect should be estimated and included in the uncertainty budget.

The same thermometer (or barometer) is used for measuring temperature (or pressure) in the first step of the calibration — measurement with the reference chamber, and in the second step — measurement with the user’s dosimeter. In this case, one should think about possible correlations between the uncertainties estimated in each step, e.g. the calibration of the thermometer cancels in the ratio of temperatures.

Besides its influence on the air mass in the cavity of open chambers, the ambient temperature might influence the response of a dosimeter by other means, e.g. by influencing the properties of some electronic components or by adsorption of air at the inner chamber wall for closed chambers. Usually these relationships are not known quantitatively, and therefore cannot be corrected in a general way. If there is evidence that such effects could play a role, then they should be investigated in more detail and included in the uncertainty budget.

2.2.2. Humidity

The portion of water vapour in the air — the humidity — influences several parameters which play a role during calibrations or dose measurements. Some of these parameters are listed below [9]:

- the air density, which is approximately 1% larger for dry air than for air saturated completely with water vapour,
- the average energy for producing an ion pair (*W* value), which is approximately 0.9% larger for dry air than for air saturated with water vapour,
- the mass stopping power for electrons, which changes by about 0.2%, depending on the radiation quality.

Fortunately these three effects normally compensate each other so that the overall effect of humidity is therefore much smaller than when considering only one of the parameters mentioned above. If the relative humidity varies between 20% and 70% at the usual operating temperatures, the response of an ionization chamber changes by less than 0.1% as a result of the combined effect of these three parameters. So usually no humidity correction is applied. However, one should include the effect of humidity in the uncertainty budget if the relative humidity during calibration is less than 20% or larger than 70%.

Under some special circumstances, changes of humidity might have some additional consequences which cannot be neglected, but must be analysed in more detail if necessary:

- At high values of relative humidity (80% and above) the leakage current of an ionization chamber might increase. This should be measured and either accounted for during the calibration procedure or included in the uncertainty budget.
- Some hygroscopic materials that are sometimes used as wall materials in ionization chambers, e.g. nylon or A150, are known to swell if used in a humid environment. As a result, the dimensions of an ionization chamber and its response might change by a certain percentage if it is used or stored in an environment where humidity varies significantly. The influence of these two effects cannot be estimated in a general way. If necessary it must be investigated separately and accounted for in the uncertainty budget.

2.3. Chamber specific influence quantities

2.3.1. Scale reading and resolution of the dosimeter

The uncertainty of the scale reading can be estimated by taking *n* measurements (at least 10), and calculating the mean value and the standard deviation of the mean (type A uncertainty).

If the observed variation of the readings is comparable to or less than the resolution of the instrument, e.g. if only one reading had been taken or if more than one reading had been taken but the observed variation was zero, then an additional contribution to the uncertainty due to limited resolution of the instrument should be included in the uncertainty budget.

If the device digitizes a continuous signal as distinct from a counter, the resolution uncertainty would be ± half the last count and is assumed to have a rectangular distribution, so that a reducing factor of 1.73 should be applied. The number of degrees of freedom in this case is infinite.

If the device is a counter, the resolution uncertainty is ± the last count (rectangular distribution).

For an analogue device, the degree to which the scale was read should be used as the resolution uncertainty — maybe ±1/10 of the smallest division. In this case, also the parallax error, i.e. uncertainty due to human bias, could be significant and should be considered in the uncertainty budget.

2.3.2. Linearity of response

The scale of a dosimeter is likely to be somewhat non-linear. This occurs more often for older type dosimeters with analogue displays than for modern dosimeters.

The dosimeter is usually calibrated with a certain reference scale reading which is typically either half or two thirds of full scale deflection. The non-linearity of response can be estimated by performing charge (or current) calibrations at a series of scale readings different from the reference scale reading. The measurements need only be carried out on a new instrument since scale linearity is unlikely to change with time.

The information about non-linearity is expressed as a correction factor which is unity at the reference scale reading. If the effect of non-linearity is less than 0.1% then it can safely be neglected; otherwise the reading should either be corrected for non-linearity or the effect should be included in the uncertainty budget.

2.3.3. Stabilization time

Before calibrations can be started, the ionization chamber and measuring assembly require a certain amount of time to warm up after having been switched on. Additionally the chamber needs some time for temperature stabilization after placing in a water phantom. Most ionization chambers also show a change in their response at the beginning of an irradiation[2]. Therefore, it is good practice to irradiate a chamber to a dose of several grays before taking the first reading for the calibration.

If possible the readings obtained during this ‘pre-calibration’ stage should be checked to ensure that the chamber and the electrometer have settled adequately.

2.3.4. Long term stability (ageing)

The response of the reference chamber might have changed slightly over time since the last calibration. This is more likely for very soft radiation than for medium and high energy radiation. It is therefore recommended [6] to check the stability of the reference chamber periodically by means of measurements in a gamma ray beam, or by using a stability check source.

The results of this constancy testing can be used to estimate the uncertainty due to a possible change of response of the reference chamber, which should be less than 0.5%.

2.3.5. Leakage current

The leakage current is that current generated by the complete measuring system in the absence of radiation. Leakage can also be radiation induced and chambers may show no leakage prior to irradiation and yet have a significant leakage after irradiation. Furthermore, the leakage current may depend on the humidity.

For therapy level calibrations (with a relative humidity less than 70%) the leakage current is usually less than 0.1% of the current measured due to irradiation and can be ignored, nevertheless it should be checked before and after each calibration. If it is found to be larger, an attempt should be made to reduce it to an acceptable level (e.g. by cleaning the connectors).

For protection level calibrations, a leakage current of up to 1% of the current measured in the calibration beam can occur at the lower end of the dose rate range. In this case either the

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[2] This is usually ascribed to the presence of electrons which are trapped in the body or on the surface of the insulator, and which change the field configuration in the collecting volume.

measurement current should be corrected for this leakage current or the leakage current should be included into the uncertainty budget, e.g. by assuming a uniform distribution of the leakage current with the measured value as its half-width.

2.3.6. Recombination loss

The ion recombination effect is generally very small for continuous (non-pulsed) beams at the output rates normally encountered from low and medium X ray sets and ^{60}Co γ ray units in an SSDL (usually less than 0.2%).

For calibrations at an SSDL, it is a common practice to ignore ion recombination and not to correct the calibration coefficient of a chamber for this effect. This is because it is expected that the user of the instrument is unlikely to account for such a small correction and, therefore, the measurement would be more accurate if the calibration coefficient was also left uncorrected.

It should be checked if the calibration coefficient for the reference instrument obtained from the PSDL is corrected for ion recombination. If this is the case, an appropriate correction should be applied during the measurement with the reference instrument [7] or the whole effect should be included in the uncertainty budget. As a general rule, calibrations done at the IAEA are not corrected for recombination.

2.3.7. Polarity effect

The response of some ionization chambers changes significantly when the sign of the polarizing potential is changed. This polarity effect often depends on the radiation energy, the magnitude of the polarizing potential, and the field size.

The reference instrument is usually calibrated at the PSDL or the IAEA with one magnitude and sign of polarizing potential, and the calibration coefficient refers to the same magnitude and sign. The polarity sign and magnitude are indicated in IAEA calibration certificates.

If the reference instrument is used at the SSDL in a beam with (nearly) the same quality as the beam at the PSDL, and with the same magnitude and sign of the polarizing potential, then the polarity effect is the same, and no correction is necessary. In this case, the uncertainty associated with the polarity effect can be neglected.

If the reference instrument is used in a beam with a quality or field size different from that during calibration or with another magnitude of the polarizing potential, then a possible change of the polarity effect should be checked by measuring the response of the reference chamber for both polarities. If the change of the polarity effect is larger than 0.1%, it should be included in the uncertainty budget.

2.4. Radiation field specific influence quantities

2.4.1. Geometrical properties

The spectral and angular distribution of radiation changes with field size; so does its quality. The influence of field size on chamber sensitivity can usually be neglected if it is close to the reference field size. It is necessary for the radiation beam to be of adequate uniformity over an area large enough for all calibration purposes. A uniformity of ±0.5% over the field area used to irradiate a single ionization chamber of not more than 1 cm^3 in volume may be regarded as being adequate[3]. The uniformity should be checked by using a small ionization chamber because film is ordinarily not adequate for checking uniformity to the accuracy needed for calibration. The uncertainty due to non-uniformity of the beam is usually neglected.

2.4.2. Beam quality

The radiation quality of an X ray beam is normally characterized by tube potential, total filtration and first half value layer. However, it is often not possible to match all these parameters of the PSDL, or the IAEA beam with the beam at the SSDL. The beam quality at the SSDL might differ slightly from the reference beam quality at the PSDL or the IAEA.

The response of a chamber designed for use in medium energy X rays, i.e. 70 kV to 300 kV, HVL from about 2 mm Al to 3 mm Cu — usually a cylindrical chamber — should not vary by more than ±2%[4] over this range of beam qualities. This is quite similar to a thin-window chamber, which is designed for measuring low energy X rays. Its response should not vary by more than ±2% in a range of half value layers from 0.05 mm to 2 mm Al, corresponding to X rays generated by tube potentials ranging from 12 kV to about 70 kV.

If possible the reference dosimeter should be calibrated at the PSDL or the IAEA with several beam qualities. From these calibrations the variation of response with beam quality can be estimated.

Gamma rays from ^{60}Co or ^{137}Cs therapy sources do not require a beam quality specifier other than the radionuclide. Although gamma ray spectra from different ^{60}Co or ^{137}Cs therapy sources might differ in the amount of low energy scattered photons, ionization chamber measurements are not expected to be influenced by such spectral differences by more than 0.1%. So this influence is usually neglected in the uncertainty budget.

2.4.3. Dose rate

In most ^{60}Co units the source is moved to a fully shielded position when radiation is switched ‘off’. The position of the source might therefore differ slightly each time the radiation is switched ‘on’, leading to a slight variation of the output rate under reference conditions.

This variation of output rate due to movement of the source can be estimated from a series of measurements of air kerma rate using the reference chamber, in which the chamber is left in place and the source is repeatedly exposed (at least 20 times), compared to a series of measurements in which the source is left in place (which will demonstrate the repeatability of chamber readings unaffected by source position uncertainty).

The standard deviation of the results obtained is dominated by the (type A) contribution from variations in source position, and represents an estimate for the uncertainty of the output rate due to a change in source position.

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[3] A poorer uniformity may have to be accepted for the much larger fields needed when calibrating the large volume chambers used in radiation protection.

[4] In diagnostic radiology, the acceptable variation is ±2.6 % [5].

2.5. Timing uncertainties

Irradiation time is of no importance when the calibration is performed tip-to-tip or by substitution using a monitor. However, the actual irradiation time needs to be known accurately when calibration by substitution is performed without a monitor, e.g. in a ^{60}Co beam. In that case, the parameter of importance is the ratio of the irradiation times of the reference chamber and the chamber to be calibrated.

Timers that have an internal oscillator (usually 1 kHz) are essentially independent of fluctuations in the mains frequency, and are very accurate. Timers that are essentially synchronous clocks that depend on the mains frequency are less reliable and can give rise to both random and systematic errors; stopwatches fall in this category. They should be checked regularly against a time signal of known accuracy. When using stopwatches, the uncertainty due to the observer (response time) should be kept in mind. It might play a significant role especially when using irradiation times of less than a few hundred seconds.

The most accurate method of timing a measurement is by the use of a dosimeter with an internal time base.

If the relative timing uncertainty is expected to be larger than 0.1%, it should be included in the uncertainty budget.

