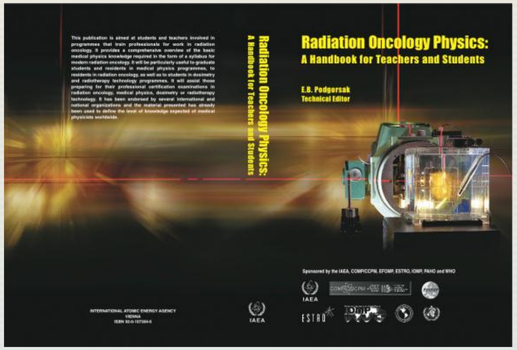
Chapter 1: Basic Radiation Physics

Slide set of 195 slides based on the chapter authored by
E.B. Podgorsak
of the IAEA publication (ISBN 92-0-107304-6):
Review of Radiation Oncology Physics:
A Handbook for Teachers and Students

Objective:

To familiarize the student with basic principles of radiation physics and modern physics used in radiotherapy.



This publication is aimed at students and teachers involved in programmes that train professionals for work in radiation oncology. It provides a comprehensive overview of the basic medical physics knowledge required in the form of a syllabus for modern radiation oncology. It will be particularly useful to graduate students and residents in medical physics programmes, to residents in radiation oncology, as well as to students in dosimetry and radiotherapy technology programmes. It will assist those preparing for their professional certification examinations in radiation oncology, medical physics, dosimetry or radiotherapy technology. It has been endorsed by several international and national organizations and the material presented has already been used to define the level of knowledge expected of medical physicists worldwide.

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Radiation Oncology Physics:
A Handbook for Teachers and Students

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Radiation Oncology Physics:
A Handbook for Teachers and Students

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Technical Editor

Sponsored by the IAEA, COMP/CCPM, EFOMP, ESTRO, IOMP, PAHO and WHO

IAEA
International Atomic Energy Agency

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CHAPTER 1. TABLE OF CONTENTS

1.1 INTRODUCTION

1.1.1 Fundamental physical constants

- Avogadro's number: $N_{\mathrm{A}} = 6.022 \times 10^{23}$ atom/mol
- Speed of light in vacuum: $c = 3 \times 10^{8}$ m/s
- Electron charge: $e = 1.6 \times 10^{19}$ As
- Electron rest mass: $m_{\mathrm{e}} = 0.511\ \mathrm{MeV}/c^{2}$
- Proton rest mass: $m_{\mathrm{p}} = 938.2\ \mathrm{MeV}/c^{2}$
- Neutron rest mass: $m_{\mathrm{n}} = 939.3\ \mathrm{MeV}/c^{2}$
- Atomic mass unit: $u = 931.5\ \mathrm{MeV}/c^{2}$

1.1 INTRODUCTION

1.1.2 Derived physical constants

- Reduced Planck's constant × speed of light in vacuum

$$\hbar c = 197\ \text{MeV} \times \text{fm} \text{ » } 200\ \text{MeV} \times \text{fm}$$

- Fine structure constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_{\text{o}}} \frac{1}{\hbar c} = \frac{1}{137}$$

- Classical electron radius

$$r_{\text{e}} = \frac{e^2}{4\pi\varepsilon_{\text{o}}} \frac{1}{m_{\text{e}} c^2} = 2.818\ \text{MeV}$$

1.1 INTRODUCTION

1.1.2 Derived physical constants

- Bohr radius:

$$a_0 = \frac{\hbar c}{a m_\text{e} c^2} = \frac{4pe_0}{e^2}\,\frac{(\hbar c)^2}{m_\text{e} c^2} = 0.529\ \text{Å}$$

- Rydberg energy:

$$E_\text{R} = \frac{1}{2} m_\text{e} c^2 \alpha^2 = \frac{1}{2}\left[\frac{e^2}{4\pi\varepsilon_\text{o}}\right]^2 \frac{m_\text{e} c^2}{(\hbar c)^2} = 13.61\ \text{eV}$$

- Rydberg constant:

$$R_¥ = \frac{E_\text{R}}{2p\hbar c} = \frac{m_\text{e} c^2 a^2}{4p\hbar c} = 109\ 737\ \text{cm}^{-1}$$

1.1 INTRODUCTION

1.1.3 Physical quantities and units

- ☐ Physical quantities are characterized by their numerical value (magnitude) and associated unit.

- ☐ Symbols for physical quantities are set in *italic type*, while symbols for units are set in roman type.

For example: $m = 21\,\mathrm{kg};\ E = 15\ \mathrm{MeV}$

1.1 INTRODUCTION

1.1.3 Physical quantities and units

- ☐ Numerical value and the unit of a physical quantity must be separated by space.

  For example:

  21 kg and NOT **21kg**; 15 MeV and NOT **15MeV**

- ☐ The currently used metric system of units is known as the Systéme International d'Unités (International system of units) or the SI system.

1.1 INTRODUCTION

1.1.3 Physical quantities and units

The SI system of units is founded on base units for seven physical quantities:

Quantity	SI unit
length ℓ	meter (m)
mass *m*	kilogram (kg)
time *t*	second (s)
electric current (*I*)	ampère (A)
temperature (*T*)	kelvin (K)
amount of substance	mole (mol)
luminous intensity	candela (cd)

1.1 INTRODUCTION

1.1.4 Classification of forces in nature

There are four distinct forces observed in interaction between various types of particles

Force	Source	Transmitted particle	Relative strength
Strong	Strong charge	Gluon	1
EM	Electric charge	Photon	1/137
Weak	Weak charge	W^+, W^-, and Z^o	10^{-6}
Gravitational	Energy	Graviton	10^{-39}

1.1 INTRODUCTION

1.1.5 Classification of fundamental particles

Two classes of fundamental particles are known:

- Quarks are particles that exhibit strong interactions

  Quarks are constituents of hadrons with a fractional electric charge (2/3 or –1/3) and are characterized by one of three types of strong charge called color (red, blue, green).

- Leptons are particles that do not interact strongly.

  Electron, muon, tau, and their corresponding neutrinos.

1.1 INTRODUCTION

1.1.6 Classification of radiation

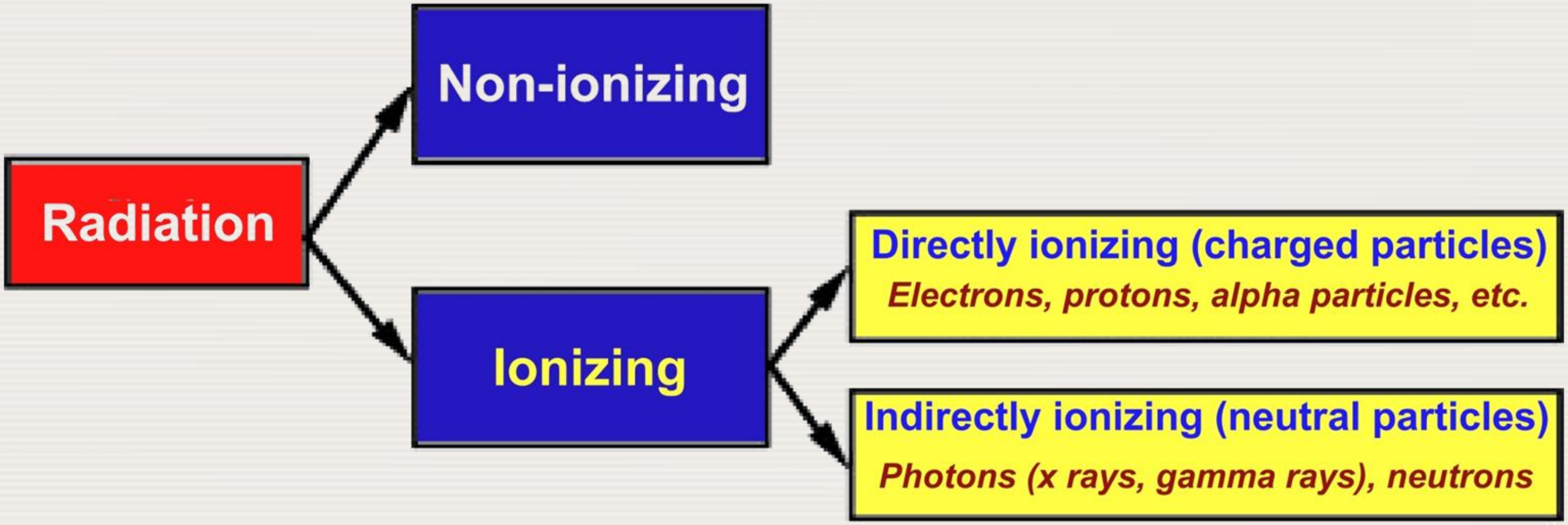
Radiation is classified into two main categories:

- ☐ Non-ionizing radiation (cannot ionise matter).
- ☐ Ionizing radiation (can ionize matter).
  - Directly ionizing radiation (charged particles)
    electron, proton, alpha particle, heavy ion
  - Indirectly ionizing radiation (neutral particles)
    photon (x ray, gamma ray), neutron

1.1 INTRODUCTION

1.1.6 Classification of radiation

Radiation is classified into two main categories:



Radiation

Non-ionizing

Ionizing

Directly ionizing (charged particles)

Electrons, protons, alpha particles, etc.

Indirectly ionizing (neutral particles)

Photons (x rays, gamma rays), neutrons

1.1 INTRODUCTION

1.1.7 Classification of ionizing photon radiation

Ionizing photon radiation is classified into four categories:

- Characteristic x ray
  - Results from electronic transitions between atomic shells.
- Bremsstrahlung
  - Results mainly from electron-nucleus Coulomb interactions.
- Gamma ray
  - Results from nuclear transitions.
- Annihilation quantum (annihilation radiation)
  - Results from positron-electron annihilation.

1.1 INTRODUCTION

1.1.8 Einstein’s relativistic mass, energy, and momentum

- Mass:

$$m(u) = \frac{m_0}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \frac{m_0}{\sqrt{1 - b^2}} = g m_0$$

- Normalized mass:

$$\frac{m(u)}{m_0} = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \frac{1}{\sqrt{1 - b^2}} = g$$

where $\beta = \frac{\upsilon}{c}$ and $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$

1.1 INTRODUCTION

1.1.8 Einstein’s relativistic mass, energy, and momentum

$$m(u) = \frac{m_0}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \frac{m_0}{\sqrt{1 - b^2}} = g m_0$$

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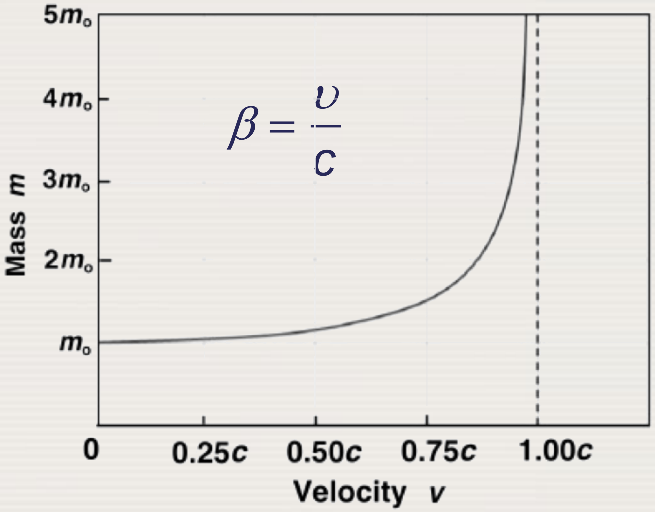

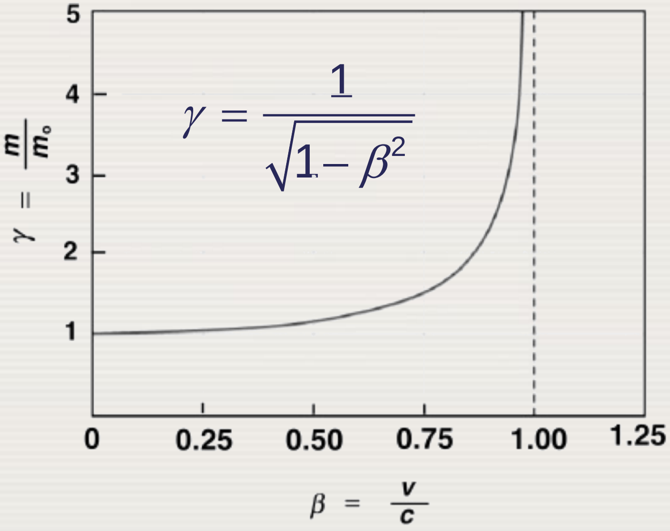

1.1 INTRODUCTION

1.1.8 Einstein’s relativistic mass, energy, and momentum

- Total energy: $E = m(\upsilon)c^2$

- Rest energy: $E_0 = m_0c^2$

- Kinetic energy: $E_K = E - E_0 = (\gamma - 1)E_0$

- Momentum: $p = \frac{1}{c}\sqrt{E^2 - E_0^2}$

with $\beta = \frac{\upsilon}{c}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

1.1 INTRODUCTION

1.1.9 Radiation quantities and units

Quantity	Definition	SI unit	Old unit	Conversion
Exposure X	$X = \dfrac{\Delta Q}{\Delta m_{\text{air}}}$	$\dfrac{2.58 \times 10^{-4}\ \text{C}}{\text{kg air}}$	$1\ \text{R} = \dfrac{1\ \text{esu}}{\text{cm}^3\ \text{air}_{\text{STP}}}$	$1\ \text{R} = \dfrac{2.58 \times 10^{-4}\ \text{C}}{\text{kg air}}$
Dose D	$D = \dfrac{\Delta E_{\text{ab}}}{\Delta m}$	$1\ \text{Gy} = 1\ \dfrac{\text{J}}{\text{kg}}$	$1\ \text{rad} = 100\ \dfrac{\text{erg}}{\text{g}}$	$1\ \text{Gy} = 100\ \text{rad}$
Equivalent dose H	$H = D w_{\text{R}}$	$1\ \text{Sv}$	$1\ \text{rem}$	$1\ \text{Sv} = 100\ \text{rem}$
Activity $\mathcal{A}$	$\mathcal{A} = \lambda N$	$1\ \text{Bq} = 1\ \text{s}^{-1}$	$1\ \text{Ci} = 3.7 \times 10^{10}\ \text{s}^{-1}$	$1\ \text{Bq} = \dfrac{1\ \text{Ci}}{3.7 \times 10^{10}}$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

❑ Constituent particles forming an atom are:

- Proton
- Neutron
- Electron

Protons and neutrons are known as nucleons and form the nucleus.

❑ Atomic number *Z*

Number of protons and number of electrons in an atom.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

☐ Atomic mass number *A*

Number of nucleons ($Z + N$) in an atom,

where

- Z is the number of protons (atomic number) in an atom.
- N is the number of neutrons in an atom.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

☐ There is no basic relation between the atomic mass number A and atomic number Z of a nucleus but the empirical relationship:

$$Z = \frac{A}{1.98 + 0.0155 A^{2/3}}$$

furnishes a good approximation for stable nuclei.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

- Atomic mole is defined as the number of grams of an atomic compound that contains exactly one Avogadro’s number of atoms, i.e.,

$$N_A = 6.022 \times 10^{23} \text{ atom/mol}$$

- Atomic mass number *A* of all elements is defined such that *A* grams of every element contain exactly N_A atoms.

- *For example*:

  - 1 mole of cobalt-60 is 60 g of cobalt-60.
  - 1 mole of radium-226 is 226 g of radium-226.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

- Molecular mole is defined as the number of grams of a molecular compound that contains exactly one Avogadro's number of molecules, i.e.,

$$N_A = 6.022 \times 10^{23} \text{ molecule/mol}$$

- Mass of a molecule is the sum of masses of all atoms that make up the molecule.

- *For example:*
  - 1 mole of water (H_2O) is 18 g of water.
  - 1 mole of carbon dioxide (CO_2) is 44 g of carbon dioxide.