2.6. Chamber positioning

2.6.1. Distance to source or depth in phantom

During the calibration process it is assumed that the output rate is known at a certain distance from the source — the reference distance, or at a certain depth in the phantom — the reference depth. This is realized at the SSDL by means of a reference dosimeter, calibrated at a PSDL or the IAEA, performing measurements at the reference distance or the reference depth, respectively.

Because of uncertainties in positioning the reference point of the reference chamber, the measured output rate might differ from the ‘true’ output rate at the reference distance (or the reference depth). The uncertainty in positioning the reference chamber causes an uncertainty in the output rate used for the calibration of the user’s dosimeter.

A similar effect appears in the second step of the calibration process when the reference point of the instrument to be calibrated is positioned at the reference distance or depth. In this case the uncertainty of positioning causes the instrument to ‘measure’ an output rate different from that (expected to be known) at the reference distance or depth.

A contribution to the uncertainty of the output rate due to uncertainties in positioning the reference chamber and the chamber under calibration must therefore be considered.

In order to quantify the uncertainties of the output rate due to positioning uncertainties, the uncertainty of the distance or of the depth in phantom must be estimated. This uncertainty strongly depends on: a) the method and tool used for positioning the chamber, b) the instrument used for measuring the distance or depth, and c) the experience of the operator.

After estimating the uncertainty of distance or depth, it can be converted to an appropriate uncertainty of the output rate by using the measured dependency of the output rate on the distance or depth, respectively, e.g. a measured depth dose curve. The uncertainty of the output rate is simply the uncertainty of the distance (or depth) multiplied by the derivative of this curve at the reference distance (or depth).

The dependency of the output rate on the distance or depth should be measured — at least in the vicinity of the reference distance or depth. If no such experimental data are available, one could use tabulated data for depth dose curves or the inverse square law for measurements in air.

> *Example:*
>
> If the relative change of absorbed dose to water in a ^{60}Co beam in the reference depth (usually 5 cm) is 0.6%/mm [10] and the uncertainty of positioning the chamber is 0.5mm, then the relative uncertainty of the dose rate would be 0.3%.

2.6.2. Rotation and tilt of chamber

A chamber intended to measure medium or high energy X radiation is usually cylindrical or thimble shaped, and should normally be positioned so that its axis is perpendicular to the beam axis. Usually a mark is made on the stem of the chamber. The mark should be directed towards the radiation source.

A thin-window chamber designed to measure low energy X radiation usually has a collecting volume in the shape of a thin disc, and its axis should be positioned coaxial with the beam axis.

Deviations from this reference orientation, e.g. by rotating or tilting the reference chamber or the chamber to be calibrated, might cause similar effects as described in the beginning of the previous section on ‘Distance to source or depth in phantom’. However, the response of a well designed cylindrical or thimble chamber in a photon beam is usually insensitive to small changes in the orientation of the chamber, i.e. rotation and tilt. Usually, therefore, no contribution to uncertainty due to rotation and tilt of the chamber has to be included in the uncertainty budget.

If there is evidence that rotation or tilt of the chamber could change its response significantly, it should be included in the uncertainty budget. This can be checked by rotating or tilting the chamber in several steps and each time measuring its response. The response should not change by more than 0.1% when the chamber is rotated or tilted by the largest amount that may occur during routine calibrations.

2.6.3. Off-axis positioning

The reference points of both the reference chamber and the chamber under calibration should be positioned on the beam axis.[5]

The uncertainty related to the displacement of the reference point from the beam axis is usually negligible, provided that the field is of adequate uniformity. This can be checked by intentionally placing the chamber ‘off axis’ and measuring the change in response which should be less than 0.1% for a lateral displacement of about 1 mm.

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[5] This does not hold when calibrating tip-to-tip.

3. MEASUREMENT UNCERTAINTY

In this section the essential concepts are introduced in the context of a series of related worked examples. Further examples that were taken from dosimetry are presented in two appendices.

3.1. Errors and uncertainties

3.1.1. Introduction

At the outset, it is important to recognize that all measurements are wrong in that the measured value (the result) and the right answer (the 'true' value) are different. The difference between these two is the measurement error, which is meant in the sense of a discrepancy, rather than an avoidable mistake. Unfortunately the true value is never precisely known and, by the same token, neither is the measurement error. Instead, one makes a statement about measurement uncertainty, whose purpose may be summarized, as follows:

A statement of uncertainty indicates how large the measurement error might be:

For instance:

$$T = T_m \pm U_T \,, \qquad (7)$$

where

T is the true value,

T_m is the measured value, and

the range $\pm U_T$ is the measurement uncertainty.

This statement can be taken to mean that the true value, T, probably lies somewhere between T_m — U_T and $T_m + U_T$. On its own, even this statement is incomplete, since there is a need to quantify what the word probably means.

Example 1: Uncertainties need confidence limits

The ambient air pressure is measured with a barometer and found to be 102.30 kPa. This result might be reported either as:

102.30 ± 0.10 kPa, with 99% confidence limits (8)

or

102.30 ± 0.07 kPa, with 95% confidence limits (9)

Note that these are two different ways of reporting *the same* measurement. There is greater confidence in a measured value when it is quoted with a large uncertainty and less confidence when the same value is quoted with a smaller uncertainty. In general, there should be a decision on the confidence probability with which the measurement result should be stated. Limiting values that achieve this should be selected.

The lesson of this example is that an uncertainty statement is ambiguous unless the level of confidence is also given — e.g. by stating, as above, the probability that the 'true' value lies within the range quoted.

3.1.2. Types of uncertainty

One measurement alone is never enough. It is essential to repeat the procedure and average the results. Even the simplest measurements are subject to external factors, which may influence the result to a varying degree each time the measurement is made. The benefit of taking the mean value is that the variation in such influences will tend to be nullified. It is also possible to work out the standard deviation of the repeated measurements, and use this to estimate the standard deviation of their mean, known as the standard uncertainty of the mean.

Example 2: Statistical analysis of uncertainty

In fact, the air pressure measurement reported in **Example 1** is based on ten (10) readings taken over a short period of time.[6] For $i = 1, \dots, 10$ the values obtained were:

$$p_i = 102.26, 102.26, 102.31, 102.41, 102.33,$$

$$102.32, 102.13, 102.45, 102.35, 102.21 \text{ kPa} \tag{10}$$

Their arithmetic mean was calculated to be:

$$\overline{p} = \frac{1}{10}\sum_{i=1}^{10} p_i = 102.303 \text{ kPa} \tag{11}$$

This is then rounded off to two decimal places in the reported result. The standard deviation of the readings[7] was calculated from:

$$s(p) = \sqrt{\frac{1}{10-1}\sum_{i=1}^{10} (p_i - \overline{p})^2} = 0.09 \,\text{kPa} \tag{12}$$

If this process is repeated several times, a series of mean air pressure values would be generated, and these mean values would themselves show a small random variation. It is possible to estimate the standard deviation of these mean values using the following formula:

$$u_p \equiv \frac{1}{\sqrt{10}} s(p) = 0.03 \,\text{kPa} \tag{13}$$

This provides a measure of the width of the distribution of mean values that would be expected, and is called the standard uncertainty of the mean. If enough readings are taken per mean value, then the distribution of mean values will be roughly Gaussian, in which case the

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[6] It is assumed that the period of time is short enough that any real variation in air pressure is negligible compared to the observed fluctuation in the barometer readings.

[7] Strictly, this is an estimate of the standard deviation of the probability distribution from which the readings come.

standard uncertainty corresponds to a confidence probability of 68%. It will be shown later how to obtain limits for other confidence probabilities, such as those quoted in Example 1.

The point of this example is merely to provide a gentle reminder of the expressions for the standard deviation of a set of data, and for the standard deviation of the mean of the set. These and other statistical results are collated for convenience in Appendix 1.

3.1.3. Standard deviation and calibration uncertainty

Individual readings in the sequence (10) can be written in terms of deviations from their mean value:

$$p_i = \overline{p} + \delta p_i \tag{14}$$

and then the standard deviation (12) reduces to:

$$s(p) = s(\delta p) = \sqrt{\frac{1}{10-1}\sum_{i=1}^{10} (\delta p_i)^2} \tag{15}$$

This is an estimate of the standard deviation σ of the distribution from which the readings are taken: $\sigma(\delta p) = \sqrt{\overline{\delta p^2}}$. Each deviation δp_i may be thought of as the 'error' of that reading, so that the uncertainty of the mean (13) is based on the statistics of these errors.

An uncertainty component derived in this way, by a statistical analysis, is referred to as a type A estimate of uncertainty; any other estimate is of type B (considered below). This terminology was introduced in the ISO GUM [3]. The type A estimate is only a *component* of uncertainty, because one must allow for the possibility that the result of our procedure is influenced by other sources of uncertainty that may affect all readings in the same way.

Example 3: Calibration uncertainty

The pressure measurement is made, of course, with a barometer that has a traceable calibration. The certificate reports this calibration as a correction of 0.20 ± 0.10 kPa, for a coverage factor $k = 2$, with a confidence probability of 95%.

This calibration uncertainty contributes directly to the uncertainty in our measurement. Its contribution is one that does not vary when measurements are repeated and so it will not be exposed by a statistical analysis. It is an example of a type B uncertainty component.

Such an additive correction Δ_p to the raw pressure reading p_{raw} can easily be recast as a multiplicative factor N_p :

$$\begin{aligned} p &= p_{raw} + \Delta_p \\ &= p_{raw}\left(1 + \frac{\Delta_p}{p_{raw}}\right) \\ &\equiv p_{raw}\, N_p \end{aligned} \tag{16}$$

In this example, the factor $N_p = 1.0020$.

It is not uncommon for a calibration certificate to quote the uncertainty with a higher degree of confidence than is represented by the standard uncertainty u (for which the confidence level is 68%, if the value comes from a Gaussian distribution). Such an expanded uncertainty U for an elevated confidence level is obtained by multiplying the standard uncertainty by a coverage factor, k, which should also be quoted in the certificate. The standard uncertainty may be recovered on dividing U by k:

$$u = \frac{U}{k}. \tag{17}$$

In this example, the standard uncertainty is 0.05 kPa. Used in this way, k is also referred to as a reducing factor.

Example 4: Resolution uncertainty

Air temperature is measured with a mercury-in-glass thermometer having 0.2°C graduations. Five readings are taken over a short period. For $i = 1 \ldots 5$, the results, after applying the thermometer calibration correction, are:

$$T_i = 23.4, 23.4, 23.4, 23.4, 23.4°\text{C} \tag{18}$$

In this case, the statistics are trivial: the mean value is $\overline{T} = 23.4\,°\text{C}$ and the standard deviation $s(T)$ vanishes. Does this mean that the temperature uncertainty is zero? Of course not! Only the type A component of uncertainty is zero. Further thought indicates that, because the thermometer only has a resolution of 0.2°C, all that can be said is that the temperature definitely lies somewhere in the range 23.3 to 23.5°C. Therefore the measured temperature could be reported as:

$$T = 23.4 \pm 0.1°\text{C} \tag{19}$$

with 100% confidence. This uncertainty estimate was arrived at by a non-statistical argument and so, like the calibration uncertainty, it is an example of a type B uncertainty component.

The lesson of this example is that even if the type A uncertainty component vanishes there will always be a type B contribution coming from somewhere, whether the calibration coefficient, or limited resolution effects. In fact, one must make type B estimates for all sources of uncertainty, and type A estimates only where repetition allows the statistical approach.

3.2. The difference between type A and type B estimates

Whether or not a source of uncertainty leads to an estimate of type A or type B depends on the context. For example, in determining the calibration of an instrument, one will normally take repeated readings of both the instrument and the reference against which it is compared. The repeatability of instrument readings makes a contribution of type A towards its calibration uncertainty. However, when the calibrated instrument is subsequently used to make another measurement, its calibration coefficient makes a type B contribution to the uncertainty of that subsequent measurement. What was originally an uncertainty contribution of type A (the repeatability of measurements during calibration) has been transformed into one of type B. This context dependence shows that the distinction between type A and type B is not absolute. This is the reason why, in the ISO-GUM [3], all sources of uncertainty are combined together in reporting a measurement result.

3.3. Indirect measurements

The examples considered up to this point are particularly simple in that the quantities of interest, air pressure and temperature are directly accessible to measurement.[8] It is more commonly the case that the quantity of interest is only accessible indirectly, having a value that must be inferred from the results of other measurements. Likewise, its uncertainty must be obtained from the uncertainties of those other measurements in a way that is explored in the next example.

Example 5: Measuring air density

The air pressure and temperature measurements considered in the examples so far are really only the means to an end, which is a determination of the air density correction k_{TP} for an ionization measurement. The correction factor is the air density under standard conditions, i.e. a pressure 101.325 kPa and a temperature 20°C, divided by the density of the air in the collecting volume of the ion chamber at the time of the ionization measurement. Assuming that air obeys the ideal gas law, this ratio is given by the expression:

$$k_{TP} = \frac{101.325}{p} \frac{(273.15 + T)}{293.15} \tag{20}$$

where

p is pressure, given in kPa,

T is temperature, in °C, and

these both include calibration corrections such as Equation (16). Based on the data given in the earlier examples, the air density correction turns out to have a value 1.0020, and now the uncertainty of this correction will be derived.

The formula, Equation (20), represents the model for measuring air density correction. In any indirect measurement such a model or equation is an essential first step in the estimation of uncertainty. It follows from an analysis of the process by which the measurement is made. The model makes explicit the consequences of assumptions in this analysis. In this example, it is assumed that the ideal gas law holds for air in the ionization chamber. Not least, one may use the model to test the validity of these assumptions.

The first step is to express the 'true' values of temperature and pressure in terms of measured values and measurement errors:

$$\begin{aligned} p &= p_m + \delta p \\ T &= T_m + \delta T \end{aligned} \tag{21}$$

The uncertainties of the measured pressure and temperature given above indicate how large these errors might be. The next step is to obtain an expression for the 'true' value of the density correction:

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[8] This is not really obvious, at least in so far as the air temperature is concerned. After all, the thermometer reading relates to the temperature of the mercury in the glass bulb, and this is unlikely to respond quickly to changes in air temperature.

$$\begin{aligned} k_{TP} &= k_{TP,m} + \delta k_{TP} \\ &= k_{TP,m}\left(1 + \frac{\delta k_{TP}}{k_{TP,m}}\right) \end{aligned} \tag{22}$$

It is written here as the sum of its measured value and error. The uncertainty of this correction will be obtained in terms of the temperature and pressure uncertainties, by making explicit the relationship between their measurement errors.

The expressions given in Equation (21), are inserted into the model Equation (20):

$$k_{TP} = \frac{101.325}{p_m + \delta p}\frac{(273.15 + T_m + \delta T)}{293.15} \tag{23}$$

$$= \frac{101.325}{p_m}\left(1 + \frac{\delta p}{p_m}\right)^{-1}\left(\frac{273.15 + T_m}{293.15}\right)\left(1 + \frac{\delta T}{273.15 + T_m}\right)$$

$$\equiv k_{TP,m}\left(1 + \frac{\delta p}{p_m}\right)^{-1}\left(1 + \frac{\delta T}{273.15 + T_m}\right). \tag{24}$$

The measured value of the correction, $k_{TP,m}$, is defined by inserting measured values of temperature and pressure into the model Equation (20). It is assumed that the measurement errors are small, allowing first order approximations in Equation (24):

$$\left(1 + \frac{\delta p}{p_m}\right)^{-1} = 1 - \frac{\delta p}{p_m} + O(\delta)^2 \tag{25}$$

$$\left(1 - \frac{\delta p}{p_m}\right)\left(1 + \frac{\delta T}{273.15 + T_m}\right) = 1 - \frac{\delta p}{p_m} + \frac{\delta T}{273.15 + T_m} + O(\delta)^2 . \tag{26}$$

Then the ‘true’ air density correction can be written as:

$$k_{TP} = k_{TP,m}\left(1 - \frac{\delta p}{p_m} + \frac{\delta T}{273.15 + T_m}\right), \tag{27}$$

in terms of the measured value and its error:

$$\frac{\delta k_{TP}}{k_{TP,m}} = -\frac{\delta p}{p_m} + \frac{\delta T}{273.15 + T_m}. \tag{28}$$

The final step is to identify the uncertainty in the measured value as the standard deviation of this error:

$$\frac{\overline{\delta k_{TP}{}^2}}{k_{TP,m}{}^2} = \frac{\overline{\delta p^2}}{p_m{}^2} + \frac{\overline{\delta T^2}}{(273.15 + T_m)^2} - \frac{2\,\overline{\delta p\,\delta T}}{p_m(273.15 + T_m)}. \tag{29}$$

If the errors in pressure and temperature are uncorrelated, then the last term in Equation (29) will vanish, giving:

$$\frac{s(\delta k_{TP})}{k_{TP,m}} = \sqrt{\frac{s(\delta p)^2}{{p_m}^2} + \frac{s(\delta T)^2}{(273.15 + T_m)^2}} \,. \tag{30}$$

In terms of the standard uncertainties, it follows that:

$$\frac{u_{k_{TP}}}{k_{TP,m}} = \sqrt{\frac{{u_p}^2}{{p_m}^2} + \frac{{u_T}^2}{(273.15 + T_m)^2}} \,. \tag{31}$$

This is the result that was sought: the standard uncertainty of the air density correction has been expressed in terms of the standard uncertainties of pressure and temperature. The standard uncertainty of the pressure is given in Equation (13), and it only remains to evaluate the standard uncertainty of the temperature from Equation (19).

In Example 4, the temperature is equally likely to take any value in the range between 23.3 and 23.5°C. The standard deviation of a uniform distribution[9] with limits $\pm a$ is $a/\sqrt{3}$, so that $s(\delta T) = 0.06\ ^\circ\mathrm{C} = 0.25\,\%$ (relative to the measured temperature, 23.4°C).

3.4. Sensitivity coefficients

Equation (30) expresses the uncertainty of the air density correction as a weighted sum, in quadrature, of uncertainty components. It is often convenient to present the component uncertainties relative to measured values. Equation (30) is already in this form for the pressure, and would be for the temperature as well, if it had been expressed as absolute temperature in Kelvin, K, from the start. However, if one persists with degrees Celsius, then the uncertainty takes the form:

$$\frac{u_{k_{TP}}}{k_{TP,m}} = \sqrt{\frac{{u_p}^2}{{p_m}^2} + \frac{{T_m}^2}{(273.15 + T_m)^2} \frac{{u_T}^2}{{T_m}^2}} \,. \tag{32}$$

The squared relative uncertainty of the temperature in °C appears with a coefficient $c(T_m)^2$ where:

$$c(T) = \frac{T}{(273.15 + T)} \tag{33}$$

is called the sensitivity coefficient of the temperature in this measurement. In the present example, $c(T)$ is 0.08. This value will be used below.

3.5. Preliminary uncertainty budget

In summary, the steps in the process are, as follows.

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[9] This and some other results from statistics and probability are discussed in Appendix A.

- The output result of a measurement depends on various inputs, termed influence quantities. Some of these are measured, and some are not. The relationship between the inputs and output is written down and defines the measurement model. In the case of the air density correction, the model is given by Equation (20):

$$k_{TP} = \frac{101.325}{p} \frac{(273.15 + T)}{293.15} \tag{34}$$

- For each influence quantity, the possible sources of uncertainty are considered. type B estimates of uncertainty are made for them all, and type A estimates are made where possible.
- The contribution of each source of uncertainty to the standard uncertainty of the measurement result is obtained using sensitivity coefficients derived from the model equation.
- These standard uncertainty contributions are summed up in quadrature.

This calculation should be presented in a table, sometimes referred to as an uncertainty budget. The various sources of uncertainty are presented, one per row, and grouped under the input quantity they affect. The value of each input quantity is given and, for each source of uncertainty, sufficient detail is given to derive a standard uncertainty. Usually this will be derived from an expanded uncertainty by the application of a reducing factor *k*. Sometimes the probability distribution ascribed to each source of uncertainty will be specified. Sensitivity coefficients are given and, in the last column, the product of the standard uncertainty and the sensitivity coefficient. This product is the contribution from each source towards the standard uncertainty of the quantity being measured. The sum in quadrature of these uncertainty components is given in the last row. The result is given in

TABLE 1, where the sum in quadrature is worked out in an extra column, which would not normally be shown. The whole calculation is conveniently performed in a spreadsheet, so that intermediate results are retained with full (usually double) precision, which is greater than the values displayed in the table. In this way, the effect of rounding is postponed until the last step.

The flow of the calculation in the table is from left to right for each influence quantity, from top to bottom in the summation, and from right to left for the measurement result.

In preparing an uncertainty budget it is essential to make clear the basis of all estimates of the component uncertainties, preferably by the addition of a note which describes how each source of uncertainty has been taken into account. When a source of uncertainty is considered, but the contribution is judged to be negligible, it is important to record the basis of this judgement, even though the contribution itself may be omitted from the table.

The combination of component uncertainties in quadrature is not always valid. Exceptions occur when the relative uncertainties are not small, when one or two component uncertainties dominate the combined uncertainty, or when their probability distributions are asymmetric. None of these cases normally arises in dosimetry.

TABLE 1. UNCERTAINTY BUDGET FOR EXAMPLE 5 MEASURING AIR DENSITY (PRELIMINARY BUDGET)

Note	Quantity, Source of uncertainty	Value of quantity	Expanded uncertainty	Uncertainty type	Confidence Level	Coverage factor	Standard uncertainty	Sensitivity coefficient	Uncertainty component	Working
			U_i			k	u_i	c_i	$\lvert c_i u_i \rvert$	$\lvert c_i u_i \rvert^2$
Pressure measurement		102.30 kPa								
1	Barometer calibration		0.1 kPa	Type B	95%	2	0.05%	1.00	0.05%	2.4×10^{-7}
2	Uncertainty of mean reading		0.03 kPa	Type A	68%	1	0.03%	1.00	0.03%	8.6×10^{-8}
3	Barometer resolution		0.005 kPa	Type B	100%	1.73	0.003%	0.32	0.001%	8.0×10^{-11}
Temperature measurement		23.4 °C								
4	Uncertainty of mean reading		-	Type A	-	-	-	-	-	-
5	Thermometer calibration		0.5 °C	Type B	95%	2	1.07%	0.08	0.08%	7.1×10^{-7}
6	Thermometer resolution		0.1 °C	Type B	100%	1.73	0.25%	0.08	0.02%	3.8×10^{-8}
Air density correction		1.0020								
7	Combined uncertainty			Combined			0.10%			1.07×10^{-6}

Notes on the preliminary uncertainty budget (see Table 1):

1. The barometer has a certificate in which the (additive) calibration is given as 0.2 kPa ± 0.1 kPa, where the uncertainty is based on a coverage factor $k = 2$, stated to correspond approximately to a confidence level of 95%. Each pressure reading is affected in the same way and so, in the present context, calibration makes a contribution to the uncertainty of type B. The reducing factor k is used to obtain the standard uncertainty (0.05 kPa), which is expressed, relative to the measured value, as 0.05%.