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1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definition for atomic structure

☐ Atomic mass $\mathcal{M}$ is expressed in atomic mass units *u*:

- 1 *u* is equal to 1/12th of the mass of the carbon-12 atom or 931.5 MeV/c^2.
- Atomic mass $\mathcal{M}$ is smaller than the sum of the individual masses of constituent particles because of the intrinsic energy associated with binding the particles (nucleons) within the nucleus.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definition for atomic structure

- Nuclear mass M is defined as the atomic mass with the mass of atomic orbital electrons subtracted, i.e.,

$$M = \mathcal{M} - Zm_{\mathrm{e}} \qquad ,$$

where $\mathcal{M}$ is the atomic mass. Binding energy of orbital electrons to the nucleus is neglected.

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1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

In nuclear physics the convention is to designate a nucleus X as ${}^{A}_{Z}X$,

where

A is the atomic mass number.

Z is the atomic number.

For example:

- Cobalt-60 nucleus with $Z = 27$ protons and $A = 33$ neutrons is identified as ${}^{60}_{27}\mathrm{Co}$.
- Radium-226 nucleus with 88 protons and 138 neutrons is identified as ${}^{226}_{88}\mathrm{Ra}$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

- ☐ Number of atoms N_{a} per mass m of an element:

$$\frac{N_{\mathrm{a}}}{m} = \frac{N_{\mathrm{A}}}{A}$$

- ☐ Number of electrons N_{e} per mass m of an element:

$$\frac{N_{\mathrm{e}}}{m} = Z\frac{N_{\mathrm{a}}}{m} = Z\frac{N_{\mathrm{A}}}{A}$$

- ☐ Number of electrons N_{e} per volume V of an element:

$$\frac{N_{\mathrm{e}}}{V} = \rho Z\frac{N_{\mathrm{a}}}{m} = \rho Z\frac{N_{\mathrm{A}}}{A}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.1 Basic definitions for atomic structure

- ☐ For all elements the ratio $Z/A \approx 0.5$ with two notable exceptions:
  - Hydrogen-1 for which $Z/A = 1.0$.
  - Helium-3 for which $Z/A = 0.67$.
- ☐ Actually, the ratio Z/A gradually decreases:
  - From 0.5 for low atomic number *Z* elements.
  - To ~0.4 for high atomic number *Z* elements.
- ☐ *For example*:

$$Z/A = 0.50 \quad \text{for} \quad {}^{4}_{2}\mathrm{He}$$

$$Z/A = 0.45 \quad \text{for} \quad {}^{60}_{27}\mathrm{Co}$$

$$Z/A = 0.39 \quad \text{for} \quad {}^{235}_{92}\mathrm{U}$$

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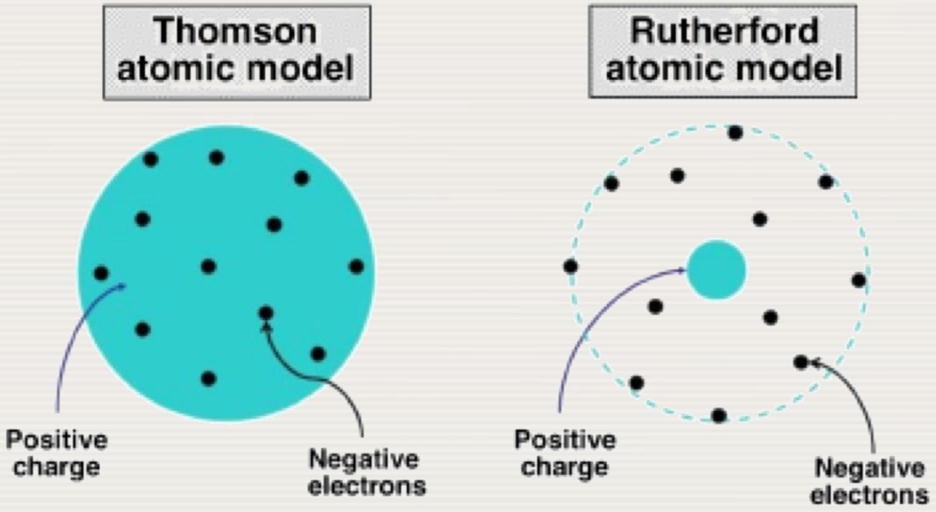
1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

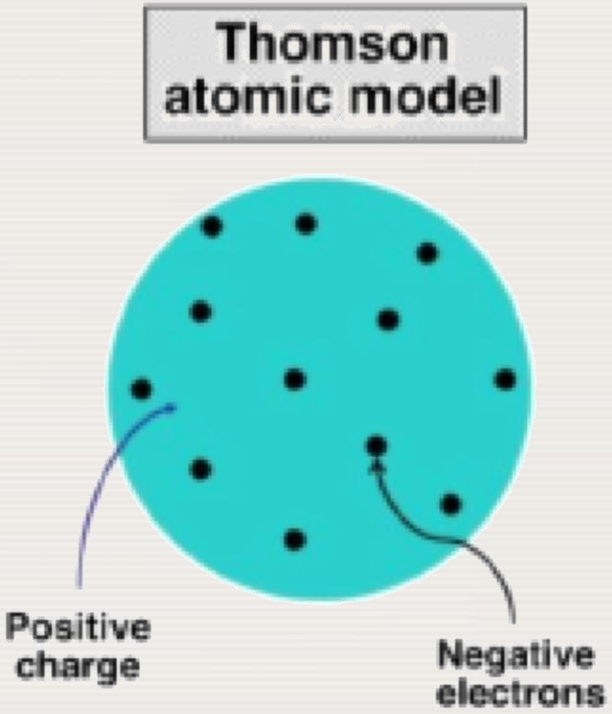
- Rutherford's atomic model is based on results of the Geiger-Marsden experiment of 1909 with 5.5 MeV alpha particles scattered on thin gold foils with a thickness of the order of 10^{-6} m.



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

- ☐ At the time of the Geiger-Marsden experiment in 1909 Thomson atomic model was the prevailing atomic model.
- ☐ Thomson model was based on an assumption that the positive and the negative (electron) charges of the atom were distributed uniformly over the atomic volume ("plum-pudding model of the atom").



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford’s model of the atom

- ☐ Geiger and Marsden found that more than 99 % of the alpha particles incident on the gold foil were scattered at scattering angles less than 3º and that the distribution of scattered alpha particles followed a Gaussian shape.
- ☐ Geiger and Marsden also found that roughly one in 10^4 alpha particles was scattered with a scattering angle exceeding 90º (probability 10^{-4})
- ☐ This finding was in drastic disagreement with the theoretical prediction of one in 10^{3500} resulting from the Thomson’s atomic model (probability 10^{-3500}).

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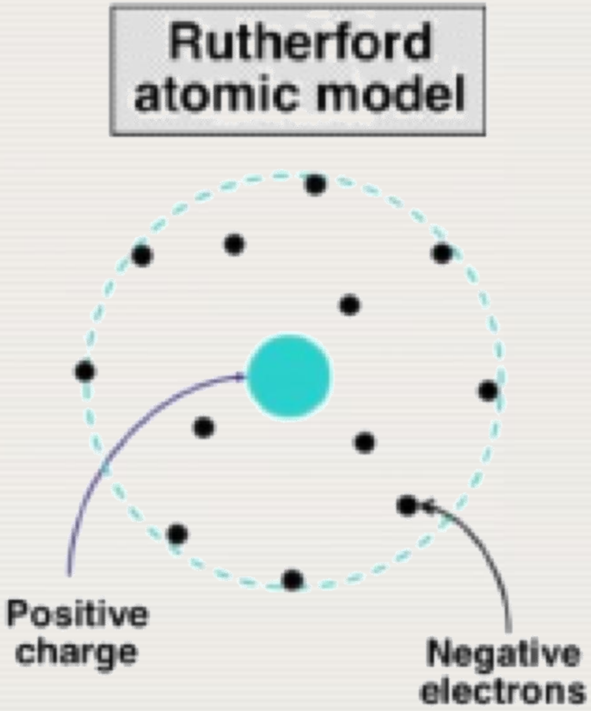
1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

- ☐ Ernest Rutherford concluded that the peculiar results of the Geiger-Marsden experiment did not support the Thomson's atomic model and proposed the currently accepted atomic model in which:
  - Mass and positive charge of the atom are concentrated in the nucleus the size of which is of the order of 10^{-15} m.
  - Negatively charged electrons revolve about the nucleus in a spherical cloud on the periphery of the Rutherford atom with a radius of the order of 10^{-10} m.



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford’s model of the atom

- ❑ Based on his model and four additional assumptions, Rutherford derived the kinematics for the scattering of alpha particles on gold nuclei using basic principles of classical mechanics.

- ❑ The four assumptions are related to:
  - Mass of the gold nucleus.
  - Scattering of alpha particles.
  - Penetration of the nucleus.
  - Kinetic energy of the alpha particles.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

☐ The four assumptions are:

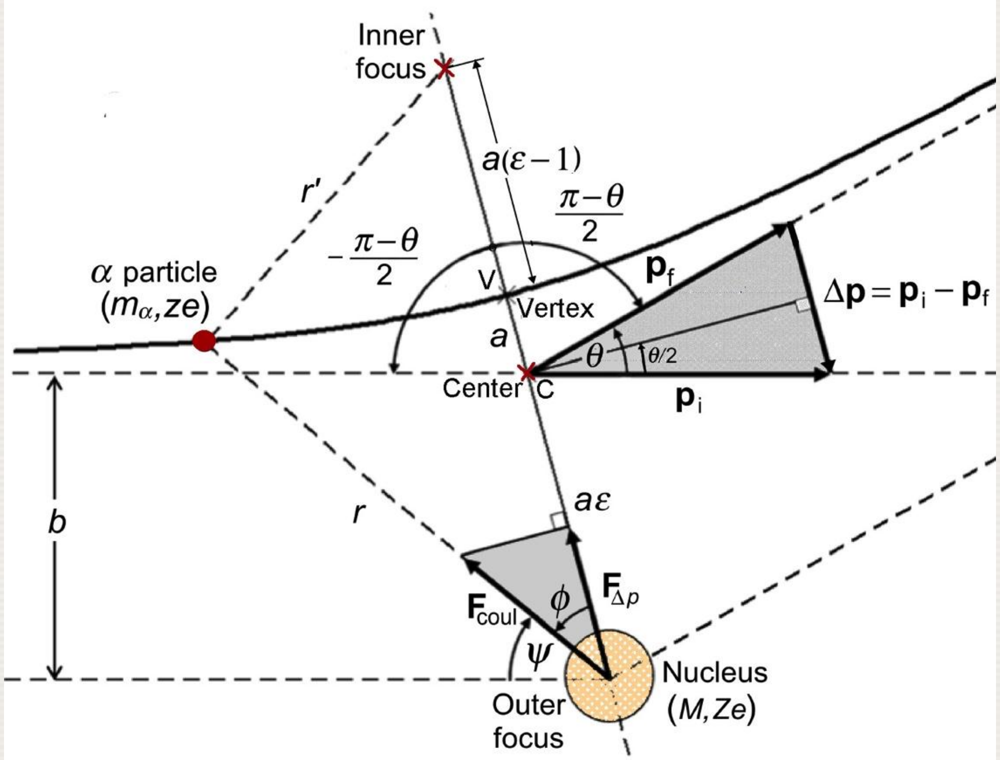
- Mass of the gold nucleus M >> mass of the alpha particle m_a.
- Scattering of alpha particles on atomic electrons is negligible.
- Alpha particle does not penetrate the nucleus, i.e., there are no nuclear reactions occurring.
- Alpha particles with kinetic energies of the order of a few MeV are non-relativistic and the simple classical relationship for the kinetic energy E_K of the alpha particle is valid:

$$E_{\mathrm{K}} = \frac{m_\alpha \upsilon^2}{2}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford’s model of the atom

As a result of the repulsive Coulomb interaction between the alpha particle (charge +2*e*) and the nucleus (charge +*Ze*) the alpha particle follows a hyperbolic trajectory.



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

- ☐ Shape of the hyperbolic trajectory and the scattering angle θ depend on the impact parameter *b.*

The limiting case is a direct hit with $b = 0$ and $\theta = \pi$ (backscattering) that, assuming conservation of energy, determines the distance of closest approach $D_{\alpha-\mathrm{N}}$ in a direct hit (backscattering) interaction.

$$E_\mathrm{K} = \frac{2Z_\mathrm{N}e^2}{4\pi\varepsilon_\mathrm{o}D_{\alpha-\mathrm{N}}} \quad \Rightarrow \quad D_{\alpha-\mathrm{N}} = \frac{2Z_\mathrm{N}e^2}{4\pi\varepsilon_\mathrm{o}E_\mathrm{K}}$$

1.2.2 Rutherford's model of the atom

- ☐ Shape of the hyperbolic trajectory and the scattering angle θ are a function of the impact parameter *b.*

- ☐ Repulsive Coulomb force between the alpha particle (charge *ze*, $z = 2$) and the nucleus (charge *Ze*) is governed by $1/r^2$ dependence:

$$F_{\text{coul}} = \frac{2Ze^2}{4\pi\varepsilon_o r^2}$$

where r is the separation between the two charged particles.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

- ☐ Relationship between the impact parameter *b* and the scattering angle θ follows from the conservation of energy and momentum considerations:

$$b = \frac{1}{2} D_{\alpha-\mathrm{N}} \cot\frac{\theta}{2}$$

- ☐ This expression is derived using:
  - Classical relationship for the kinetic energy of the α particle:

    $$E_{\mathrm{K}} = m_{\alpha}\upsilon^2/2$$

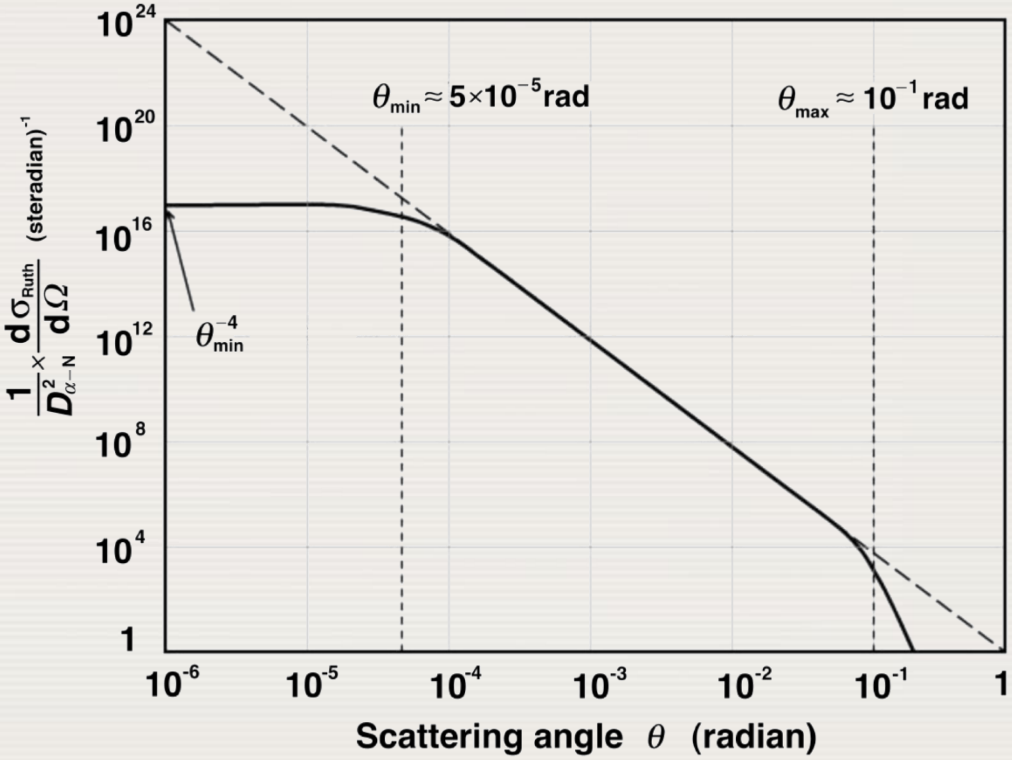
  - Definition of $D_{\alpha-\mathrm{N}}$ in a direct hit head-on collision for which the impact parameter $b = 0$ and the scattering angle $\theta = \pi$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.2 Rutherford's model of the atom

Differential Rutherford scattering cross section is:

$$\frac{ds}{dW}=\left[\frac{D_{a-\mathrm{N}}}{4}\right]^2\frac{1}{\sin^4(q/2)}$$

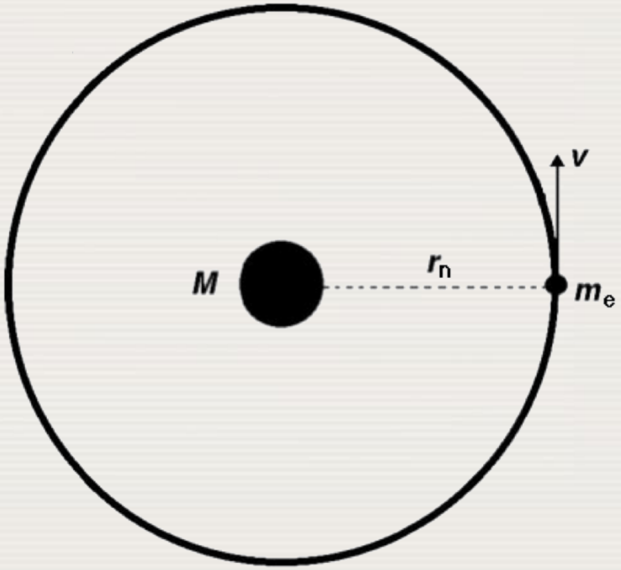


1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

❑ Niels Bohr in 1913 combined Rutherford's concept of nuclear atom with Planck's idea of quantized nature of the radiation process and developed an atomic model that successfully deals with one-electron structures, such as hydrogen atom, singly ionized helium, etc.

- M nucleus with mass M
- m_e electron with mass m_e
- r_n radius of electron orbit



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr’s model of the hydrogen atom

☐ Bohr’s atomic model is based on four postulates:

- *Postulate* 1: Electrons revolve about the Rutherford nucleus in well-defined, allowed orbits (planetary-like motion).
- *Postulate* 2: While in orbit, the electron does not lose any energy despite being constantly accelerated (no energy loss while electron is in allowed orbit).
- *Postulate* 3: The angular momentum of the electron in an allowed orbit is quantized (quantization of angular momentum).
- *Postulate* 4: An atom emits radiation only when an electron makes a transition from one orbit to another (energy emission during orbital transitions).

1.2 ATOMIC AND NUCLEAR STRUCTURE

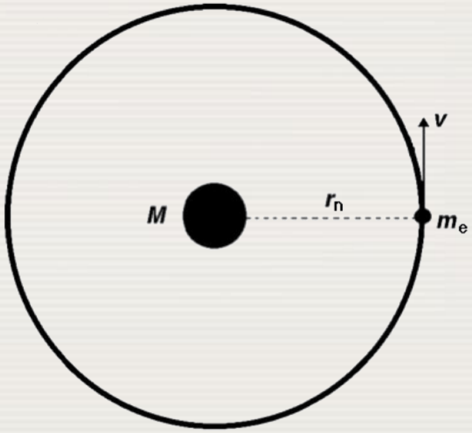
1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

Postulate 1: Planetary motion of electrons

- Electrons revolve about the Rutherford nucleus in well-defined, allowed orbits.
- Coulomb force of attraction between the electron and the positively charged nucleus is balanced by the centrifugal force

$$F_{\mathrm{coul}} = \frac{1}{4\pi\varepsilon_{\mathrm{o}}}\frac{Ze^2}{r_{\mathrm{e}}^2} \equiv F_{\mathrm{cent}} = \frac{m_{\mathrm{e}}\upsilon_{\mathrm{e}}^2}{r_{\mathrm{e}}}$$



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

Postulate 2: No energy loss while electron is in orbit.

- While in orbit, the electron does not lose any energy despite being constantly accelerated.
- This is a direct contravention of the basic law of nature (Larmor's law) which states that:

  "Any time a charged particle is accelerated or decelerated part of its energy is emitted in the form of photon (bremsstrahlung)".

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

Postulate 3: Quantization of angular momentum

- Angular momentum $L = m_e \upsilon r$ of the electron in an allowed orbit is quantized and given as $L = n\hbar$, where n is an integer referred to as the principal quantum number and $\hbar = h/2p$.
- Lowest possible angular momentum of electron in an allowed orbit is $L = \hbar$.
- All atomic orbital electron angular momenta are integer multiples of $\hbar$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

Postulate 4: Emission of photon during atomic transition.

- Atom emits radiation only when an electron makes a transition from an initial allowed orbit with quantum number n_i to a final orbit with quantum number n_f.
- Energy of the emitted photon equals the difference in energy between the two atomic orbits.

$$h\nu = E_\mathrm{i} - E_\mathrm{f}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr’s model of the hydrogen atom

- Radius r_n of a one-electron Bohr atom is:

$$r_n = a_0 \left[\frac{n^2}{Z} \right] = (0.53\ \text{Å}) \times \left[\frac{n^2}{Z} \right]$$

- Velocity υ_n of the electron in a one-electron Bohr atom is:

$$u_n = ac \left[\frac{Z}{n} \right] = \frac{c}{137} \left[\frac{Z}{n} \right] \approx 7 \times 10^{-3} c \left[\frac{Z}{n} \right]$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

- ☐ Energy levels E_n of orbital electron shells in a one-electron Bohr atom are:

$$E_n = -E_R \left[\frac{Z}{n} \right]^2 = (-13.6\ \text{eV}) \times \left[\frac{Z}{n} \right]^2$$

- ☐ Wave number k for transition from shell n_i to shell n_f :

$$k = R_\infty Z^2 \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\} = 109\ 737\ \text{cm}^{-1}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

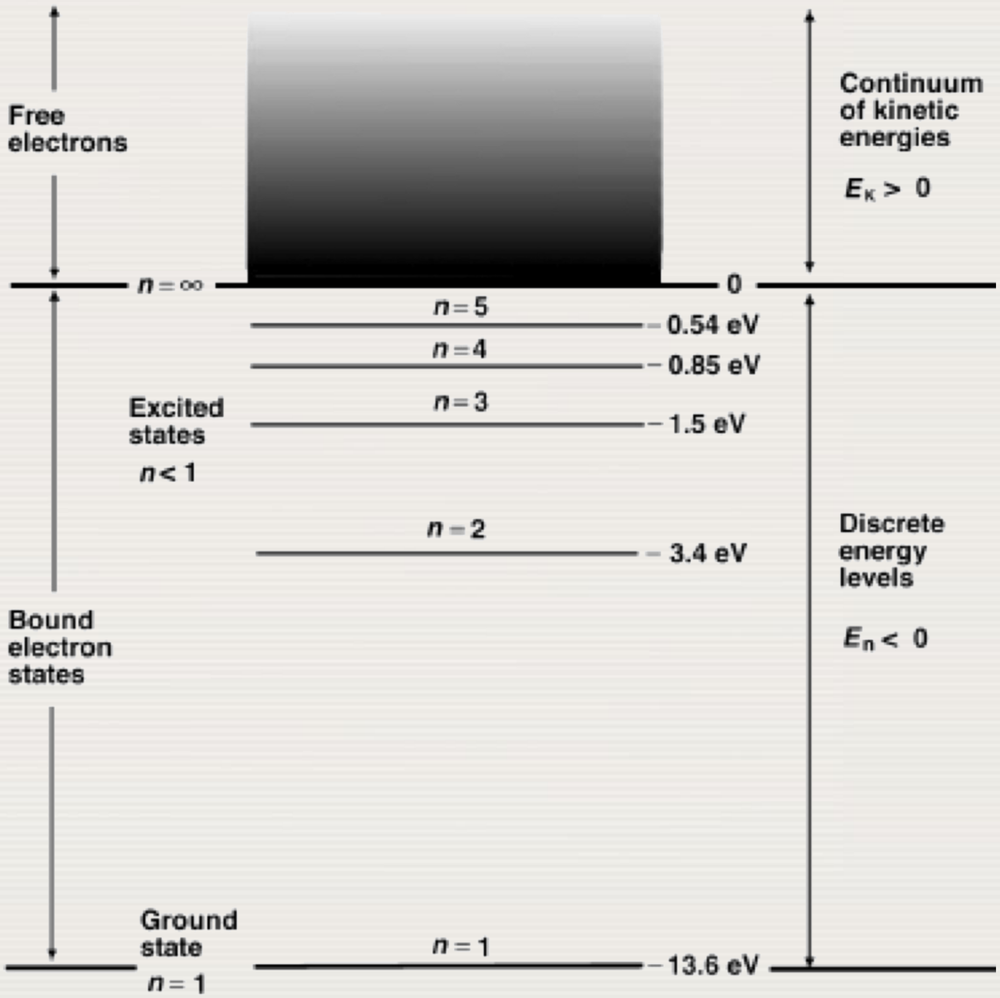
1.2.3 Bohr’s model of the hydrogen atom

- Energy levels E_n of orbital electron shells in a one-electron Bohr atom are:

$$E_n = -E_R\left[\frac{Z}{n}\right]^2$$

$$= (-13.6\text{ eV}) \times \left[\frac{Z}{n}\right]^2$$

- E_R = Rydberg energy



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

Velocity of the orbital electron in the ground state $n = 1$ is less than 1 % of the speed of light for the hydrogen atom with $Z = 1$.

$$\frac{u_n}{c} = a\left[\frac{Z}{n}\right] = \frac{1}{137}\left[\frac{Z}{n}\right] \approx (7 \times 10^{-3}) \times \left[\frac{Z}{n}\right]$$

Therefore, the use of classical mechanics in the derivation of the kinematics of the Bohr atom is justified.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

- Both Rutherford and Bohr used classical mechanics in their discoveries of the atomic structure and the kinematics of the electronic motion, respectively.
- Rutherford introduced the idea of atomic nucleus that contains most of the atomic mass and is 5 orders of magnitude smaller than the atom.
- Bohr introduced the idea of electronic angular momentum quantization.

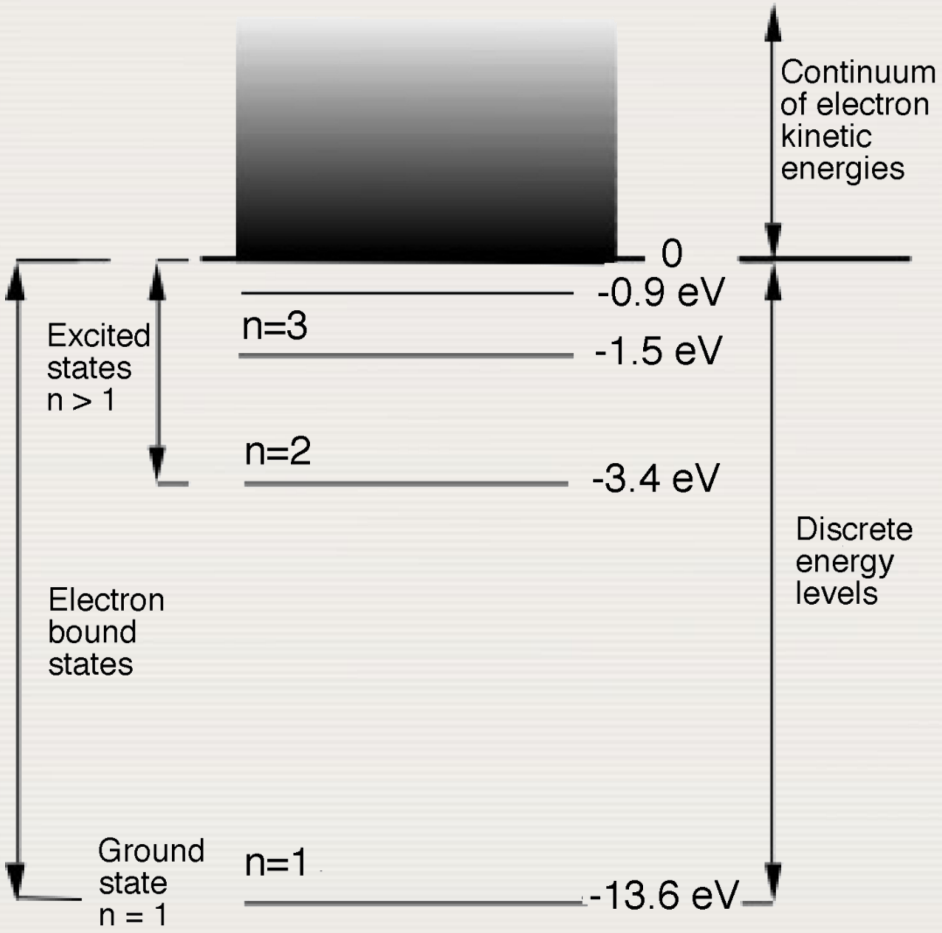
1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom

- ☐ Nature provided Rutherford with an atomic probe (naturally occurring alpha particles) having just the appropriate energy (few MeV) to probe the atom without having to deal with relativistic effects and nuclear penetration.
- ☐ Nature provided Bohr with the hydrogen one-electron atom in which the electron can be treated with simple classical relationships.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.3 Bohr's model of the hydrogen atom



Energy level diagram for the hydrogen atom.

n = 1 ground state

n > 1 excited states

Wave number of emitted photon

$$k = \frac{1}{\lambda} = R_{\infty} Z^2 \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\}$$

$$R_{\infty} = 109\ 737\ \mathrm{cm}^{-1}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.4 Multi-electron atom

- Bohr theory works well for one-electron structures but does not apply directly to multi-electron atoms because of the repulsive Coulomb interactions among the atomic electrons.

- Electrons occupy allowed shells; however, the number of electrons per shell is limited to $2n^2$.

- Energy level diagrams of multi-electron atoms resemble those of one-electron structures, except that inner shell electrons are bound with much larger energies than E_{R}.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.4 Multi-electron atoms

- Douglas Hartree proposed an approximation that predicts the energy levels and radii of multi-electron atoms reasonably well despite its inherent simplicity.

- Hartree assumed that the potential seen by a given atomic electron is

$$V(r) = -\frac{Z_{\text{eff}}\, e^2}{4\pi\varepsilon_{o}} \frac{1}{r}$$

where Z_{eff} is the effective atomic number that accounts for the potential screening effects of orbital electrons $(Z_{\text{eff}} < Z)$.

  - Z_{eff} for K-shell ($n = 1$) electrons is $Z - 2$.
  - Z_{eff} for outer shell electrons is approximately equal to n.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.4 Multi-electron atom

Hartree’s expressions for atomic radii and energy level

- Atomic radius

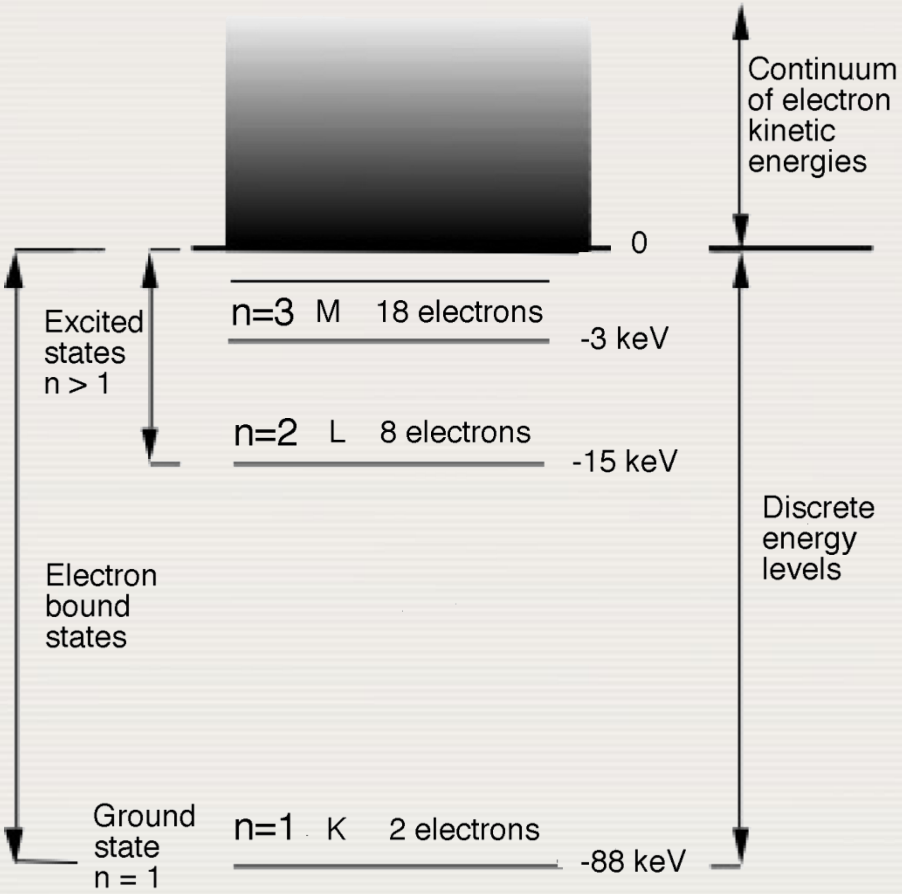
In general	**For the K shell**	**For the outer shell**
$r_n = a_0 \dfrac{n^2}{Z_{\text{eff}}}$	$r(\text{K shell}) = r_1 = a_0 \dfrac{n^2}{Z - 2}$	$r_{\text{outer shell}} \text{ » } na_0$

- Binding energy

In general	**For the K shell**	**For outer shell**
$E_n = -E_R \dfrac{Z_{\text{eff}}^2}{n^2}$	$E(\text{K shell}) = E_1 = -E_R (Z-2)^2$	$E_{\text{outer shell}} \approx -E_R$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.4 Multi-electron atom



Energy level diagram for multi-electron atom (lead)

Shell (orbit) designations:

$n = 1$ K shell (2 electrons)

$n = 2$ L shell (8 electrons)

$n = 3$ M shell (18 electrons)

$n = 4$ N shell (32 electrons)

……

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.5 Nuclear structure

- Most of the atomic mass is concentrated in the atomic nucleus consisting of Z protons and $A - Z$ neutrons where Z is the atomic number and A the atomic mass number (Rutherford-Bohr atomic model).