2. The mean pressure reading has an uncertainty that makes a type A contribution, as worked out in Example 2 above.

3. The resolution of the barometer, which reads to the nearest 0.01 kPa, is such that it makes a negligible contribution to the uncertainty, unlike the thermometer (note 6 below). The sensitivity coefficient for this contribution is equal to $1/\sqrt{N}$, where N is the number of pressure readings taken, for reasons which are discussed below in the section on correlations in uncertainty.

4. In this example the mean value of temperature readings has vanishing type A uncertainty.

5. The thermometer's (additive) calibration, 0.6°C ± 0.5°C (based on a coverage factor $k = 2$, and stated to correspond approximately to a 95% confidence level), is handled similarly at the barometer calibration. It is reduced to a standard uncertainty, relative to the measured value in °C, and the required sensitivity coefficient obtained.

6. The effect of limited resolution is not negligible in the temperature measurement, and an uncertainty contribution of type B arises. In this case the coverage factor is $\sqrt{3}$.

7. Each uncertainty contribution is squared (the working is shown here in the last column, but would normally be omitted) and summed to produce the squared relative uncertainty of the measurement result (in the same column). The standard uncertainty (0.10%) of the result follows on taking the square root and rounding appropriately.

3.6. Higher levels of confidence — the coverage factor k

The standard uncertainty obtained for the air density measurement corresponds to a confidence probability of about 68%, at least if the measured value has a Gaussian distribution. But what if an uncertainty is required with a greater degree of confidence, i.e. an expanded uncertainty U: how is the value of k to be determined?

If the measured quantity is known to have a Gaussian distribution, and if there is no doubt about the value of its standard uncertainty u, then a 95% confidence interval would be obtained by taking $k = 1.96$, normally rounded to 2.

If, on the other hand, there is some doubt about the value of u, then a little thought shows that taking $k = 2$ would produce an interval for which the level of confidence is less than 95%. A value of k that is somewhat larger than 2 is required to compensate for this doubt. The question emerging here is: What is the uncertainty of the uncertainty?

The answer to this question, about the combined standard uncertainty in the bottom line of the uncertainty budget, depends on the reliability of each of the component uncertainties appearing in the lines above.

3.7. Degrees of freedom

In general, an uncertainty is fully characterized by specifying its probability distribution. The model equation, which defines the measurement result in terms of the various influence quantities, can be used directly to derive the probability distribution of the output from the probability distributions of all the inputs. In practice this approach would, in general, require a numerical solution using Monte Carlo sampling. Such sophistication is not generally required provided that any dominant sources of uncertainty can be assumed to have a Gaussian distribution, and provided also that the relative uncertainty from each source is small, as is the case in dosimetry.

The ISO GUM [3] approach may be motivated by considering the reliability of a type A estimate of uncertainty. The larger the sample of data analysed statistically, the more reliable is the resulting uncertainty estimate. In Example 2, the uncertainty given in Equation (7) arises from an average of N contributions of comparable magnitude, and so may be expected itself to have a relative standard deviation of the order $1/\sqrt{N}$.

This number, or rather $N - 1$, is used as an indication of the reliability of the uncertainty value itself, referred to as the 'effective number of degrees of freedom' ν_{eff}. The reliability of the combined uncertainty will follow, provided that an effective number of degrees of freedom can also be assigned to type B uncertainty estimates. Bentley [11] advocates a pragmatic approach. It turns out that, once the degrees of freedom for uncertainty contributions from all sources have been combined, the effective number of degrees of freedom of the result is insensitive, in nearly all cases, to the precise values of ν_{eff} for most of the components. Provided that type B contributions do not dominate the combined uncertainty, it is sufficient to distinguish between uncertainty estimates that are judged to be 'excellent', 'good', 'reasonable' or 'rough', to assign them effective degrees of freedom equal to 100, 30, 10 and 3, respectively, and to make no finer distinction than this.

Equipped with such a scheme for assigning effective degrees of freedom to each uncertainty component, the next step is to consider how these effective degrees of freedom may be combined.

3.8. Combining effective degrees of freedom

The uncertainty contributed by each influence quantity is $c_i u_i$, and these are summed in quadrature to give the combined uncertainty, u, according to

$$u^2 = \sum_i (c_i u_i)^2 \ . \qquad (35)$$

In the example, Equation (35) takes this form with the index *i* corresponding to the two quantities pressure *p* and temperature *T*. At risk of confusion, but just for a moment, it is possible to regard Equation (3) as a measurement model in its own right, and ask how the uncertainty of the result (i.e. of the left-hand-side) is determined by the uncertainties of the terms in the sum on the right. Each term, as suggested above, has an uncertainty proportional to $\sqrt{\frac{1}{\nu_i}}$, where ν_i is the effective number of degrees of freedom for that source of uncertainty. In fact the uncertainty in $(c_i u_i)^2$ is estimated to be equal to $(c_i u_i)^2 / \sqrt{\nu_i}$.

Since uncertainties add in quadrature, this implies that the squared uncertainty of the RHS in Equation (35) is $\sum_i (c_i u_i)^4 / \nu_i$. This sum may be identified with the squared uncertainty of the LHS, which is written in the same form, u^4 / ν_{eff} :

$$u^4 / \nu_{eff} = \sum_i (c_i u_i)^4 / \nu_i \, , \qquad (36)$$

from which it follows that the effective number of degrees of freedom of the combined uncertainty is given by:

$$\nu_{eff} = u^4 / \left(\sum_i (c_i u_i)^4 / \nu_i \right), \qquad (37)$$

a formula due to Welch and Satterthwaite [3].

3.9. Final uncertainty budget

The complete process leading to the final uncertainty consists of the following steps:

- The output result of a measurement depends on various inputs, termed influence quantities. Some of these are measured, and some are not. The relationship between the inputs and output is written down and defines the measurement model equation.
- For each influence quantity, the possible sources of uncertainty are considered. type B estimates of uncertainty are made for them all, and type A estimates are made where possible.
- The contribution of each source of uncertainty to the standard uncertainty of the measurement result is obtained using sensitivity coefficients derived from the model equation.
- These standard uncertainty contributions are summed in quadrature to obtain the combined standard uncertainty for the measurement.
- Effective degrees of freedom are assigned to each source of

uncertainty, and combined using the Welch–Satterthwaite formula, Equation (30).

- A combined and expanded uncertainty, for the required confidence probability, is obtained on multiplying the standard uncertainty by a coverage factor k, taken from a Student's t-distribution with the required number of effective degrees of freedom.

The initial version of the uncertainty budget can now be extended to include a column which records the effective number of degrees of freedom for each source of uncertainty. The calculation leading to the effective number of degrees of freedom for the measurement result, ν_{eff}, may conveniently be carried out in further columns to the right, as in Table 2. The use of the t-distribution is described in more detail in Appendix A. As before, the right hand part of the table would not normally be shown and is given here only by way of clarification.

3.10. Rounding

In the final expression of measurement uncertainty, the value and its uncertainty should be rounded off to the same precision, for example:

> The air density correction was measured to be $k_{TP} = 1.0019 \pm 0.0021$, with a confidence probability of 95%. The coverage factor for the interval is $k = 2$.

In many cases, one significant digit in the uncertainty is enough. Only where the leading digit is a 1 or 2 is there a strong case for greater precision in the uncertainty. Rounding should only be applied to the final result and its uncertainty, not at intermediate steps in the analysis.

TABLE 2. UNCERTAINTY BUDGET FOR EXAMPLE 5 MEASURING AIR DENSITY (FINAL VERSION)

Note	Quantity, Source of uncertainty	Value of quantity	Expanded uncertainty	Uncertainty type	Confidence level	Coverage factor	Standard uncertainty	Sensitivity coefficient	Uncertainty component	Effective degrees of freedom	Working			
			U_i			k	u_i	c_i	$\lvert c_i u_i \rvert$	ν_{eff}	$\lvert c_i u_i \rvert^2$	$1/\nu_{eff}$	$\lvert c_i u_i \rvert^4$	$\lvert c_i u_i \rvert^4 / \nu_{ef}$
Pressure measurement		102.30 kPa												
1	Barometer calibration		0.1 kPa	Type B	95%	2	0.05%	1.00	0.05%	30	2.4×10^{-7}	3.3×10^{-2}	5.7×10^{-14}	1.9×10^{-15}
2	Uncertainty of mean reading		0.03 kPa	Type A	68%	1	0.03%	1.00	0.03%	9	8.6×10^{-8}	1.1×10^{-1}	7.4×10^{-15}	8.2×10^{-16}
3	Barometer resolution		0.005 kPa	Type B	100%	1.73	0.003%	0.32	0.001%	100	8.0×10^{-11}	1.0×10^{-2}	6.3×10^{-21}	6.3×10^{-23}
Temperature measurement		23.4 °C												
4	Uncertainty of mean reading		-	Type A	-	-	-	-	-	-	-	-	-	-
5	Thermometer calibration		0.5 °C	Type B	95%	2	1.07%	0.08	0.08%	30	7.1×10^{-7}	3.3×10^{-2}	5.1×10^{-13}	1.7×10^{-14}
6	Thermometer resolution		0.1 °C	Type B	100%	1.73	0.25%	0.08	0.02%	100	3.8×10^{-8}	1.0×10^{-2}	1.4×10^{-15}	1.4×10^{-17}
Air density correction		1.0019												
7	Combined uncertainty		0.21%	Combined	95%	2.0	0.10%			59	1.07×10^{-6}	1.7×10^{-2}	1.1×10^{-12}	2.0×10^{-14}

Notes on the final uncertainty budget (see Table 2)

All of the remarks made in the notes to Table 1 apply to the final version of the uncertainty budget in Table 2. In addition:

1. If the number of degrees of freedom is stated explicitly in the calibration certificate, that value should be used. If it is not one can, in principle, take the confidence level (95%) and coverage factor ($k = 2$) at face value and work backwards, using the tables of Student's t-distribution, and infer a value of 60. Here the same approach is adopted as for the other type B contributions, and the value 30 is chosen, which represents a 'good' estimate of uncertainty.[10]
2. The effective number of degrees of freedom for the uncertainty on the mean of N readings, a type A estimate, is just $N-1$, in this case equal to 9.
3. The barometer resolution is not in doubt, thus the uncertainty estimate is 'excellent'. Any increase in ν_{eff} for this contribution beyond the conventional value of 100 would have a completely negligible effect on the final uncertainty and its number of degrees of freedom.
4. The same remarks apply here as for note (1) above.
5. The same remarks apply as for note (3), except that here the contribution is merely small, rather than completely negligible.
6. The formula for the effective number of degrees of freedom will in general yield a non-integer value. It must be rounded off to an integer in order to use the t-distribution, either tabulated as in Appendix A, or using a formula in a spreadsheet program.[11] This is the only point in the calculation where it is necessary to round off an intermediate result.

3.11. Correlations in uncertainty

One should always consider carefully whether correlations may be present in a measurement, because the rule that component uncertainties should be summed up in quadrature assumes that those components are uncorrelated. In Example 5, for instance, it was necessary in passing from Equation (23) to Equation (24) to assume that the measurement errors in temperature and pressure were uncorrelated so that the mean value of their product could be set to zero. Otherwise, correlated uncertainty components contribute additional terms (which can be positive or negative) to the combined uncertainty of the measurement, which may be increased or decreased as a result.

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[10] The implied small doubt about the information in the certificate has no particular foundation and should not be taken too seriously.

[11] In the present example, the number of degrees of freedom came out close to 60 purely by accident. This is the number of degrees of freedom for which a coverage probability of 95% makes the t_p value closest to the commonly adopted value of 2.

If the same instrument is used to measure more than one of the input quantities, then the sources of uncertainty associated with that instrument will appear more than once in the uncertainty budget, and may lead to correlated contributions.

For instance, the calibration, by substitution, of a user's ionization chamber against a reference chamber involves two air density corrections, one for each ionization measurement. This is the case in each of the examples presented in Appendix II. The measurement model, for example Equation (I.9), involves a ratio of the 'user' and 'reference' pressures multiplied by a ratio of the 'reference' and 'user' absolute temperatures. The following discussion of the uncertainty in absolute temperature applies in a similar way to the uncertainty in pressure.

The thermometer calibration, expressed as a coefficient multiplying the raw absolute temperature reading, appears in both the numerator and denominator of the temperature ratio. Assuming that the two temperatures are not vastly different, this calibration coefficient cancels in the ratio. Any error in the thermometer calibration would also affect numerator and denominator in the same way, leaving the ratio of absolute temperatures unaffected.

However, not all of the effects of sources of uncertainty associated with the thermometer cancel. The effect of the resolution uncertainty on one reading may be different from the effect on the other reading. In the examples, it has been assumed that there is no correlation between the effects of thermometer resolution on the air density corrections of the user and reference chambers.

As a special case of repeated use of an instrument, one may consider the series of N readings from which the mean and standard deviation are calculated. Each of these readings is subject to a resolution uncertainty and, in principle, these N contributions must be allowed for in arriving at a combined uncertainty of the mean. There are two extreme cases. If the readings are identical, then we may assume that the resolution uncertainties for the different readings are completely correlated, as in the temperature measurement considered in Example 4. In this case, the N contributions add linearly and contribute the same relative uncertainty to the mean as to any one reading. At the opposite extreme, the readings may have a standard deviation that is much greater than the instrument resolution, as in the pressure measurement considered in Example 2. In this case, the resolution uncertainties for the different readings will be uncorrelated, and should be added in quadrature. The resolution uncertainty of the mean will therefore be reduced by a factor $1/\sqrt{N}$ compared to the resolution uncertainty of any one reading.

3.12. Practical use of uncertainty budgets

A useful analogy exists between an uncertainty budget and a financial budget: it is an allowance to be used, but not necessarily to be used up, and it should definitely not be exceeded. It is not essential to go through a fresh uncertainty analysis every time a routine measurement is performed. Rather, uncertainties should be analysed in the way described here when the measurement procedure is drawn up.[12] As experience is gained in following the procedure and as measurement data accumulate, it becomes possible, and even preferable, to replace some of the component uncertainty estimates (ones based on actual readings taken during a particular measurement) by typical, if conservative, values based on experience with many instruments of the type in use. This experience should be incorporated into acceptance

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[12] The uncertainty analysis should also be reviewed whenever the measurement procedure is changed.

criteria (tolerances) for the measurements which, if they are not met, will indicate that a problem exists. These criteria may be refined as further data are acquired.

Note that where a calibration laboratory achieves accreditation, this will be for the provision of a service to a specified uncertainty. It would be a non-compliance for an accredited laboratory to issue a calibration certificate with an uncertainty which is better than that specified in the accreditation. In the terms of the analogy above one should in such a case, rather than issue a certificate with an uncertainty which is ‘too good’, report a conventional value for the uncertainty (for which the service has been accredited) knowing that on this occasion at least, the budget has not been exhausted.

APPENDIX I
STATISTICS FORMULAS

A set of data, for example repeated readings of an instrument:

$$x_1, x_2, ..., x_n \tag{I.1}$$

has an average, or arithmetic mean value given by

$$m(x) = \bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i \tag{I.2}$$

The fact that the readings vary indicates the presence of randomly varying influences, and the point of taking the mean value is to estimate what the reading would be in the absence of these random variations. The sample mean, given by Equation (I.2) is an estimator for the mean value of the probability distribution *p(x)* associated with the reading *x*.

In a similar way, the standard deviation of the probability distribution *p(x)* can be estimated using the expression

$$s(x) = \sqrt{\frac{1}{N-1}\sum_i (x_i - \bar{x})^2} \tag{I.3}$$

Note that this expression is not defined in the case $N = 1$, for the simple reason that no conclusion can be drawn about the width of the distribution *p(x)* when only one reading is taken. That is why one divides by $N-1$ rather than by N.

Of course, the estimator (I.2) is not completely reliable: the actual value obtained is still subject to those randomly varying influences, though their effect on the mean is smaller than on individual readings. An estimate of the uncertainty of (I.2) is the standard deviation of the mean, which is best referred to as the standard uncertainty of the mean [to avoid confusion with the standard deviation (I.3)]:

$$u(x) = \frac{s(x)}{\sqrt{N}} = \sqrt{\frac{1}{N(N-1)}\sum_i (x_i - \bar{x})^2} \tag{I.4}$$

The number of degrees of freedom associated with this standard uncertainty is

$$\nu_x = N - 1 \tag{I.5}$$

An indirect measurement, whose result depends on a set of different input quantities (such as temperatures, pressures, etc.) has a standard uncertainty *u* which can be written as a sum in quadrature, over the input quantities *x*, of their standard uncertainties u_x:

$$u^2 = \sum_x (c_x u_x)^2 \tag{I.6}$$

in which each component uncertainty is multiplied by a sensitivity coefficient c_x.

The effective number of degrees of freedom associated with this combined standard uncertainty is given by the Welch-Satterthwaite formula [3]:

$$\nu_{eff} = u^4 / (\sum_x (c_x u_x)^4 / \nu_x) \tag{I.7}$$

The Equations (I.6) and (I.7) must be generalized in the case that the component uncertainties are correlated [3].

I.1. Probability distributions

The essential features of three probability distributions relevant to the analysis of uncertainty are summarized below.

I.1.1. Gaussian

A Gaussian distribution is appropriate for instrument readings subject to small random errors, setting aside the effects of limited resolution. It is specified in terms of its mean μ and standard deviation σ. The probability density:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} \right) \tag{I.8}$$

is shown, for the case $\mu = 0$, $\sigma = 1$ in Fig. I.1.

I.1.2. Uniform

When limited instrument resolution has a dominant effect, the associated uncertainty has a uniform, or rectangular distribution, defined in terms of its limiting (100% confidence) values, say $\mu - a$ and: $\mu + a$

$$p(x) = \frac{1}{2a} \text{ for } |x-\mu| < a \tag{I.9}$$

otherwise:

$$p(x) = 0 \tag{I.10}$$

The summation in quadrature of uncertainty contributions requires the standard uncertainty, i.e. the standard deviation σ of the distribution. This is calculated from the definition, as:

$$\sigma(x)^2 = \overline{(x - x^2)} = \frac{1}{2a}\int_{\mu-a}^{\mu+a} (x-\mu)^2 dx = \frac{a^2}{3} \tag{I.11}$$

so that $\sigma(x) = \frac{a}{\sqrt{3}}$. The value $\sqrt{3} \approx 1.73$ is used in the examples given in Appendices II and III.

I.1.3. Student's t-distribution

Finally, the t-distribution is needed to determine the expanded uncertainty of a measurement, corresponding to the required confidence probability (often 95%), as follows. Given N instrument readings x_i, which are assumed to come from a Gaussian distribution (whose mean μ and standard deviation σ are unknown), the sample mean $m(x)$ and standard deviation $s(x)$ are calculated. The mean μ can be asserted to lie in the range $m(x) \pm t_p \frac{s(x)}{\sqrt{N}}$, with a confidence probability P, where $\pm t_p$ is the width of the Student's t-distribution, on $N-1$ degrees of freedom, that includes the probability P. In the context of uncertainty analysis, this width t_p is referred to as the coverage factor k.

The shape of the t-distribution is similar to that of a Gaussian distribution, especially when the number of degrees of freedom is large. Figure I.1 also shows the probability density of a Student's t-distribution on 3 degrees of freedom for comparison with the Gaussian distribution.

Accreditation bodies commonly require that calibration certificates report uncertainty with a confidence interval of 95%: Table I.1 gives the coverage factor for this probability, for various numbers of degrees of freedom.

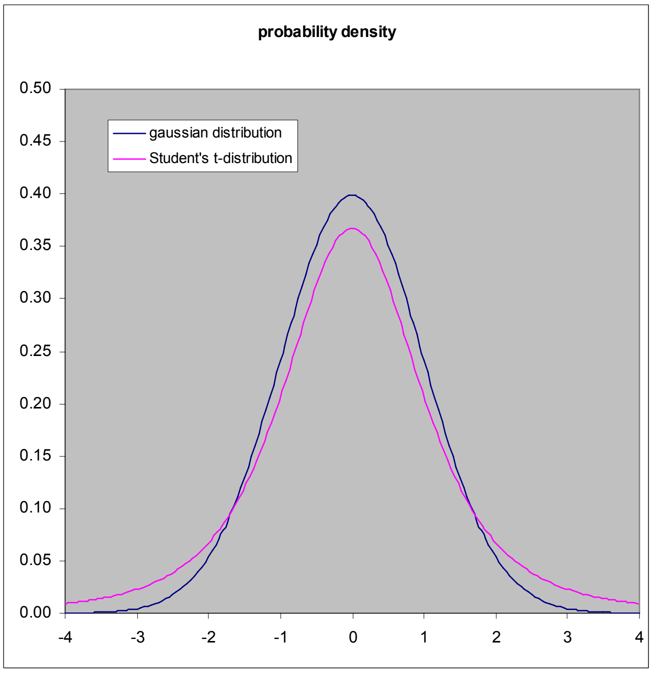

Figure I.1. Probability densities of the Gaussian distribution with mean μ = 0 and standard deviation σ = 1 and Student's t distribution on 3 degrees of freedom.

TABLE I.1. COVERAGE FACTORS FOR A CONFIDENCE INTERVAL OF 95% FOR DIFFERENT DEGREES OF FREEDOM

Number of degrees of freedom	Coverage factor k for 95% confidence probability (Student's t value $t_p = 95\%$)
∞	1.96
100	1.98
30	2.04
20	2.09
15	2.13
10	2.23
7	2.36
5	2.57
4	2.78
3	3.18
2	4.30
1	12.71

APPENDIX II
EXAMPLE OF UNCERTAINTY ANALYSIS FOR AIR KERMA CALIBRATION FOR ^{60}CO GAMMA BEAM

The overall aim of Appendices II and III is to help the user in the analysis and reporting of measurement uncertainty. The essential uncertainty components are introduced in the context of a measurement procedure and a series of numerical values. An example is illustrated in Tables II.1 and III.1. Although efforts have been made to provide realistic values based on experience, readers should not use any values given in this example unless they are consistent with their own experience.

Values are provided in these examples only for the purpose of illustration and must, of course, be replaced by values arising in practice.