- Protons and neutrons are commonly called nucleons and are bound to the nucleus with the strong force.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.5 Nuclear structure

- ☐ In contrast to the electrostatic and gravitational forces that are inversely proportional to the square of the distance between two particles, the strong force between two particles is a very short range force, active only at distances of the order of a few femtometers.
- ☐ Radius r of the nucleus is estimated from: $r = r_0 \sqrt[3]{A}$, where r_0 is the nuclear radius constant (1.25 fm).

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.5 Nuclear structure

- ☐ Sum of masses of the individual components of a nucleus that contains Z protons and $(A - Z)$ neutrons is larger than the mass of the nucleus M.

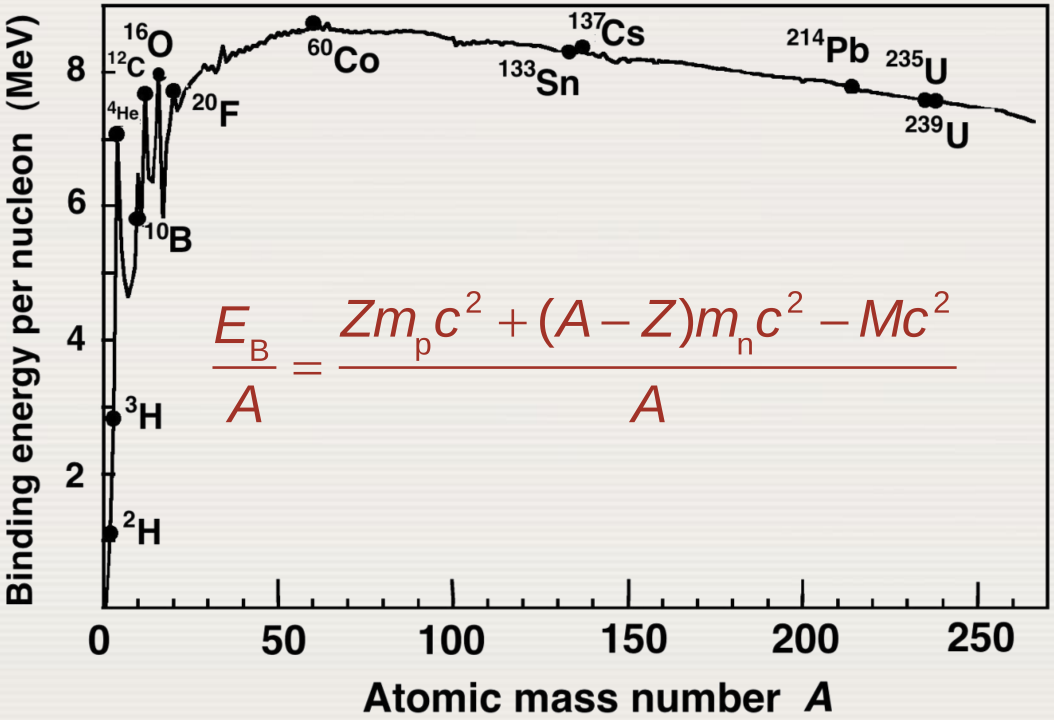
- ☐ This difference in masses is called the mass defect (deficit) Δm and its energy equivalent Δmc^2 is called the total binding energy E_B of the nucleus:

$$E_\mathrm{B} = Zm_\mathrm{p}c^2 + (A - Z)m_\mathrm{n}c^2 - Mc^2$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.5 Nuclear structure

Binding energy per nucleon (E_B/A) in a nucleus varies with the number of nucleons A and is of the order of 8 MeV per nucleon.



$$\frac{E_B}{A} = \frac{Zm_pc^2 + (A-Z)m_nc^2 - Mc^2}{A}$$

Nucleus	E_B/A (MeV)
$^{2}_{1}$H	1.1
$^{3}_{1}$H	2.8
$^{3}_{1}$He	2.6
$^{4}_{1}$He	7.1
$^{60}_{27}$Co	8.8
$^{238}_{92}$U	7.3

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.6 Nuclear reactions

☐ Nuclear reaction: $A + a = B + b$ or A(a,b)B

Projectile a bombards target A

which is transformed into reactants B and b.

☐ The most important physical quantities that are conserved in a nuclear reaction are:

- Charge
- Mass number
- Linear momentum
- Mass-energy

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.6 Nuclear reactions

- Threshold kinetic energy $(E_\mathrm{K})^\mathrm{a}_\mathrm{thr}$ for a nuclear reaction is calculated from the relativistic invariant and is the smallest value of projectile's kinetic energy at which the reaction will take place:

$$(E_\mathrm{K})^\mathrm{a}_\mathrm{thr} = \frac{(m_\mathrm{B}c^2 + m_\mathrm{b}c^2)^2 - (m_\mathrm{A}c^2 + m_\mathrm{a}c^2)^2}{2m_\mathrm{A}c^2}$$

- Threshold total energy $E^\mathrm{a}_\mathrm{thr}$ for a nuclear reaction to occur is:

$$E^\mathrm{a}_\mathrm{thr} = \frac{(m_\mathrm{B}c^2 + m_\mathrm{b}c^2)^2 - (m_\mathrm{A}^2c^4 + m_\mathrm{a}^2c^4)}{2m_\mathrm{A}c^2}$$

m_A, m_a, m_B, and m_b

are rest masses of A, a, B, and b, respectively.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- Radioactivity is a process by which an unstable nucleus (parent) decays into a new nuclear configuration (daughter) that may be stable or unstable.

- If the daughter is unstable, it will decay further through a chain of decays until a stable configuration is attained.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- ☐ Henri Becquerel discovered natural radioactivity in 1896.
- ☐ Other names used for radioactive decay are:
  - Nuclear decay.
  - Nuclear disintegration.
  - Nuclear transformation.
  - Nuclear transmutation.
  - Radioactive decay.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- ☐ Radioactive decay involves a transition from the quantum state of the parent P to a quantum state of the daughter D.
- ☐ Energy difference between the two quantum states is called the decay energy *Q.*
- ☐ Decay energy *Q* is emitted:
  - In the form of electromagnetic radiation (gamma rays)

    or
  - In the form of kinetic energy of the reaction products.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

☐ All radioactive processes are governed by the same formalism based on:

- Characteristic parameter called the decay constant λ
- Activity $\mathcal{A}(t)$ defined as $\lambda N(t)$ where $N(t)$ is the number of radioactive nuclei at time t

$$\mathcal{A}(t) = \lambda N(t)$$

☐ Specific activity a is the parent's activity per unit mass:

$$a = \frac{\mathcal{A}(t)}{M} = \frac{\lambda N(t)}{M} = \frac{\lambda N_A}{A}$$

N_A is Avogadro's number.

A is atomic mass number.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- ☐ Activity represents the total number of disintegrations (decays) of parent nuclei per unit time.

- ☐ SI unit of activity is the becquerel (1 Bq = 1 s^{-1}).

  Both becquerel and hertz correspond to s^{-1} yet hertz expresses frequency of periodic motion, while becquerel expresses activity.

- ☐ The older unit of activity is the curie $(1\,\text{Ci} = 3.7 \times 10^{10}\ \text{s}^{-1})$, originally defined as the activity of 1 g of radium-226.

  Currently, the activity of 1 g of radium-226 is 0.988 Ci.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- Decay of radioactive parent P into stable daughter D:

$$\mathrm{P} \xrightarrow{\lambda_\mathrm{P}} \mathrm{D}$$

- Rate of depletion of the number of radioactive parent nuclei $N_\mathrm{P}(t)$ is equal to the activity $\mathcal{A}_\mathrm{P}(t)$ at time t:

$$\frac{\mathrm{d}N_\mathrm{P}(t)}{\mathrm{d}t} = -\mathcal{A}_\mathrm{P}(t) = -\lambda_\mathrm{P} N_\mathrm{P}(t)$$

$$\int_{N_\mathrm{P}(0)}^{N_\mathrm{P}(t)} \frac{\mathrm{d}N_\mathrm{P}(t)}{N_\mathrm{P}} = -\int_0^t \lambda_\mathrm{P}\,\mathrm{d}t$$

where $N_\mathrm{P}(0)$ is the initial number of parent nuclei at time $t = 0$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- ☐ Number of radioactive parent nuclei $N_{\mathrm{P}}(t)$ as a function of time t is:

$$N_{\mathrm{P}}(t) = N_{\mathrm{P}}(0)e^{-\lambda_{\mathrm{P}}t}$$

- ☐ Activity of the radioactive parent $\mathcal{A}_{\mathrm{P}}(t)$ as a function of time t is:

$$\mathcal{A}_{\mathrm{P}}(t) = \lambda_{\mathrm{P}}N_{\mathrm{P}}(t) = \lambda_{\mathrm{P}}N_{\mathrm{P}}(0)e^{-\lambda_{\mathrm{P}}t} = \mathcal{A}_{\mathrm{P}}(0)e^{-\lambda_{\mathrm{P}}t}$$

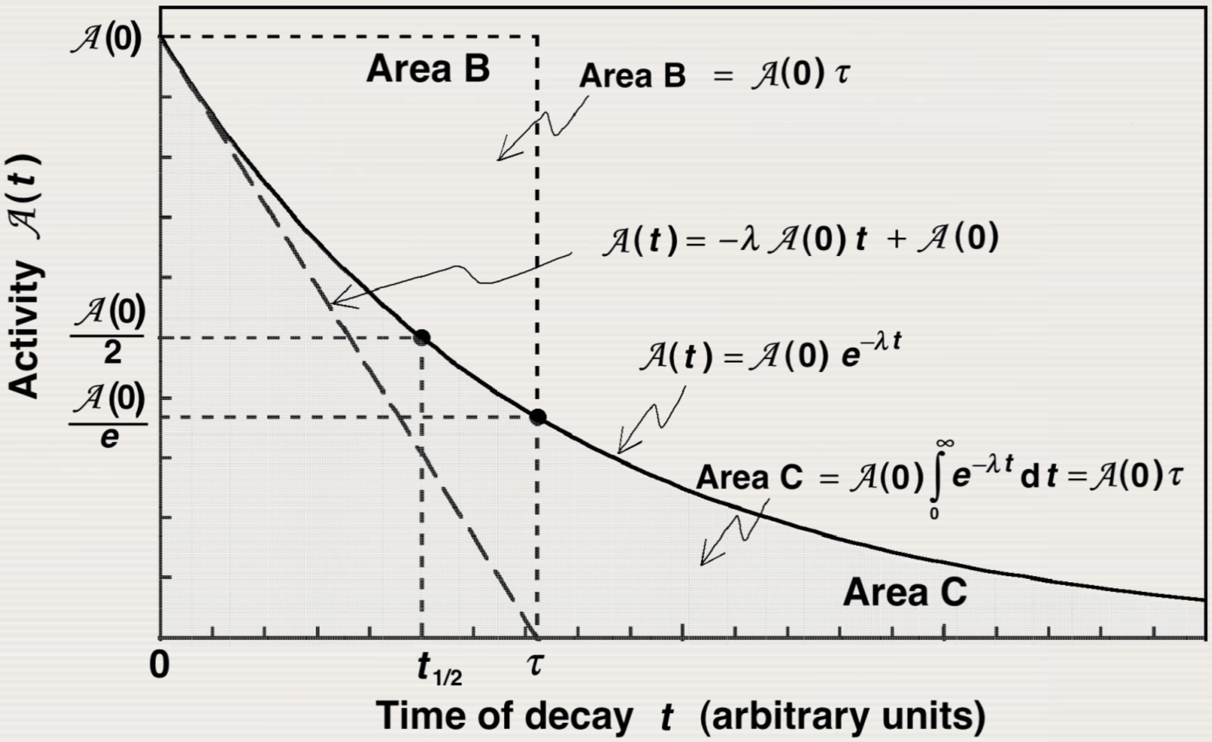
where $\mathcal{A}_{\mathrm{P}}(0)$ is the initial activity at time $t = 0$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

Parent activity $\mathcal{A}_P(t)$ plotted against time t illustrating:

- Exponential decay of the activity.
- Concept of half life.
- Concept of mean life.



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- Half life $(t_{1/2})_{\mathrm{P}}$ of radioactive parent P is the time during which the number of radioactive parent nuclei decays from the initial value $N_{\mathrm{P}}(0)$ at time $t = 0$ to half the initial value

$$N_{\mathrm{P}}(t = t_{1/2}) = (1/2)N_{\mathrm{P}}(0) = N_{\mathrm{P}}(0)e^{-\lambda_{\mathrm{P}}(t_{1/2})_{\mathrm{P}}}$$

- Decay constant λ_{P} and the half life $(t_{1/2})_{\mathrm{P}}$ are related as follows

$$\lambda_{\mathrm{P}} = \frac{\ln 2}{(t_{1/2})_{\mathrm{P}}}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- ☐ Decay of radioactive parent P into unstable daughter D which in turn decays into granddaughter G:

$$\mathrm{P} \xrightarrow{\lambda_\mathrm{P}} \mathrm{D} \xrightarrow{\lambda_\mathrm{D}} \mathrm{G}$$

- ☐ Rate of change $\mathrm{d}N_\mathrm{D}/\mathrm{d}t$ in the number of daughter nuclei D equals to supply of new daughter nuclei through decay of P given as $\lambda_\mathrm{P} N_\mathrm{P}(t)$ and the loss of daughter nuclei D from the decay of D to G given as $-\lambda_\mathrm{D} N_\mathrm{D}(t)$

$$\frac{\mathrm{d}N_\mathrm{D}}{\mathrm{d}t} = \lambda_\mathrm{P} N_\mathrm{P}(t) - \lambda_\mathrm{D} N_\mathrm{D}(t) = \lambda_\mathrm{P} N_\mathrm{P}(0)\, e^{-\lambda_\mathrm{P} t} - \lambda_\mathrm{D} N_\mathrm{D}(t)$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

- Number of daughter nuclei is:

$$N_{\mathrm{D}}(t) = N_{\mathrm{P}}(0)\frac{\lambda_{\mathrm{P}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left\{e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}\right\}$$

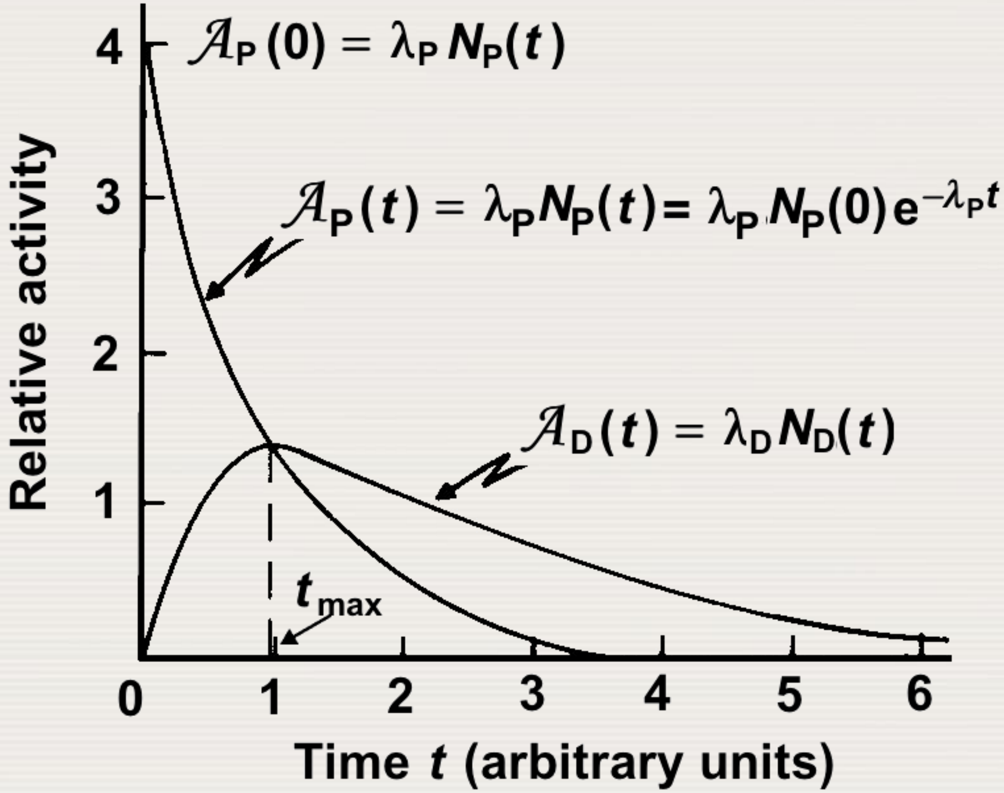
- Activity of the daughter nuclei is:

$$\mathcal{A}_{\mathrm{D}}(t) = \frac{N_{\mathrm{P}}(0)\lambda_{\mathrm{P}}\lambda_{\mathrm{D}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left\{e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}\right\} = \mathcal{A}_{\mathrm{P}}(0)\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left\{e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}\right\} =$$

$$= \mathcal{A}_{\mathrm{P}}(0)\frac{1}{1 - \dfrac{\lambda_{\mathrm{P}}}{\lambda_{\mathrm{D}}}}\left\{e^{-\lambda_{\mathrm{P}} t} - e^{-\lambda_{\mathrm{D}} t}\right\} = \mathcal{A}_{\mathrm{P}}(t)\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}}}\left\{1 - e^{-(\lambda_{\mathrm{D}} - \lambda_{\mathrm{P}})t}\right\},$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity



At $t = t_{max}$ parent and daughter activities are equal and the daughter activity reaches its maximum

$$\left.\frac{d\mathcal{A}_D}{dt}\right|_{t=t_{max}} = 0$$

and

$$t_{max} = \frac{\ln\dfrac{\lambda_D}{\lambda_P}}{\lambda_D - \lambda_P}$$

Parent and daughter activities against time for $P \xrightarrow{\lambda_P} D \xrightarrow{\lambda_D} G$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.7 Radioactivity

Special considerations for the $P \xrightarrow{\lambda_P} D \xrightarrow{\lambda_D} G$ relationship:

- For $\lambda_D < \lambda_P$ or $(t_{1/2})_D > (t_{1/2})_P$

  General relationship (no equilibrium)

$$\frac{\mathcal{A}_D}{\mathcal{A}_P} = \frac{\lambda_D}{\lambda_D - \lambda_P}\left\{1 - e^{-(\lambda_D - \lambda_P)t}\right\}$$

- For $\lambda_D > \lambda_P$ or $(t_{1/2})_D < (t_{1/2})_P$

  Transient equilibrium for $t >> t_{max}$

$$\frac{\mathcal{A}_D}{\mathcal{A}_P} = \frac{\lambda_D}{\lambda_D - \lambda_P}$$

- For $\lambda_D >> \lambda_P$ or $(t_{1/2})_D << (t_{1/2})_P$

  Secular equilibrium

$$\frac{\mathcal{A}_D}{\mathcal{A}_P} \approx 1$$

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1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.8 Activation of nuclides

- ☐ Radioactivation of nuclides occurs when a parent nuclide P is bombarded with thermal neutrons in a nuclear reactor and transforms into a radioactive daughter nuclide D that decays into a granddaughter nuclide G.

$$P \xrightarrow{s\dot{j}} D \xrightarrow{/_{\mathrm{D}}} G$$

- ☐ Probability for radioactivation to occur is governed by the cross section σ for the nuclear reaction and the neutron fluence rate $\dot{j}$.
  - Unit of σ is barn per atom where $1\,\mathrm{barn} = 1\,\mathrm{b} = 10^{-24}\ \mathrm{cm}^2$.
  - Unit of $\dot{j}$ is $\mathrm{cm}^{-2} \cdot \mathrm{s}^{-1}$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.8 Activation of nuclides

- Daughter activity $\mathcal{A}_D(t)$ in radioactivation is described by an expression similar to that given for the series decay except that λ_P is replaced by the product $\sigma\dot{\varphi}$

$$\mathcal{A}_D(t) = \frac{\sigma\dot{\varphi}\lambda_D}{\lambda_D - \sigma\dot{\varphi}} N_P(0)\left[e^{-\sigma\dot{\varphi}t} - e^{-\lambda_D t}\right]$$

- Time at which the daughter activity reaches its maximum value is given by

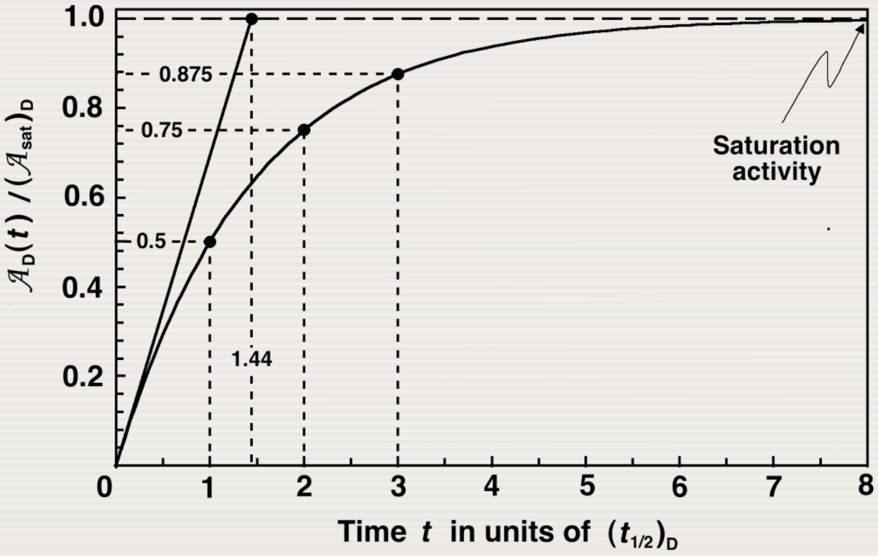
$$t_{\max} = \frac{\ln[\lambda_D/(\sigma\dot{\varphi})]}{\lambda_D - \sigma\dot{\varphi}}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.8 Activation of nuclides

- ☐ When $s\dot{j} << /_{\mathrm{D}}$, the daughter activity expression transforms into a simple exponential growth expression

$$\mathcal{A}_{\mathrm{D}}(t) = s\,\dot{j}\,N_{\mathrm{P}}(0)\left\{1 - e^{-/_{\mathrm{D}}t}\right\} = \mathcal{A}_{\mathrm{sat}}\left\{1 - e^{-/_{\mathrm{D}}t}\right\}$$



1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.8 Activation of nuclides

- An important example of nuclear activation is the production of the cobalt-60 radionuclide through bombarding stable cobalt-59 with thermal neutrons

$$ {}^{59}_{27}\mathrm{Co} + \mathrm{n} \rightarrow {}^{60}_{27}\mathrm{Co} + \gamma \qquad \text{or} \qquad {}^{59}_{27}\mathrm{Co}(\mathrm{n},\gamma){}^{60}_{27}\mathrm{Co} $$

  - For cobalt-59 the cross section σ is 37 b/atom.
  - Typical reactor fluence rates $\dot{\varphi}$ are of the order of $10^{14}\ \mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}$.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- ☐ Radioactive decay is a process by which unstable nuclei reach a more stable configuration.
- ☐ There are four main modes of radioactive decay:
  - Alpha decay
  - Beta decay
    - Beta plus decay
    - Beta minus decay
    - Electron capture
  - Gamma decay
    - Pure gamma decay
    - Internal conversion
  - Spontaneous fission

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- Nuclear transformations are usually accompanied by emission of energetic particles (charged particles, neutral particles, photons, neutrinos)

Radioactive decay	Emitted particles
Alpha decay	α particle
Beta plus decay	β^{+} particle (positron), neutrino
Beta minus decay	β^{-} particle (electron), antineutrino
Electron capture	Neutrino
Pure gamma decay	Photon
Internal conversion	Orbital electron
Spontaneous fission	Fission products

1.2.9 Modes of radioactive decay

- ☐ In each nuclear transformation a number of physical quantities must be conserved.

- ☐ The most important conserved physical quantities are:
  - Total energy
  - Momentum
  - Charge
  - Atomic number
  - Atomic mass number (number of nucleons)

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- ☐ Total energy of particles released by the transformation process is equal to the net decrease in the rest energy of the neutral atom, from parent P to daughter D.

- ☐ Decay energy (Q value) is given as:

$$Q = \left\{ M(\mathrm{P}) - \left[M(\mathrm{D}) + m \right] \right\} c^2$$

M(P), M(D), and m are the nuclear rest masses of the parent, daughter and emitted particles.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- ☐ Alpha decay is a nuclear transformation in which:
  - Energetic alpha particle (helium-4 ion) is emitted.
  - Atomic number *Z* of the parent decreases by 2.
  - Atomic mass number *A* of the parent decreases by 4.

$$ {}^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^{4}_{2}\mathrm{He} $$

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1.2 ATOMIC AND NUCLEAR STRUCTURE

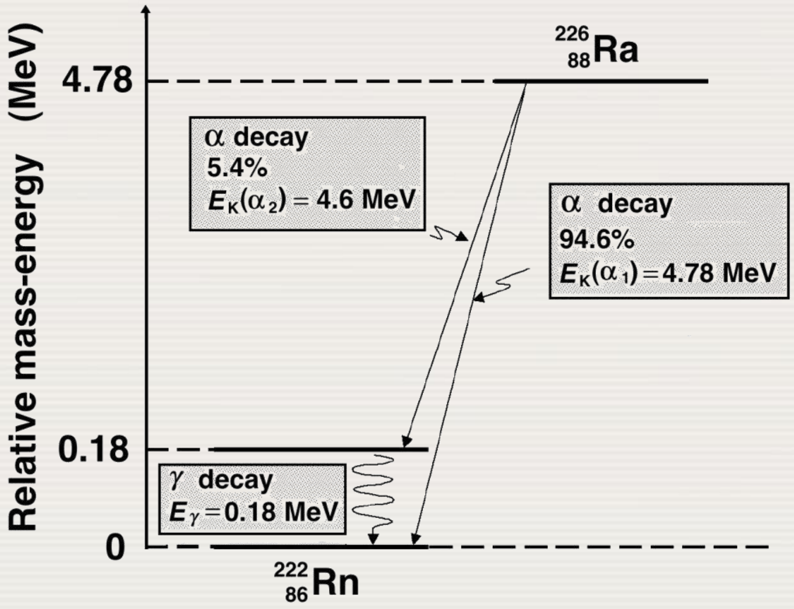
1.2.9 Modes of radioactive decay

- ☐ Henri Becquerel discovered alpha decay in 1896; George Gamow explained its exact nature in 1928 using the quantum mechanical effect of tunneling.
- ☐ Hans Geiger and Ernest Marsden used 5.5 MeV alpha particles emitted by radon-222 in their experiment of alpha particle scattering on a gold foil.
- ☐ Kinetic energy of alpha particles released by naturally occurring radionuclides is between 4 MeV and 9 MeV.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- Best known example of alpha decay is the transformation of radium-226 into radon-222 with a half life of 1602 years.



$$^{A}_{Z}\mathrm{P} \rightarrow {}^{A-4}_{Z-2}\mathrm{D} + {}^{4}_{2}\mathrm{He}$$

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + \alpha$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

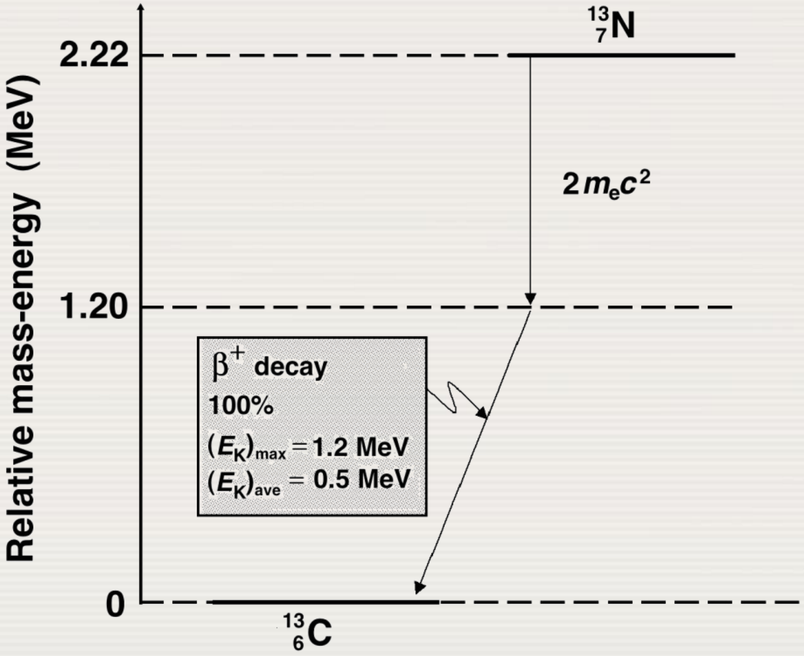
- Beta plus decay is a nuclear transformation in which:
  - Proton-rich radioactive parent nucleus transforms a proton into a neutron.
  - Positron and neutrino, sharing the available energy, are ejected from the parent nucleus.
  - Atomic number *Z* of the parent decreases by one; the atomic mass number *A* remains the same.
  - Number of nucleons and total charge are conserved in the beta decay process and the daughter D can be referred to as an isobar of the parent P.

$$\mathrm{p} \rightarrow \mathrm{n} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \qquad {}^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z-1}\mathrm{D} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- *Example* of a beta plus decay is the transformation of nitrogen-13 into carbon-13 with a half life of 10 min.



β^+ decay
100%
$(E_K)_{max} = 1.2$ MeV
$(E_K)_{ave} = 0.5$ MeV

$$p \rightarrow n + e^+ + \nu_e$$

$$^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + e^+ + \nu_e$$

$$^{13}_{7}N \rightarrow {}^{13}_{6}C + e^+ + \nu_e$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- Beta minus decay is a nuclear transformation in which:
  - Neutron-rich radioactive parent nucleus transforms a neutron into a proton.
  - Electron and anti-neutrino, sharing the available energy, are ejected from the parent nucleus.
  - Atomic number *Z* of the parent increases by one; the atomic mass number *A* remains the same.
  - Number of nucleons and total charge are conserved in the beta decay process and the daughter D can be referred to as an isobar of the parent P.