II.1. Outline measurement procedure

For this example it is assumed that the SSDL reference chamber and measuring assembly have been calibrated by the IAEA in terms of air kerma within the last three years, and checked for stability by making weekly measurements with a ^{90}Sr stability check source. The ^{60}Co unit is of the type where the source moves to the fully shielded position between measurements with different chambers. The collimator setting is fixed throughout the calibration procedure. The reference and user chambers are set up one after the other in air at 1 m from the nominal source position to within 0.1 mm, and their positions checked using a telescope. The chamber orientation is checked visually, aligning the mark on the stem towards the source. In all measurements with both the reference and user ionization chambers, the chamber is allowed to settle for at least 15 minutes in the measurement position (temperature stabilization), the chamber polarizing potential is verified and the chamber exposed to an air kerma of 4 Gy before the first reading. The leakage and radiation-induced leakage currents are required to be less than 0.1% of the current during measurements. Ten readings are taken, keeping the source exposed. Air density corrections to the readings with the reference and user instruments are based on measurements with the same barometer and thermometer. Ambient humidity is measured with a traceably calibrated humidity meter: no correction for humidity is made provided that it lies within the range 20% to 70% relative humidity, over which the humidity correction is constant to within 0.1%. Ionization measurements are subject to acceptance criteria on the standard deviation of the readings and on their rate of drift. The standard deviation must be less than 0.1% for the reference instrument and less than 0.2% for the user instrument; the mean rate of drift must be less than 0.05% over ten readings.

- Step 1. Measurements are made with the reference standard dosimeter to determine air kerma rate under reference conditions at the time of calibration.
- Step 2. Measurements are made with the user instrument following [6], for calibration by substitution.

II.2. Model equation

In the **first step** the air kerma rate is determined with the reference instrument according to the equation:

$$\dot{K}_a = N_K^{\mathrm{ref}} M_{corr}^{\mathrm{ref}} k_{Q,Q_0} \qquad \text{(II.1)}$$

In the **second step** the calibration coefficient of the user's instrument is obtained according to:

$$N_K^{\text{user}} = \frac{\dot{K}_a \cdot k_{source}}{M_{corr}^{\text{user}}} \qquad \text{(II.2)}$$

where

N_K^{ref}	is the air kerma calibration coefficient of the reference instrument, determined at the IAEA, in the reference quality Q_0,
N_K^{user}	is the air kerma calibration coefficient of the user instrument in the calibration quality Q,
M_{corr}^{ch}	is the mean reading, after correction for influence quantities, obtained with either the reference chamber (ch = ref), or the user chamber (ch = user),
$\dot{K}_a$	is the air kerma rate determined with the reference instrument,
k_{source}	is a correction for the effect of a change in source position,
k_{Q,Q_0}	is a correction for the effect of any difference between the qualities of the beams at the IAEA and at the SSDL. This factor is unity since both are ^{60}Co beams.

The corrected chamber reading is given by

$$M_{corr}^{\text{ch}} = M_{raw}^{\text{ch}} k_{TP}^{\text{ch}} k_{dist}^{\text{ch}} k_{stab}^{\text{ch}} k_{other}^{\text{ch}} \qquad \text{(II.3)}$$

where

ch = ref or ch = user, and

M_{raw}^{ch}	is the mean value of the readings taken after the instrument settled,
k_{TP}^{ch}	is a factor to correct for departure of air density from reference conditions,
k_{dist}^{ch}	is a factor to correct for deviation of chamber position (distance from the source) from the reference position,
k_{stab}^{ch}	is a factor to correct for the stability of the SSDL reference instrument (only for ch = ref),
k_{other}^{ch}	is a factor including all the corrections whose uncertainties are too small to consider individually in the uncertainty budget (because they are estimated to be much less than 0.1%).

k_{other}^{ch} is given by:

$$k_{other}^{\mathrm{ch}} = k_{elec}^{\mathrm{ch}} k_{lin}^{\mathrm{ch}} k_{s}^{\mathrm{ch}} k_{leak}^{\mathrm{ch}} k_{h}^{\mathrm{ch}} k_{pol}^{\mathrm{ch}} k_{rot}^{\mathrm{ch}} k_{fs}^{\mathrm{ch}} k_{\mathrm{hom}}^{\mathrm{ch}} \tag{II.4}$$

where

k_{elec}^{ch}	is the calibration coefficient of the measuring assembly, in case the chamber and measuring assembly are calibrated separately,
k_{lin}^{ch}	is a factor to correct for non-linearity of the measuring assembly sensitivity,
k_{s}^{ch}	is a factor to correct for the lack of saturation due to ion recombination,
k_{leak}^{ch}	is a factor to correct for leakage current (possibly converted from an additive correction),
k_{h}^{ch}	is a factor to correct for any departure of humidity from the reference condition, 50% relative humidity,
k_{pol}^{ch}	is a factor to correct for any departure of the chamber polarizing voltage from its value at calibration,
k_{rot}^{ch}	is a factor to correct for any misalignment (rotation, tilt) of the chamber in use,
k_{fs}^{ch}	is a factor to correct for departure of field size from reference condition,
$k_{\mathrm{hom}}^{\mathrm{ch}}$	is a factor to correct for radial non-homogeneity of the beam.

Combining Equations (II.1), (II.2) and (II.3) gives

$$N_K^{\mathrm{user}} = k_{source} N_K^{\mathrm{ref}} k_{stab}^{\mathrm{ref}} k_{Q,Q_0} \frac{M_{raw}^{\mathrm{ref}}}{M_{raw}^{\mathrm{user}}} \frac{k_{TP}^{\mathrm{ref}}}{k_{TP}^{\mathrm{user}}} \frac{k_{dist}^{\mathrm{ref}}}{k_{dist}^{\mathrm{user}}} \frac{k_{other}^{\mathrm{ref}}}{k_{other}^{\mathrm{user}}} \tag{II.5}$$

Replacing k_{TP} by the explicit expression in terms of temperature and pressure one obtains

$$N_K^{\mathrm{user}} = k_{source} N_K^{\mathrm{ref}} k_{stab}^{\mathrm{ref}} k_{Q,Q_0} \frac{M_{raw}^{\mathrm{ref}}}{M_{raw}^{\mathrm{user}}} \left(\frac{273.15 + T^{\mathrm{ref}}}{273.15 + T^{\mathrm{user}}} \right) \left(\frac{p^{\mathrm{user}}}{p^{\mathrm{ref}}} \right) \left(\frac{k_{dist}^{\mathrm{ref}}}{k_{dist}^{\mathrm{user}}} \right) \left(\frac{k_{other}^{\mathrm{ref}}}{k_{other}^{\mathrm{user}}} \right) \tag{II.6}$$

The air kerma rate, as a function of distance d from the source obeys an inverse square law:

$$\dot{K}(d) = \dot{K}(1) \cdot d^{-2}$$

To first order in the deviation from the reference distance, $d = 1\,\mathrm{m}$, this is:

$$\dot{K}(1 + \delta d) = \dot{K}(1) \cdot (1 - 2 \cdot \delta d) \tag{II.7}$$

So that the ratio $\frac{k_{dist}^{\text{ref}}}{k_{dist}^{\text{user}}}$ appearing in Equation (II.6) reduces to

$$\frac{k_{dist}^{\text{ref}}}{k_{dist}^{\text{user}}} = 1 - 2(\delta d^{\text{ref}} - \delta d^{\text{user}}) \qquad \text{(II.8)}$$

where

δd^{ch} is the deviation in chamber positioning from the reference distance.

Finally the model equation takes the form

$$N_K^{\text{user}} = k_{source} N_K^{\text{ref}} k_{stab}^{\text{ref}} k_{Q,Q_0} \frac{M_{raw}^{\text{ref}}}{M_{raw}^{\text{user}}} \left(\frac{273.15 + T^{\text{ref}}}{273.15 + T^{\text{user}}}\right) \left(\frac{p^{\text{user}}}{p^{\text{ref}}}\right) \left(1 - 2(\delta d^{\text{ref}} - \delta d^{\text{user}})\right) \left(\frac{k_{other}^{\text{ref}}}{k_{other}^{\text{user}}}\right) \qquad \text{(II.9)}$$

The combined uncertainty $u_c\left(N_K^{user}\right)$ is obtained from the component uncertainties which arise from the influence quantities on the right hand side of Equation (II.9). These are discussed in turn, and the analysis is presented in the following table.

II.3. Notes on influence quantities and sources of uncertainty

Each source of uncertainty is considered individually. Some have a negligible effect on the measurement; the others make some contribution which is included as an item in Table I.1.

II.3.1. Change in source position k_{source}

This uncertainty was estimated from a series of measurements of air kerma rate made on a previous occasion, in which the reference chamber was left in place and the source was repeatedly stowed and exposed between measurements. The standard deviation of these air kerma rate measurements was found to be larger than when measurements were repeated without moving the source. The increased standard deviation is attributed to variations in source position from one exposure to another. The effective number of degrees of freedom was about 20. In the present context, this influence quantity makes a contribution to the overall uncertainty of type B. The source is exposed once for the reference measurement and once for the user measurement. Therefore, this uncertainty enters twice but in an uncorrelated fashion, and the sensitivity of the overall calibration to the change in air kerma rate arising from a change in source position has a coefficient equal to $\sqrt{2}$. See item 1 in Table II.1.

II.3.2. Uncertainty of calibration N_K^{ref}

The uncertainty stated on the calibration certificate from the IAEA is 0.4% with a coverage factor *k* = 1, corresponding to a confidence level of approximately 68%. The effective number of degrees of freedom (30) corresponds to a “good” estimate. See item 2 in Table II.1.

II.3.3. Stability k_{stab}^{ref}

Measurements with the same ^{90}Sr stability check source over an extended period show that the relative change of the calibration coefficient is less than 0.3% (100% coverage). This is taken to be a 'reasonable' estimate, and so the effective number of degrees of freedom has been set to 10. See item 3 in Table II.1.

II.3.4. Quality dependence k_{Q,Q_0}

The difference between the qualities of the ^{60}Co beams at the IAEA and at the SSDL is negligible, as is the resulting contribution to the uncertainty.

II.3.5. Raw measurements with reference instrument M_{raw}^{ref}

Repeatability

Ten readings were taken and the mean and standard deviation obtained. The standard uncertainty of the mean of the actual readings taken is given in the table. The number of effective degrees of freedom in this approach is one less than the number of readings. See item 4 in Table II.1.

Another approach is possible in laboratories that have extensive records of measurements with reference instruments. It is possible to extract from these data a mean value for this standard deviation, i.e. an average over many measurements equivalent to the one made here. In this alternative approach the effective number of degrees of freedom would be multiplied by the number of mean values included.

Resolution

The reference instrument has a digital display and, for readings around 1.7 nC shows the value to the nearest 0.001 nC, giving a resolution uncertainty for each reading of only 0.03%, which does not need to be considered separately.

II.3.6. Raw measurements with user instrument M_{raw}^{user}

Repeatability

Ten readings were taken, and the mean and standard deviation obtained. The standard uncertainty of the mean of the actual readings taken is given in the table. The number of effective degrees of freedom in this approach is one less than the number of readings. See item 5 in Table II.1.

Resolution

The user's instrument has a digital display and, for readings around 4 nC shows the value to the nearest 0.01 nC, giving a resolution uncertainty of 0.13% in the individual readings, which is large enough that it needs to be considered explicitly. The sensitivity coefficient is $1/\sqrt{N}$

since the standard deviation of the N readings (which is 0.6%) is much larger than this resolution. See item 6 in Table II.1.

II.3.7. Air temperature in chamber cavity T^{ch}

Thermometer calibration

The same thermometer is used for the temperature measurements for the reference and user ionization measurements, and so its calibration cancels in the ratio of temperatures, provided that these temperatures are not vastly different.

Repeatability

The observed temperature did not change over the course of readings taken with either reference or user instrument, and so a Type A uncertainty does not arise.

Resolution

In the absence of any temperature variations, the Type B uncertainty arising from the thermometer resolution has a sensitivity coefficient of unity. See items 7 and 9 in Table II.1.