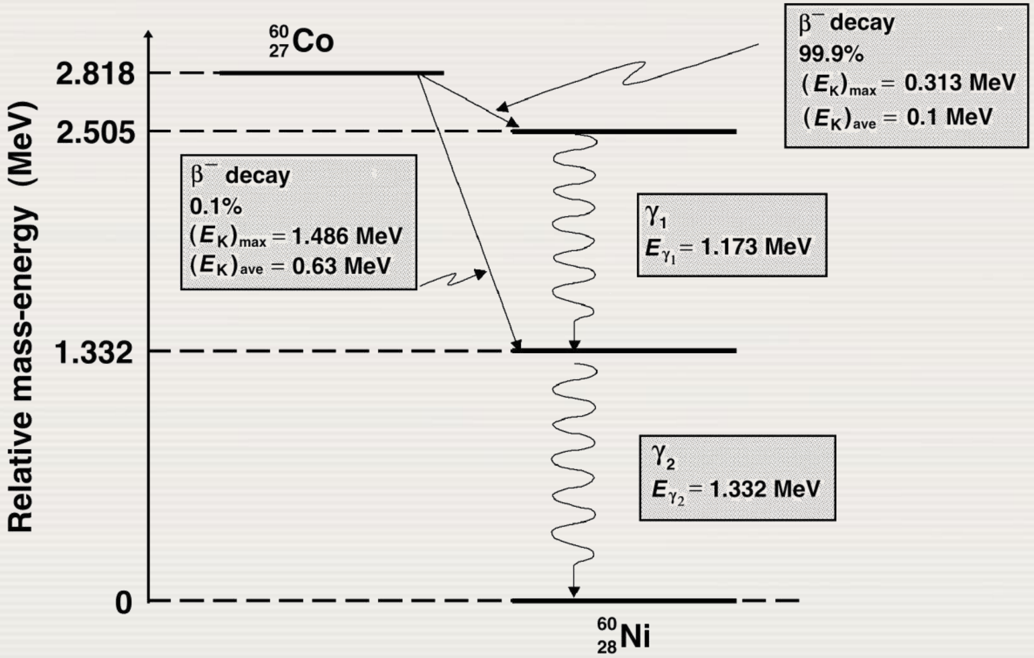
$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}$$

$$^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z+1}\mathrm{D} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- ☐ *Example* of beta minus decay is the transformation of cobalt-60 into nickel-60 with a half life of 5.26 y.



$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + e^- + \bar{\nu}_e$$

$$^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}_e$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

☐ Electron capture decay is a nuclear transformation in which:

- Nucleus captures an atomic orbital electron (usually K shell).
- Proton transforms into a neutron.
- Neutrino is ejected.
- Atomic number *Z* of the parent decreases by one; the atomic mass number *A* remains the same.
- Number of nucleons and total charge are conserved in the beta decay process and the daughter D can be referred to as an isobar of the parent P.

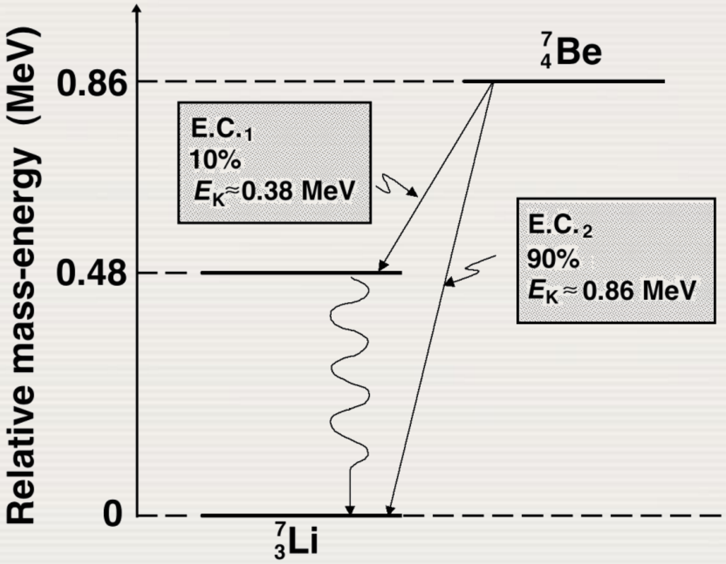
$$p + e^{-} = n + \nu_e$$

$${}^{A}_{Z}P + e^{-} = {}^{A}_{Z-1}D + \nu_e$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- *Example* of nuclear decay by electron capture is the transformation of berillium-7 into lithium-7



$$p + e^{-} = n + \nu_e$$

$$^{A}_{Z}P + e^{-} = {}^{A}_{Z+1}D + \nu_e$$

$$^{7}_{4}\text{Be} + e^{-} = {}^{7}_{3}\text{Li} + \nu_e$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- Gamma decay is a nuclear transformation in which an excited parent nucleus P, generally produced through alpha decay, beta minus decay or beta plus decay, attains its ground state through emission of one or several gamma photons.

- Atomic number Z and atomic mass number A do not change in gamma decay.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

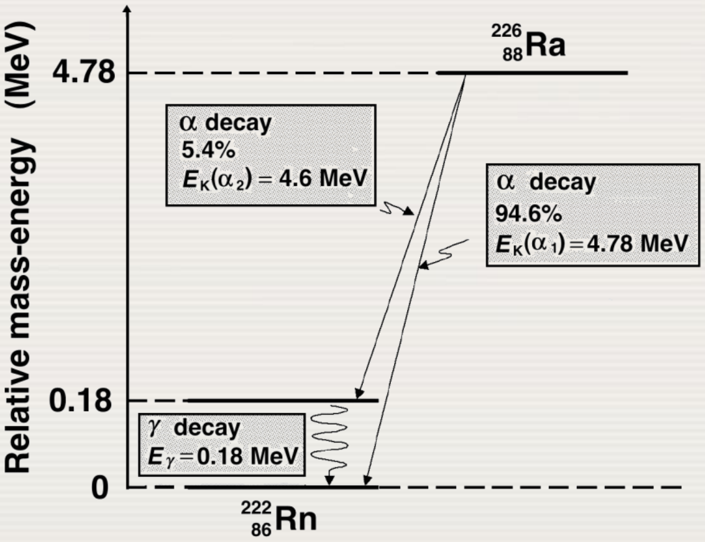
- In most alpha and beta decays the daughter de-excitation occurs instantaneously, so that we refer to the emitted gamma rays as if they were produced by the parent nucleus.

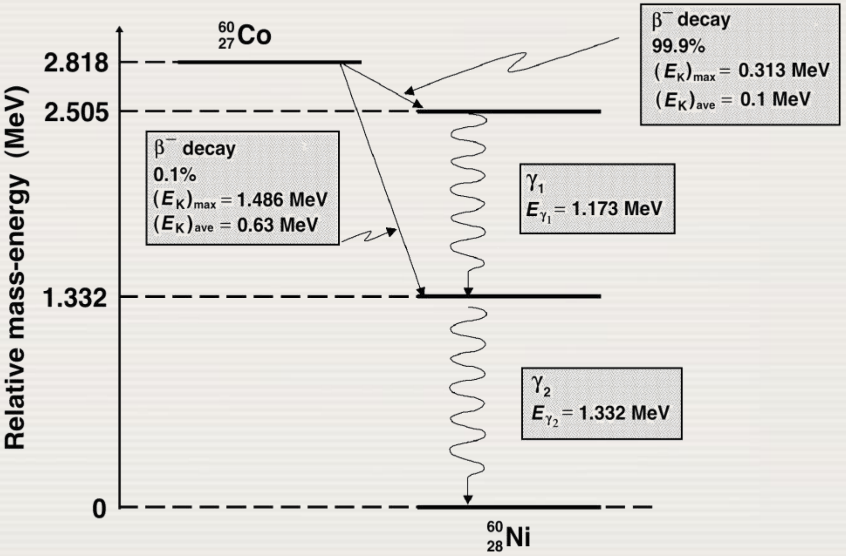
- If the daughter nucleus de-excites with a time delay, the excited state of the daughter is referred to as a metastable state and process of de-excitation is called an isomeric transition.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- *Examples* of gamma decay are the transformation of cobalt-60 into nickel-60 by beta minus decay, and transformation of radium-226 into radon-222 by alpha decay.





1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

☐ Internal conversion is a nuclear transformation in which:

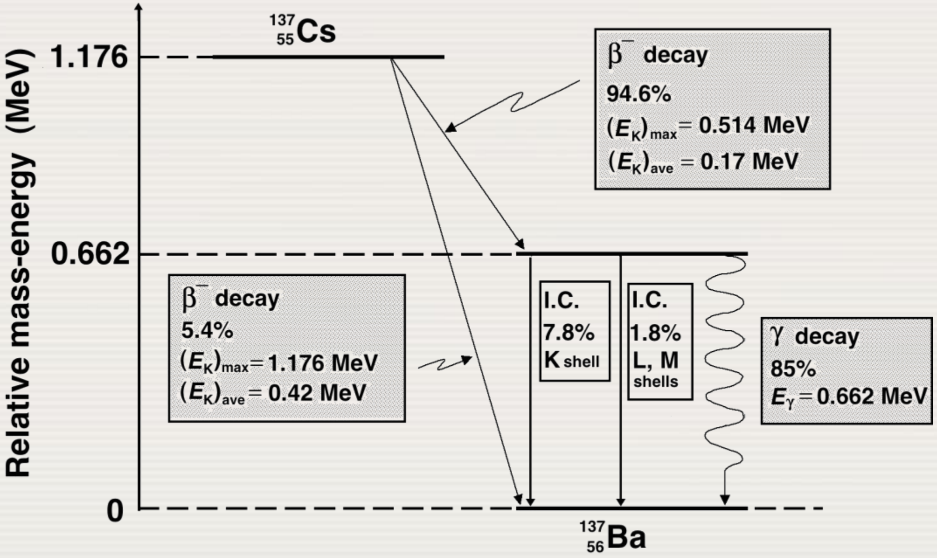
- Nuclear de-excitation energy is transferred to an orbital electron (usually K shell) .
- Electron is emitted form the atom with a kinetic energy equal to the de-excitation energy less the electron binding energy.
- Resulting shell vacancy is filled with a higher-level orbital electron and the transition energy is emitted in the form of characteristic photons or Auger electrons.

$$ {}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X^{+} + e^{-} $$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- ☐ *Example* for both the emission of gamma photons and emission of conversion electrons is the beta minus decay of cesium-137 into barium-137 with a half life of 30 years.



$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + e^- + \bar{\nu}_e$$

$$^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba + e^- + \bar{\nu}_e$$

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- Spontaneous fission is a nuclear transformation by which a high atomic mass nucleus spontaneously splits into two nearly equal fission fragments.
- Two to four neutrons are emitted during the spontaneous fission process.
- Spontaneous fission follows the same process as nuclear fission except that it is not self-sustaining, since it does not generate the neutron fluence rate required to sustain a “chain reaction”.

1.2 ATOMIC AND NUCLEAR STRUCTURE

1.2.9 Modes of radioactive decay

- In practice, spontaneous fission is only energetically feasible for nuclides with atomic masses above 230 u or with $Z^2/A \geq 235$.
- Spontaneous fission is a competing process to alpha decay; the higher is A above uranium-238, the more prominent is the spontaneous fission in comparison with the alpha decay and the shorter is the half-life for spontaneous fission.

1.3 ELECTRON INTERACTIONS

- ☐ As an energetic electron traverses matter, it undergoes Coulomb interactions with absorber atoms, i.e., with:
  - Atomic orbital electrons.
  - Atomic nuclei.

- ☐ Through these collisions the electrons may:
  - Lose their kinetic energy (collision and radiation loss).
  - Change direction of motion (scattering).

1.3 ELECTRON INTERACTIONS

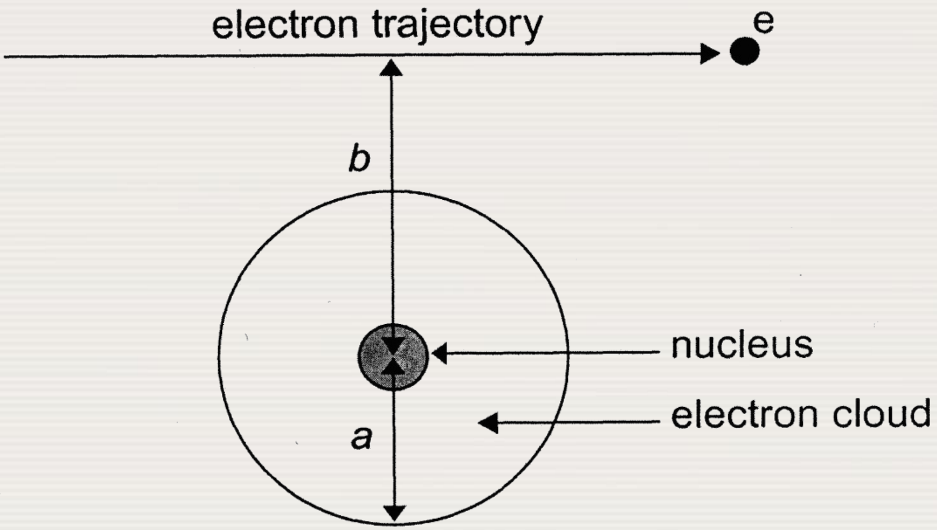
- Energy losses are described by stopping power.
- Scattering is described by angular scattering power.
- Collision between incident electron and absorber atom may be:
  - Elastic
  - Inelastic

1.3 ELECTRON INTERACTIONS

- ☐ In an elastic collision the incident electron is deflected from its original path but no energy loss occurs.
- ☐ In an inelastic collision with orbital electron the incident electron is deflected from its original path and loses part of its kinetic energy.
- ☐ In an inelastic collision with nucleus the incident electron is deflected from its original path and loses part of its kinetic energy in the form of bremsstrahlung.

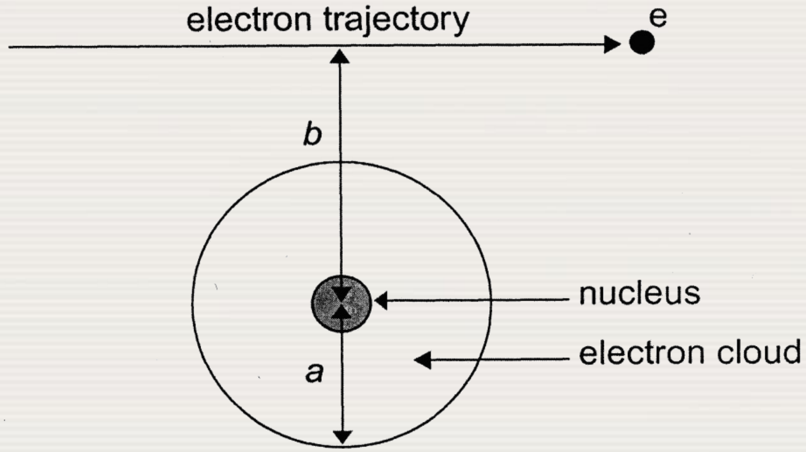
1.3 ELECTRON INTERACTIONS

Type of inelastic interaction that an electron undergoes with a particular atom of radius *a* depends on the impact parameter *b* of the interaction.



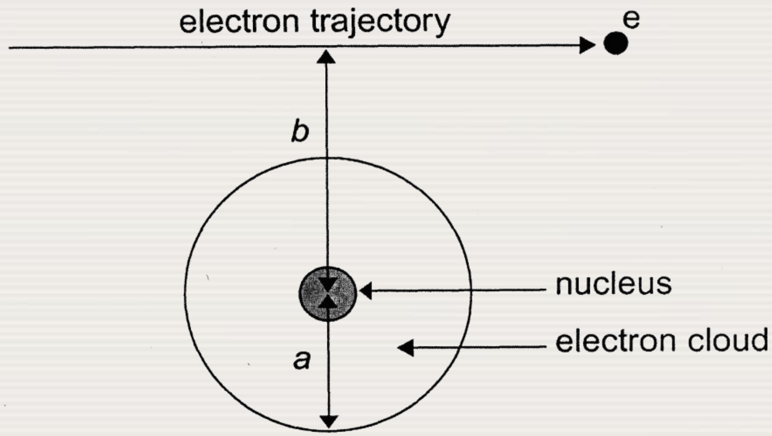
1.3 ELECTRON INTERACTIONS

- For $b \gg a$, incident electron will undergo a soft collision with the whole atom and only a small amount of its kinetic energy (few %) will be transferred from the incident electron to orbital electron.



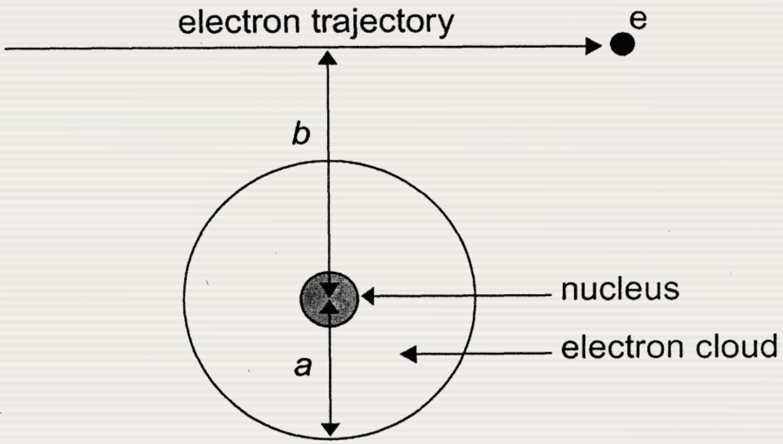
1.3 ELECTRON INTERACTIONS

- For $b \approx a$, incident electron will undergo a hard collision with an orbital electron and a significant fraction of its kinetic energy (up to 50 %) will be transferred to the orbital electron.



1.3 ELECTRON INTERACTIONS

- For $b \ll a$, incident electron will undergo a radiation collision with the atomic nucleus and emit a bremsstrahlung photon with energy between 0 and the incident electron kinetic energy.



1.3 ELECTRON INTERACTIONS

1.3.1 Electron-orbital electron interactions

☐ Inelastic collisions between the incident electron and an orbital electron are Coulomb interactions that result in:

- Atomic ionization:

  Ejection of the orbital electron from the absorber atom.
- Atomic excitation:

  Transfer of an atomic orbital electron from one allowed orbit (shell) to a higher level allowed orbit.

☐ Atomic ionizations and excitations result in collision energy losses experienced by the incident electron and are characterized by collision (ionization) stopping power.

1.3 ELECTRON INTERACTIONS

1.3.2 Electron-nucleus interaction

☐ Coulomb interaction between the incident electron and an absorber nucleus results in:

- Electron scattering and no energy loss (elastic collision):

  characterized by angular scattering power.

- Electron scattering and some loss of kinetic energy in the form of bremsstrahlung (radiation loss):

  characterized by radiation stopping power.

1.3 ELECTRON INTERACTIONS

1.3.2 Electron-nucleus interaction

☐ Bremsstrahlung production is governed by the Larmor relationship:

$$P = \frac{q^2 a^2}{6\pi\varepsilon_o c^3} \, .$$

Power *P* emitted in the form of bremsstrahlung photons from a charged particle with charge *q* accelerated with acceleration *a* is proportional to:

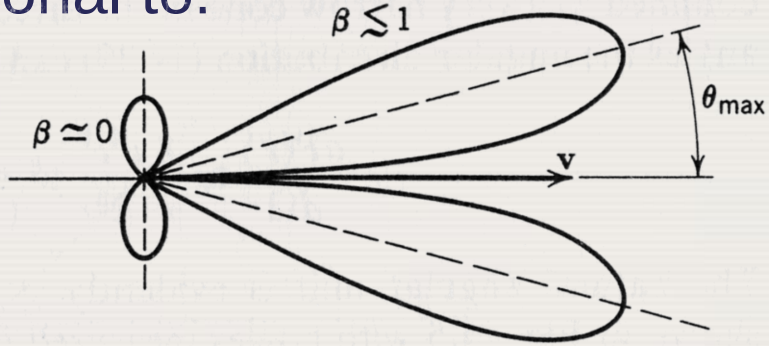
- Square of the particle acceleration *a.*
- Square of the particle charge *q.*

1.3 ELECTRON INTERACTIONS

1.3.2 Electron-nucleus interactions

☐ Angular distribution of the emitted bremsstrahlung photons is in general proportional to:

$$\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$



- At small particle velocity $(u \ll c$, i.e., $\beta = (u/c) \to 0)$ the angular distribution of emitted photons is proportional to $\sin^2\theta$.

- Angle $\theta_{\max}$ at which the photon intensity is maximum is:

$$\theta_{\max} = \arccos\left[\frac{1}{3\beta}\left(\sqrt{1+15\beta}-1\right)\right]$$

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- ☐ Energy loss by incident electron through inelastic collisions is described by total linear stopping power S_{tot} which represents kinetic energy E_{K} loss by the electron per unit path length x:

$$S_{\mathrm{tot}} = \frac{\mathrm{d}E_{\mathrm{K}}}{\mathrm{d}x} \quad \text{in} \quad \mathrm{MeV/cm}$$

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- Total mass stopping power $(S/\rho)_{\mathrm{tot}}$ is defined as the linear stopping power divided by the density of the absorbing medium.

$$\left(\frac{S}{\rho}\right)_{\mathrm{tot}} = \frac{1}{\rho}\frac{\mathrm{d}E_{\mathrm{K}}}{\mathrm{d}x} \quad \text{in} \quad \mathrm{MeV}\cdot\mathrm{cm}^2/\mathrm{g}$$

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- Total mass stopping power $(S/\rho)_{\text{tot}}$ consists of two components:
  - Mass collision stopping power $(S/\rho)_{\text{col}}$

    resulting from electron-orbital electron interactions (atomic ionizations and atomic excitations)
  - Mass radiation stopping power $(S/\rho)_{\text{rad}}$

    resulting mainly from electron-nucleus interactions (bremsstrahlung production)
  - Total mass stopping power is the sum of the two components

$$\left(\frac{S}{\rho}\right)_{\text{tot}} = \left(\frac{S}{\rho}\right)_{\text{col}} + \left(\frac{S}{\rho}\right)_{\text{rad}}$$

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- ☐ For heavy charged particles the radiation stopping power $(S/\rho)_{\mathrm{rad}}$ is negligible thus $(S/\rho)_{\mathrm{tot}} \approx (S/\rho)_{\mathrm{col}}$
- ☐ For light charged particles both components contribute to the total stopping power thus $(S/\rho)_{\mathrm{tot}} = (S/\rho)_{\mathrm{col}} + (S/\rho)_{\mathrm{rad}}$
  - Within a broad range of kinetic energies below 10 MeV collision (ionization) losses are dominant $(S/\rho)_{\mathrm{col}} > (S/\rho)_{\mathrm{rad}}$; however, the situation is reversed at high kinetic energies.
  - Cross over between the two modes occurs at a critical kinetic energy $(E_{\mathrm{K}})_{\mathrm{crit}}$ where the two stopping powers are equal

$$(E_{\mathrm{K}})_{\mathrm{crit}} \approx \frac{800\ \mathrm{MeV}}{Z}$$

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- ☐ Electrons traversing an absorber lose their kinetic energy through ionization collisions and radiation collisions.

- ☐ Rate of energy loss per gram and per cm^2 is called the mass stopping power and it is a sum of two components:
  - Mass collision stopping power
  - Mass radiation stopping power

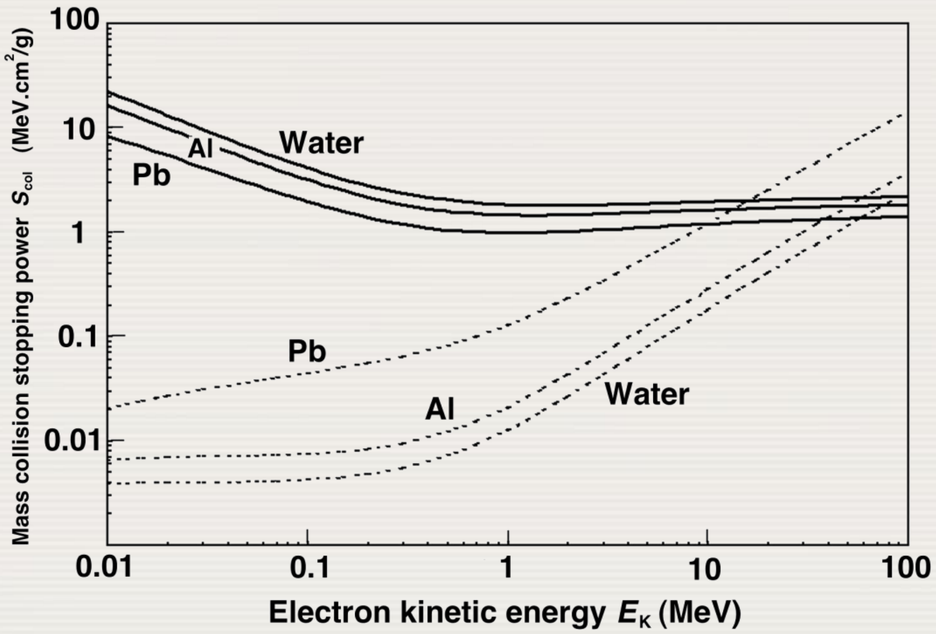
- ☐ Rate of energy loss for a therapy electron beam in water and water-like tissues, averaged over the electron’s range, is about 2 MeV/cm.

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

☐ Rate of energy loss for collision interactions depends on:

- Kinetic energy of the electron.
- Electron density of the absorber.



Rate of collision energy loss is greater for low atomic number *Z* absorbers than for high *Z* absorbers, because high *Z* absorbers have lower electron density (fewer electrons per gram).

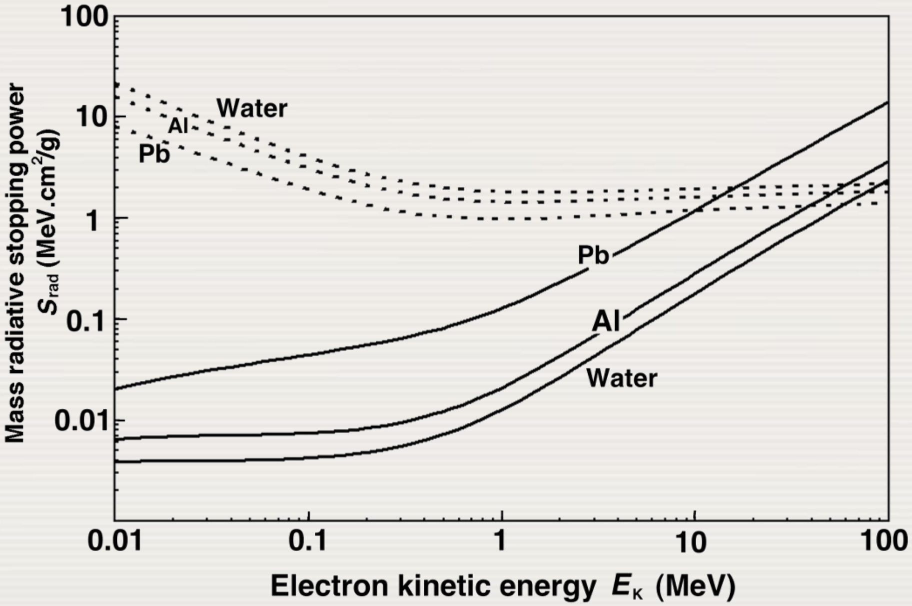
Solid lines: mass collision stopping power

Dotted lines: mass radiation stopping power

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- Rate of energy loss for radiation interactions (bremsstrahlung) is approximately proportional to:
  - Kinetic energy of the electron.
  - Square of the atomic number of the absorber.



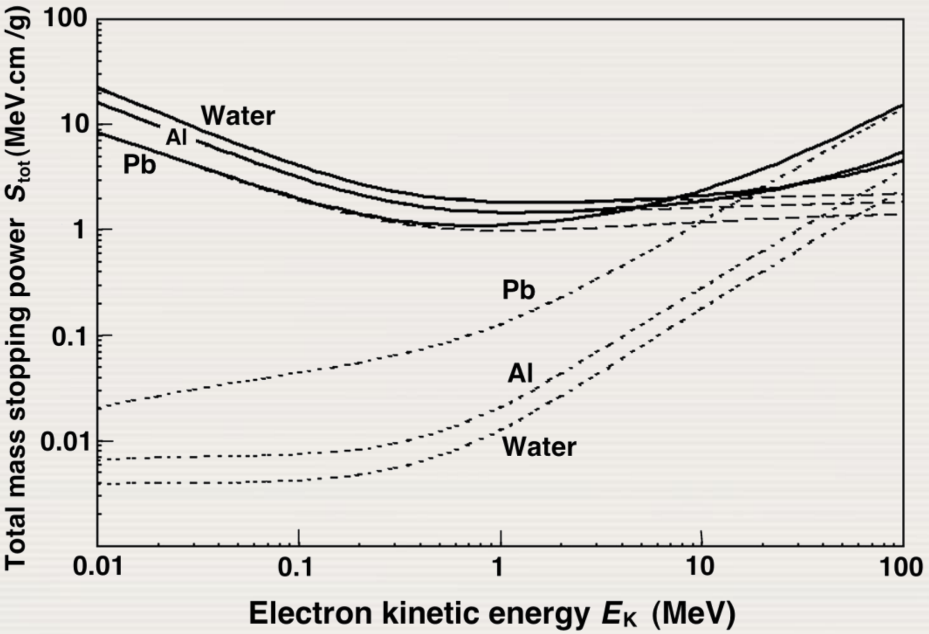
Bremsstrahlung production through radiation losses is more efficient for higher energy electrons and higher atomic number absorbers

Solid lines: mass radiation stopping power

Dotted lines: mass collision stopping power

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power



Solid lines: total mass stopping power
Dashed lines: mass collision stopping power
Dotted lines: mass radiation stopping power

Total energy loss by electrons traversing an absorber depends upon:

- Kinetic energy of the electron
- Atomic number of the absorber
- Electron density of the absorber

Total mass stopping power is the sum of mass collision and mass radiation stopping powers

$$\left(\frac{S}{\rho}\right)_{\text{tot}} = \left(\frac{S}{\rho}\right)_{\text{col}} + \left(\frac{S}{\rho}\right)_{\text{rad}}$$

1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- Total mass stopping power $(S/\rho)_{tot}$ for electrons in water, aluminum and lead against the electron kinetic energy (solid curves).

Solid lines:

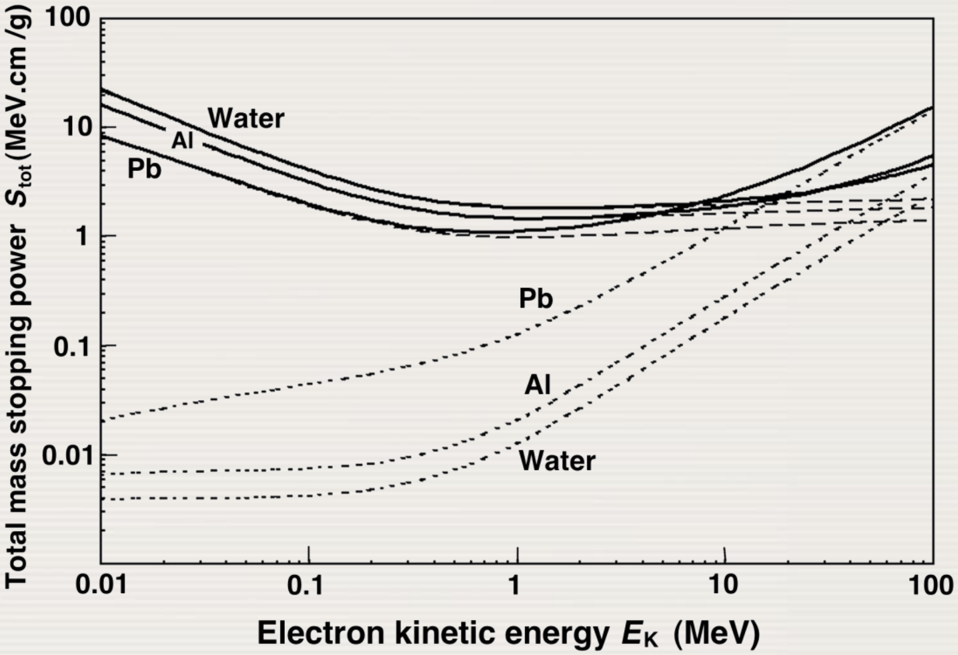
  total mass stopping power

Dashed lines:

  mass collision stopping power

Dotted lines:

  mass radiation stopping power



1.3 ELECTRON INTERACTIONS

1.3.3 Stopping power

- ☐ $(S/\rho)_{\text{tot}}$ is used in the calculation of particle range R

$$R = \int_0^{E_K} \left(\frac{S}{\rho}(E_K) \right)_{\text{tot}}^{-1} \mathrm{d}E_K$$

- ☐ Both $(S/\rho)_{\text{tot}}$ and $(S/\rho)_{\text{rad}}$ are used in the determination of radiation yield $Y(E_K)$

$$Y = \frac{1}{E_K} \int_0^{E_K} \frac{(S/\rho)_{\text{rad}}}{(S/\rho)_{\text{tot}}} \mathrm{d}E_K$$

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1.3 ELECTRON INTERACTIONS

1.3.4 Mass angular scattering power

- Angular and spatial spread of a pencil electron beam traversing an absorbing medium can be approximated with a Gaussian distribution.