Air cavity-thermometer difference

The observed temperature, i.e. that of the mercury in the glass bulb of the thermometer which is placed in air adjacent to the chamber in the ^{60}Co beam, may differ from that of the air in the chamber cavity. This difference is expected to be zero, but with an uncertainty which is estimated to be 0.2°C (a ‘rough’ estimate, so the number of effective degrees of freedom is set to 3). See items 8 and 10 in Table II.1.

II.3.8. Air pressure p^{ch}

Barometer calibration

The same barometer is used for the pressure measurements for the reference and user ionization measurements, and so its calibration cancels in the ratio of pressures, provided that these pressures are not vastly different.

Repeatability

The observed pressure did not change over the course of readings taken with either a reference or user instrument, and so a Type A uncertainty does not arise.

Resolution of the barometer

In the absence of any pressure variations, the Type B uncertainty arising from the barometer resolution has a sensitivity coefficient of unity. See items 11 and 12 in Table II.1.

II.3.9. Deviation of chamber from reference distance k_{dist}^{ch}

The reference and user chambers are set up independently, and the uncertainties in their distances from the source are uncorrelated. The air kerma rate obeys an inverse square law as a function of distance, and so the sensitivity coefficient is 2. See items 13 and 14 in Table I.1.

II.4. Other influence quantities k_{other}^{ch}

II.4.1. Measuring assembly calibration k_{elec}^{ch}

The reference instrument consists of ion chamber and measuring assembly, and the IAEA calibration coefficient N_K^{ref} applies to the instrument as a whole so that $k_{elec}^{\mathrm{ref}} = 1.0$ by definition. Therefore, no separate uncertainty contribution arises from the reference measuring assembly calibration.

The user instrument also displays charge in coulombs, and this procedure provides a calibration of the instrument as a whole, in grays per *displayed* charge value. Therefore, $k_{elec}^{user} = 1.0$ by definition and no separate uncertainty contribution arises from the user measuring assembly calibration.

II.4.2. Saturation correction k_s^{ch}

Ion recombination is likely to be essentially the same for all air kerma rates likely to be encountered in practice with non-pulsed beams, provided that the chamber is always used with the same polarizing potential. Therefore, chamber readings are not corrected for lack of saturation, neither during calibration nor during use. The air kerma calibration coefficients of both the reference and user instruments are derived from and applied to readings which have not been corrected for ion recombination effects. In this way, measured air kerma values correctly account for ion recombination,[13] $k_s^{ref} = k_s^{user} = 1.0$ and the contribution to uncertainty from variations in ion recombination is negligible.

II.4.3. Measuring assembly linearity k_{lin}^{ch}

Measurements with both the reference and user instruments are made to the instrument reference point at which the linearity correction is defined to be $k_{lin}^{\mathrm{ch}} = 1.0$. No contribution to uncertainty arises.

II.4.4. Leakage current k_{leak}^{ch}

Leakage current was measured with both the reference and user instruments and found to be less than 0.01% of the ionization current. No correction was applied and the resulting uncertainty is negligible.

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[13] Ion recombination ***is*** corrected for in the absolute measurement at the PSDL.

II.4.5. Humidity correction k_h^{ch}

The reference chamber calibration from the IAEA is for air in the chamber having a humidity of 50% RH. The humidity during the calibration at the SSDL was measured to be 50% RH, with a standard uncertainty of 10% RH. This is within the range 20% RH to 70% RH, so that no additional humidity correction is required. The resulting uncertainty is negligible.

II.4.6. Polarity correction k_{pol}^{ch}

The same sign polarizing potential was applied to the reference chamber during use at the SSDL as was applied at the IAEA during calibration. The resulting uncertainty is negligible. For ionization measurements in a photon beam, the polarity effect is negligible in any case.

II.4.7. Rotation and tilt of the chamber k_{rot}^{ch}

Both reference and user chambers were aligned perpendicularly to the beam axis to within one degree, and oriented towards the source to within five degrees. The resulting uncertainty is negligible.

II.4.8. Field size k_{fs}^{ch}

The collimator was set to produce a square field, 100 mm on a side. This field size matches that used for the calibration of the reference chamber at the IAEA, and the resulting uncertainty is negligible.

II.4.9. Radial field homogeneity $k_{\mathrm{hom}}^{\mathrm{ch}}$

The radial field non-uniformity has been measured at the SSDL previously. The air kerma rate was found to vary by much less than 0.1% within 10 mm of the beam axis. The resulting uncertainty is negligible.

II.5. Combined and expanded uncertainty

Calculation of the uncertainty proceeds in stages. For each source of uncertainty, the standard uncertainty and sensitivity coefficient are obtained and their product gives the contribution to the combined uncertainty. These contributions are summed up in quadrature to give the combined standard uncertainty of the result. The effective number of degrees of freedom for this uncertainty is calculated according to the Welch-Satterthwaite formula from the uncertainty components and effective degrees of freedom for each source of uncertainty. See item 15 in Table II.1.

The only intermediate value which is rounded off in the calculation is the effective degrees of freedom for the combined uncertainty. Intermediate values in Table II.1 are rounded off for display purposes only.

TABLE II.1. UNCERTAINTY ANALYSIS — ^{60}CO AIR KERMA CALIBRATION

#	Quantity, source of uncertainty	Value of quantity	Expanded uncertainty	Uncertainty type	Confidence level	Coverage factor	Standard uncertainty	Sensitivity coefficient	Uncertainty component	Effective degrees of freedom
			U_i			k	u_i	c_i	$\lvert c_i u_i \rvert$	υ_i
	Change in air kerma rate	1								
1	Change in source position		0.12%	Type B	68%	1	0.12%	1.41	0.16%	20
	Calibration coefficient for the reference instrument	9.12E+07 Gy/C								
2	Uncertainty of calibration		0.4%	Type B	68%	1	0.4%	1.0	0.40%	30
3	Stability		0.3%	Type B	100%	1.73	0.17%	1.0	0.17%	10
	Raw measurements with the reference instrument	1.6687E-09 C								
4	Repeatability		0.10%	Type A	-	1	0.10%	1.0	0.10%	9
	Raw measurements with the user instrument	3.7659E-09 C								
5	Repeatability		0.20%	Type A	-	1	0.20%	1.00	0.20%	9
6	Resolution		0.13%	Type B	100%	1.73	0.08%	0.32	0.02%	100
	Temperature during reference measurement	23.4 °C								
7	Resolution of the thermometer		0.1 K	Type B	100%	1.73	0.25%	0.08	0.02%	100
8	Cavity-thermometer difference		0.2 K	Type B	-	1	0.85%	0.08	0.07%	3
	Temperature during user measurement	23.4 °C								
9	Resolution of the thermometer		0.1 K	Type B	100%	1.73	0.25%	0.08	0.02%	100
10	Cavity-thermometer difference		0.2 K	Type B	-	1	0.85%	0.08	0.07%	3
	Pressure during reference measurement	102.3 kPa								

11	resolution of the barometer		0.1 kPa	Type B	100%	1.73	0.06%	1.0	0.06%	100
Pressure during user measurement		102.3 kPa								
12	resolution of the barometer		0.1 kPa	Type B	100%	1.73	0.06%	1.0	0.06%	100
Reference chamber positioning		mm								
13	deviation from reference distance		0.2 mm	Type B	95%	2	0.01%	2.0	0.02%	10
User chamber positioning		mm								
14	deviation from reference distance		0.2 mm	Type B	95%	2	0.01%	2.0	0.02%	10
Other influence quantities during reference measurement		1								
Other influence quantities during user measurement		1								
Calibration coefficient of user instrument		**4.041E+07 Gy/C**								
15	uncertainty of the calibration coefficient		**1.1%**	Combined	95%	2.0	0.53%			68

II.6. Air kerma calibration for the user instrument

The values and uncertainties provided in the table apply only to this example and must, of course, be replaced by values and uncertainties obtained at the SSDL. They are included here for the purpose of illustration only.

The calibration coefficient for the user instrument is obtained by evaluating the model equation directly. For the values given in this example, the result is:

$$N_K^{user} = 4.041 \cdot 10^7 \text{ Gy/C.}$$

The combined standard uncertainty $u_c\left(N_K^{user}\right) = 0.53\%$ has been multiplied by a coverage factor $k = 2$ to obtain an expanded uncertainty $U = 1.1\%$. This expanded uncertainty has a confidence probability of 95%, calculated from the effective number of degrees of freedom, $\nu_{eff} = 68$.

APPENDIX III
EXAMPLE OF UNCERTAINTY ANALYSIS FOR ABSORBED DOSE TO WATER CALIBRATION FOR ^{60}CO GAMMA BEAM

III.1. Outline of measurement procedure

For this example it is assumed that the SSDL reference chamber and measuring assembly have been calibrated by the IAEA in terms of absorbed dose to water within the last three years, and checked for stability by making weekly measurements with a ^{90}Sr stability check source. The ^{60}Co unit is of the type where the source moves to the fully shielded position between measurements with different chambers. The collimator setting is fixed throughout the calibration procedure. The reference and user chambers are set up one after the other in a water phantom with source-to-surface distance the same for both chambers (95 cm) at a depth of 5 cm, to within 0.1 mm, and their positions checked using a telescope. The chamber orientation is checked visually, aligning the mark on the stem towards the source. In all measurements with both the reference and user ionization chambers, the chamber is allowed to settle for at least 15 minutes in the measurement position (temperature stabilisation), the chamber polarizing potential is verified and the chamber exposed to an absorbed dose of 5 Gy before the first reading. The leakage and radiation-induced leakage currents are required to be less than 0.1% of the current during measurements. Ten readings are taken, keeping the source exposed. Air density corrections to the readings with the reference and user instruments are based on measurements with the same barometer and thermometer. Ambient humidity is measured with a traceably calibrated humidity meter: no correction for humidity is made provided that it lies within the range 20% to 70% RH, over which the humidity correction is constant to within 0.1%. Ionization measurements are subject to acceptance criteria on the standard deviation of the readings and on their rate of drift. The standard deviation must be less than 0.1% for the reference instrument and less than 0.2% for the user instrument; the mean rate of drift must be less than 0.05% over ten readings.

- Step 1. Measurements are made with the reference standard dosimeter to determine absorbed dose rate under reference conditions at the time of calibration.
- Step 2. Measurements are made with the user instrument following [6], for calibration by substitution.

III.2. Model equation

In the **first step**, the absorbed dose to water rate is determined with the reference instrument according to the equation:

$$\dot{D}_w = N_{D,w}^{\text{ref}} M_{corr}^{\text{ref}} \qquad \text{(III.1)}$$

In the **second step**, the calibration coefficient of the user's instrument is obtained according to:

$$N_{D,w}^{\text{user}} = \frac{\dot{D}_w \cdot k_{source}}{M_{corr}^{\text{user}}} \qquad \text{(III.2)}$$

where

$N_{D,w}^{\mathrm{ref}}$ is the calibration coefficient for absorbed dose to water of the reference instrument,

$N_{D,w}^{\mathrm{user}}$ is the absorbed dose to water calibration coefficient of the user instrument,

$\dot{D}_w$ is the absorbed dose to water rate determined with the reference instrument,

k_{source} is a correction for the effect of a change in source position.

and the following is the reading obtained with either the reference chamber or the user chamber (already corrected for influence quantities).

$$M_{corr} = M_{raw} k_{TP} k_{dist} k_{other} \tag{III.3}$$

where:

M_{raw} is the mean value of the readings taken after the instrument settled,

k_{TP} is a factor to correct for departure of air density from reference conditions,

k_{dist} is a factor to correct for deviation of chamber position (depth in phantom) from the reference position,

k_{other} is a factor including all the corrections whose uncertainties are too small to consider individually in the uncertainty budget, because they are estimated to be much less than 0.1%.

k_{other} is given by:

$$k_{other} = k_{elec} k_{lin} k_s k_{leak} k_h k_{pol} k_Q k_{rot} k_{fs} k_{\mathrm{hom}} \tag{III.4}$$

where

k_{elec} is the calibration coefficient of the measuring assembly, in case the chamber and measuring assembly are calibrated separately,

k_{lin} is a factor to correct for non-linearity of the measuring assembly sensitivity,

k_s is a factor to correct for the lack of saturation due to recombination,

k_{leak} is a factor to correct for leakage current (possibly converted from an additive correction),

k_h is a factor to correct for any departure of humidity from the reference condition, 50% relative humidity,

k_{pol} is a factor to correct for any departure of the reading due to changing the polarizing voltage from its value at calibration,

k_Q is a factor to correct for any change in beam quality from the calibration beam quality,

k_{rot} is a factor to correct for any misalignment (rotation, tilt) of the chamber in use,

k_{fs} is a factor to correct for departure of the field size from the reference condition, and

k_{hom} is a factor to correct for radial non-homogeneity of the beam.

Combining the Equations (III.1), (III.2) and (III.3) gives:

$$N_{D,w}^{\mathrm{user}} = k_{source} N_{D,w}^{\mathrm{ref}} \frac{M_{raw}^{\mathrm{ref}}}{M_{raw}^{\mathrm{user}}} \frac{k_{TP}^{\mathrm{ref}}}{k_{TP}^{\mathrm{user}}} \frac{k_{dist}^{\mathrm{ref}}}{k_{dist}^{\mathrm{user}}} \frac{k_{other}^{\mathrm{ref}}}{k_{other}^{\mathrm{user}}} \qquad \text{(III.5)}$$

By replacing k_{TP} by the explicit expression in terms of temperature and pressure, one obtains:

$$N_{D,w}^{\mathrm{user}} = k_{source} N_{D,w}^{\mathrm{ref}} \frac{M_{raw}^{\mathrm{ref}}}{M_{raw}^{\mathrm{user}}} \left(\frac{273.15 + T^{\mathrm{ref}}}{273.15 + T^{\mathrm{user}}} \right) \frac{p^{\mathrm{user}}}{p^{\mathrm{ref}}} \frac{k_{dist}^{\mathrm{ref}}}{k_{dist}^{\mathrm{user}}} \frac{k_{other}^{\mathrm{ref}}}{k_{other}^{\mathrm{user}}} \qquad \text{(III.6)}$$

Replacing each k_{dist} by the expression:

$$k_{dist} = 1 + \alpha_D \, \delta d \qquad \text{(IIII.7)}$$

where

α_D is the gradient of the normalized depth dose curve at the reference depth,

δd is the deviation in chamber positioning from the reference depth.

The ratio $\frac{k_{dist}^{\mathrm{ref}}}{k_{dist}^{\mathrm{user}}}$ appearing in Equation (III.6) reduces (in first order) to:

$$\frac{k_{dist}^{\mathrm{ref}}}{k_{dist}^{\mathrm{user}}} = 1 + \alpha_D (\delta d^{\mathrm{ref}} - \delta d^{\mathrm{user}}) \qquad \text{(III.8)}$$

Finally, the model equation takes the form of:

$$N_{D,w}^{\mathrm{user}} = k_{source} N_{D,w}^{\mathrm{ref}} \frac{M_{raw}^{\mathrm{ref}}}{M_{raw}^{\mathrm{user}}} \left(\frac{273.15 + T^{\mathrm{ref}}}{273.15 + T^{\mathrm{user}}} \right) \frac{p^{\mathrm{user}}}{p^{\mathrm{ref}}} \left(1 + \alpha_D (\delta d^{\mathrm{ref}} - \delta d^{\mathrm{user}}) \right) \frac{k_{other}^{\mathrm{ref}}}{k_{other}^{\mathrm{user}}} \qquad \text{(III.9)}$$

The combined uncertainty of $N_{D,w}^{\mathrm{user}}$ is obtained from the component uncertainties that arise from the influence quantities on the right hand side of Equation (III.9) in the following Table

III.1. For ^{60}Co radiation, the gradient α_D has a negligible uncertainty and enters only as a sensitivity coefficient for the deviations in chamber positioning. The values of the quantities used for calculating the calibration coefficient of the user's instrument are also given in the table.

III.3. Notes on influence quantities and sources of uncertainty

Each source of uncertainty is considered individually. Some have a negligible effect on the measurement; the others make some contribution which is included as an item in Table III.1.

Most of the influence quantities and their effect are explained in the section about 'Influence quantities' and Appendix II. The main difference between the in-water and in-air measurements is the influence of positioning an ion chamber in air and inside a water phantom. A given deviation in the ion chamber positioning will have a greater impact on the positioning uncertainty due to the gradient of depth dose curves (see Equation (III.7)).

III.4. Combined and expanded uncertainty

Calculation of the uncertainty proceeds in stages. For each source of uncertainty, the standard uncertainty and sensitivity coefficient are obtained and their product gives the contribution to the combined uncertainty. These contributions are summed up in quadrature to give the combined standard uncertainty of the result. The effective number of degrees of freedom for this uncertainty is calculated according to the Welch-Satterthwaite formula from the uncertainty components and effective degrees of freedom for each source of uncertainty. See item 15 in Table III.1. The only intermediate value which is rounded off in the calculation is the effective degrees of freedom for the combined uncertainty. Intermediate values in Table III.1 are rounded off for display purposes only.

III.5. Absorbed dose to water calibration for the user instrument

The values and uncertainties provided in the table apply only to this example and must, of course, be replaced by values and uncertainties obtained at the SSDL. They are included here for the purpose of illustration only.

The calibration coefficient for the user instrument is obtained by evaluating the model equation directly. For the values given in this example, the result is:

$$N_{D,w}^{user} = 4.524 \cdot 10^7 \text{ Gy/C}$$

The combined standard uncertainty $u_c\left(N_{D,w}^{user}\right) = 0.52\%$ has been multiplied by a coverage factor $k = 2$ to obtain an expanded uncertainty of $U = 1.0\%$. This expanded uncertainty has a confidence probability of 95%, calculated from the effective number of degrees of freedom, $\upsilon_{eff} = 88$.

TABLE III. 1. UNCERTAINTY ANALYSIS — ^{60}Co ABSORBED DOSE CALIBRATION

Quantity, source of uncertainty	Value of quantity	Expanded uncertainty	Uncertainty type	Confidence level	Coverage factor	Standard uncertainty	Sensitivity coefficient	Uncertainty component	Effective degrees of freedom
		U_i			k	u_i	c_i	$\lvert c_i u_i \rvert$	υ_i
Correction for change in source position	1								
1 Change in source position		0.12%	Type B	68%	1.00	0.12%	1.41	0.16%	20
Calibration coefficient for the reference instrument	1.021E+08 Gy/C								
2 Uncertainty of calibration at IAEA		0.4%	Type B	68%	1.00	0.4%	1.00	0.40%	50
3 Stability of the reference instrument		0.3%	Type B	100%	1.73	0.173%	1.00	0.173%	8
Raw reading of the reference instrument	1.6687E-09 C								
4 Uncertainty of the mean reading		0.10%	Type A	68%	1.00	0.10%	1.00	0.10%	9
Raw reading of the instrument to be calibrated	3.7659E-09 C								
5 Uncertainty of the mean reading		0.20%	Type A	68%	1.00	0.20%	1.00	0.20%	9
6 Resolution of the instrument		0.10%	Type B	100%	1.73	0.058%	1.00	0.058%	50
Temperature during reference measurement	23.4 °C								
7 Resolution of the thermometer		0.1 K	Type B	100%	1.73	0.247%	0.08	0.019%	1000
Temperature during user measurement	23.4 °C								
8 Resolution of the thermometer		0.1 K	Type B	100%	1.73	0.247%	0.08	0.019%	1000
Pressure during reference measurement	101.3 kPa								
9 Resolution of the barometer		0.1 kPa	Type B	100%	1.73	0.056%	0.27	0.015%	1000
Pressure during user measurement	101.3 kPa								
10 Resolution of the barometer		0.1 kPa	Type B	100%	1.73	0.056%	0.27	0.015%	1000

Reference chamber positioning (depth)		mm								
11	deviation in chamber depth in phantom		0 mm	Type B	100%	2.00	0 mm	0.50 %/mm	0,00%	1000
User chamber positioning (depth)		mm								
12	deviation in chamber depth in phantom		0.2 mm	Type B	95%	2.00	0.1 mm	0.50 %/mm	0,05%	8
Other influence quantities during reference measurement		1								
Other influence quantities during user measurement		1								
Calibration coefficient of user instrument		**4.524E+07 Gy/C**								
13	uncertainty of the calibration coefficient		**1.0%**	Combined	95%	2.00	0.5232%			88

REFERENCES

[1] INTERNATIONAL ATOMIC ENERGY AGENCY, A Charter for the SSDLs, IAEA, Vienna (1999) http://www-naweb.iaea.org/nahu/dmrp/ssdlcharter.asp

[2] MEGHZIFENE, A., CZAP, L., SHORTT, K.R., ANDREO. P., "Comparison of calibration coefficients in the IAEA/WHO network of SSDLs", Standards and Codes of Practice in Medical Radiation Dosimetry (Proc. Int. Symp. Vienna, 2002), IAEA, Vienna (2003).

[3] INTERNATIONAL ORGANIZATION FOR STANDARDIZATION, Guide to the Expression of Uncertainty in Measurement, ISO, Geneva (1995).

[4] INTERNATIONAL ATOMIC ENERGY AGENCY, Report of the Eleventh Meeting of the SSDL Scientific Committee, Vienna, Austria, 2–5 March 2004, SSDL Newsletter No. 50, IAEA, Vienna (2005).

[5] INTERNATIONAL ATOMIC ENERGY AGENCY, Dosimetry in Diagnostic Radiology: An International Code of Practice, Technical Reports Series No. 457, IAEA, Vienna (2007).

[6] INTERNATIONAL ATOMIC ENERGY AGENCY, Calibration of Dosimeters used in Radiotherapy, Technical Reports Series No. 374, IAEA, Vienna (1994).

[7] INTERNATIONAL ATOMIC ENERGY AGENCY, Absorbed Dose Determination in External Beam Radiotherapy: An international Code of Practice for Dosimetry Based Standards of Absorbed Dose to Water, Technical Reports Series No. 398, IAEA, Vienna (2000).

[8] INTERNATIONAL ATOMIC ENERGY AGENCY, Implementation of the International Code of Practice on Dosimetry in Radiotherapy (TRS 398): Review of Testing Results, IAEA-TECDOC-11455, Vienna (2005).

[9] ROGERS, D.W.O., ROSS, C.K., The role of humidity and other correction factors in the AAPM TG-21 dosimetry protocol, Med. Phys. **15** (1988), 40–48

[10] BRITISH INSTITUTE OF RADIOLOGY, Central Axis Depth Dose Data for Use in Radiotherapy, BJR Supplement 25, London (1996)

[11] BENTLEY, R.E., Uncertainty in Measurement: The ISO Guide, Monograph 1: NML Technology Transfer Series, National Measurement Laboratory, Sydney (2003).

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