- Multiple Coulomb scattering of electrons traversing a path length ℓ is commonly described by the mean square scattering angle $\overline{\theta^2}$ proportional to the mass thickness $r\ell$.

- Mass angular scattering power T/ρ is defined as

$$\frac{T}{r} = \frac{1}{r}\frac{\mathrm{d}\overline{q^2}}{\mathrm{d}\ell} = \frac{\overline{q^2}}{r\ell}$$

1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

Ionizing photon radiation is classified into four categories:

- Characteristic x ray
  - Results from electronic transitions between atomic shells
- Bremsstrahlung
  - Results mainly from electron-nucleus Coulomb interactions
- Gamma ray
  - Results from nuclear transitions
- Annihilation quantum (annihilation radiation)
  - Results from positron-electron annihilation

1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

- In penetrating an absorbing medium, photons may experience various interactions with the atoms of the medium, involving:
  - Absorbing atom as a whole
  - Nuclei of the absorbing medium
  - Orbital electrons of the absorbing medium.

1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

☐ Interactions of photons with nuclei may be:

- Direct photon-nucleus interactions (photodisintegration)

  or

- Interactions between the photon and the electrostatic field of the nucleus (pair production).

☐ Photon-orbital electron interactions are characterized as interactions between the photon and either

- Loosely bound electron (Compton effect, triplet production)

  or

- Tightly bound electron (photoelectric effect).

1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

- Loosely bound electron is an electron whose binding energy E_B to the nucleus is small compared to the photon energy $h\nu$.

$$E_B << h\nu$$

- Interaction between a photon and a loosely bound electron is considered to be an interaction between a photon and a free (unbound) electron.

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1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

- ☐ Tightly bound electron is an electron whose binding energy E_B is comparable to, larger than, or slightly smaller than the photon energy $h\nu$.

- ☐ For a photon interaction to occur with a tightly bound electron, the binding energy E_B of the electron must be of the order of, but slightly smaller, than the photon energy.

$$E_B \leq h\nu$$

- ☐ Interaction between a photon and a tightly bound electron is considered an interaction between a photon and the atom as a whole.

1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

❑ As far as the photon fate after the interaction with an atom is concerned there are two possible outcomes:

- Photon disappears (i.e., is absorbed completely) and a portion of its energy is transferred to light charged particles (electrons and positrons in the absorbing medium).
- Photon is scattered and two outcomes are possible:
    - The resulting photon has the same energy as the incident photon and no light charged particles are released in the interaction.
    - The resulting scattered photon has a lower energy than the incident photon and the energy excess is transferred to a light charged particle (electron).

1.4 PHOTON INTERACTIONS

1.4.1 Types of indirectly ionizing photon irradiations

☐ Light charged particles produced in the absorbing medium through photon interactions will:

  - Either deposit their energy to the medium through Coulomb interactions with orbital electrons of the absorbing medium (collision loss also referred to as ionization loss).
  - Or radiate their kinetic energy away through Coulomb interactions with the nuclei of the absorbing medium (radiation loss).

1.4 PHOTON INTERACTIONS

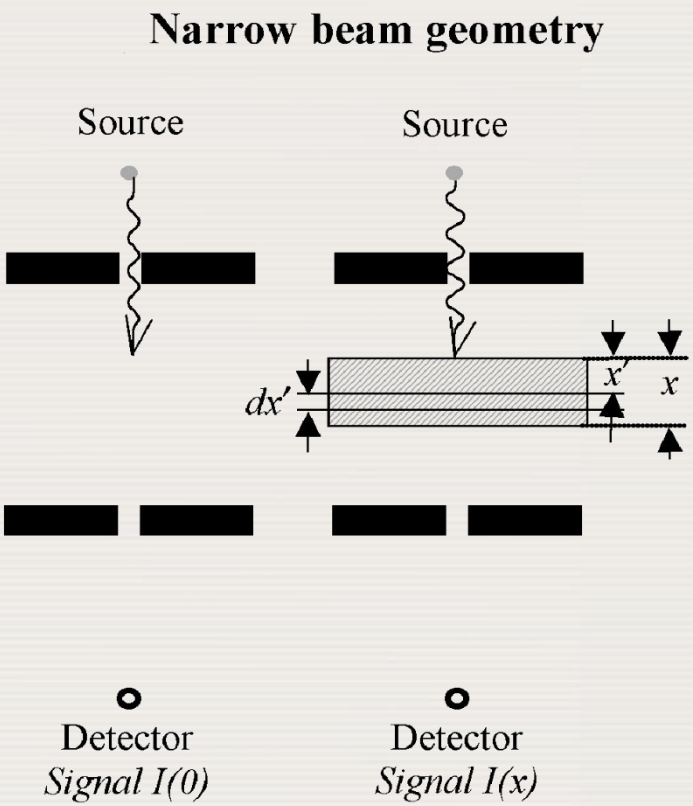
1.4.2 Photon beam attenuation

- The most important parameter used for characterization of x-ray or gamma ray penetration into absorbing media is the linear attenuation coefficient μ.
- Linear attenuation coefficient μ depends upon:
  - Energy $h\nu$ of the photon beam
  - Atomic number Z of the absorber
- Linear attenuation coefficient may be described as the probability per unit path length that a photon will have an interaction with the absorber.

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- Attenuation coefficient μ is determined experimentally using the so-called narrow beam geometry technique that implies a narrowly collimated source of mono-energetic photons and a narrowly collimated detector.
  - x represents total thickness of the absorber
  - x' represents the thickness variable.



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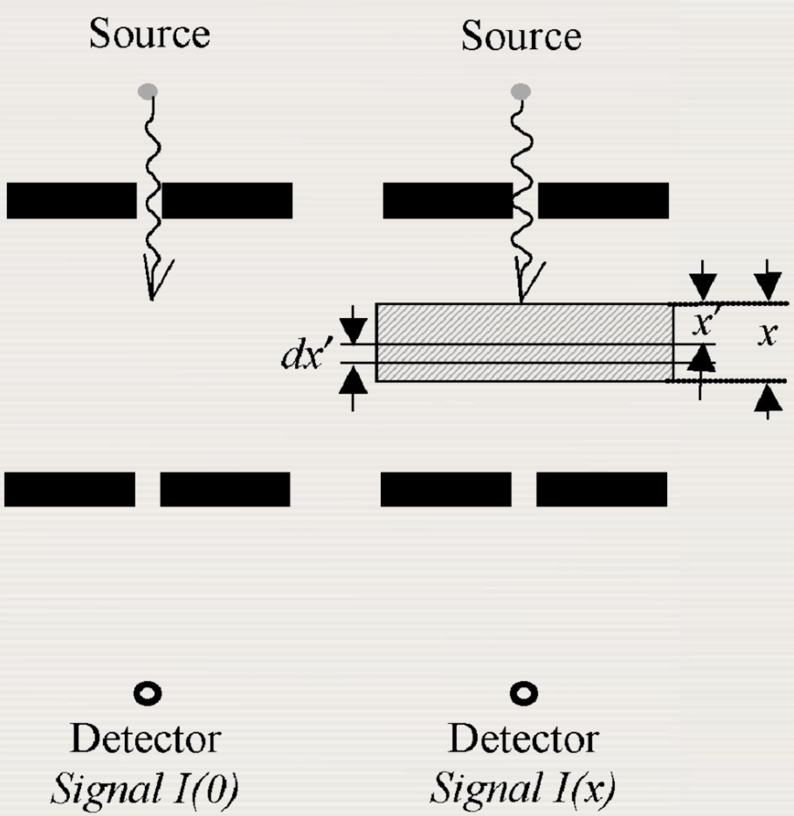
1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- ☐ A slab of absorber material of thickness *x* decreases the detector signal intensity from $I(0)$ to $I(x)$.
- ☐ A layer of thickness d*x*' reduces the beam intensity by dI and the fractional reduction in intensity, -dI/I is proportional to
  - Attenuation coefficient μ .
  - Layer thickness d*x*'.

Narrow beam geometry



1.4 PHOTON INTERACTIONS

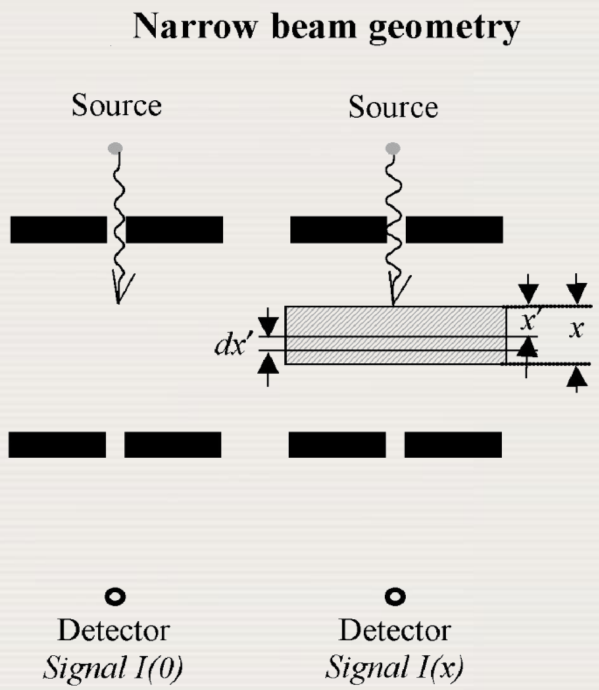
1.4.2 Photon beam attenuation

- Fractional reduction in intensity is given as:

$$-\frac{\mathrm{d}I}{I} = \mu x$$

- After integration from 0 to x we obtain

$$\int_{I(0)}^{I(x)} \frac{\mathrm{d}I}{I} = -\int_{0}^{x} \mu dx' \quad \text{or} \quad I(x) = I(0)e^{-\int_{0}^{x} \mu dx'}$$


Narrow beam geometry

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- For a homogeneous medium $\mu = $ const and one gets the standard exponential relationship valid for monoenergetic photon beams:

$$I(x) = I(0)e^{-\mu x}$$

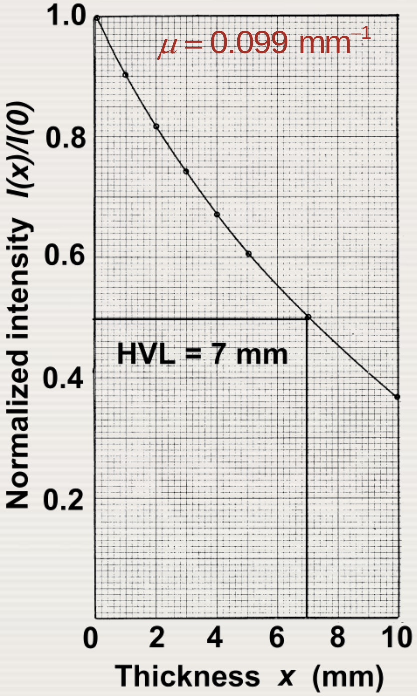
or

$$I(x)/I(0) = e^{-\mu x}$$

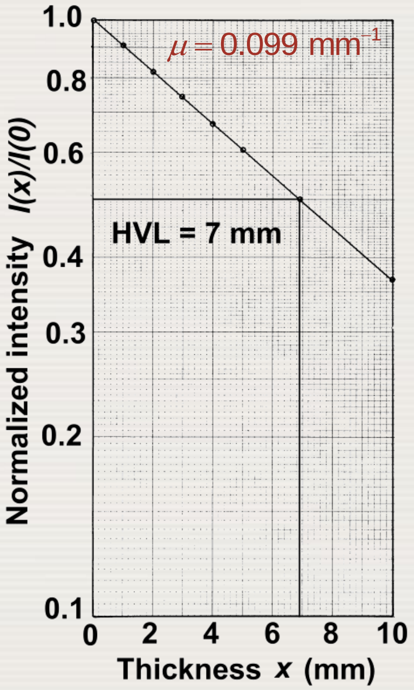
For x = HVL

$$\frac{I(x)}{I(0)} = 0.5$$

Linear graph paper



Semi-log graph paper



1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

☐ Several thicknesses of special interest are defined as parameters for mono-energetic photon beam characterization in narrow beam geometry:

- Half-value layer (HVL$_1$ or $x_{1/2}$)

  Absorber thickness that attenuates the original intensity to 50 %.

- Mean free path (MFP or $\bar{x}$)

  Absorber thickness which attenuates the beam intensity to 1/e = 36.8 %.

- Tenth-value layer (TVL or $x_{1/10}$)

  Absorber thickness which attenuates the beam intensity to 10 %.

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- ☐ The relationship for $x_{1/2}$, $\bar{x}$, and $x_{1/10}$ is:

$$\mu = \frac{\ln 2}{x_{1/2}} = \frac{1}{\bar{x}} = \frac{\ln 10}{x_{1/10}}$$

or

$$x_{1/2} = (\ln 2)\bar{x} = \frac{\ln 2}{\ln 10} x_{1/10} \approx 0.3 x_{1/10}$$

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1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- ☐ In addition to the linear attenuation coefficient μ other related attenuation coefficients and cross sections are in use for describing photon beam attenuation:
  - Mass attenuation coefficient μ_m
  - Atomic cross section ${}_a\mu$
  - Electronic cross section ${}_e\mu$

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- Basic relationships:

$$m = r\, m_{m} = n^{\square}\, {}_{a}m = n^{\square} Z\, {}_{e}m$$

$$n^{\square} = \frac{N_{a}}{V} = r \frac{N_{a}}{m} = r \frac{N_{A}}{A}$$

where $n^{\square}$ is the number of atoms per volume of absorber with density ρ and atomic mass A.

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

	Symbol	***Relationship to*** μ	***Units***
Linear attenuation coefficient	μ	μ	cm^{-1}
Mass attenuation coefficient	μ_m	$\frac{\mu}{\rho}$	cm^2 / g
Atomic cross section	${}_a\mu$	$\frac{\mu}{n^{\square}}$	cm^2 / atom
Electronic cross section	${}_e\mu$	$\frac{\mu}{Zn^{\square}}$	cm^2 / electron

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1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- ☐ Energy transfer coefficient $\mu_{\text{tr}} = \mu \frac{\bar{E}_{\text{tr}}}{h\nu}$,

  with $\bar{E}_{\text{tr}}$ the average energy transferred from the primary photon with energy $h\nu$ to kinetic energy of charged particles (e^- and e^+).

- ☐ Energy absorption coefficient $\mu_{\text{ab}} = \mu \frac{\bar{E}_{\text{ab}}}{h\nu}$,

with $\bar{E}_{\text{ab}}$ the average energy absorbed in the volume of interest in the absorbing medium.

In the literature, μ_{en} is usually used instead of μ_{ab} , however, the the use of subscript “ab” for energy absorbed compared to the subscript “tr” for energy transferred seems more logical.

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- Average (mean) energy absorbed in the volume of interest

$$\bar{E}_{ab} = \bar{E}_{tr} - \bar{E}_{rad}$$

with $\bar{E}_{rad}$ the average energy component of $\bar{E}_{tr}$ which the charged particles lose in the form of radiation collisions (bremsstrahlung) and is not absorbed in the volume of interest.

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- Linear energy absorption coefficient is

$$\mu_{\mathrm{ab}} = \mu \frac{\bar{E}_{\mathrm{ab}}}{h\nu} = \mu \frac{\bar{E}_{\mathrm{tr}} - \bar{E}_{\mathrm{rad}}}{h\nu} = \mu_{\mathrm{tr}} - \mu_{\mathrm{tr}} \frac{\bar{E}_{\mathrm{rad}}}{\bar{E}_{\mathrm{tr}}} = \mu_{\mathrm{tr}}(1 - \bar{g})$$

where $\bar{g}$ is the so-called radiation fraction (the average fraction of the energy lost in radiation interactions by the secondary charged particles, as they travel through the absorber).

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

☐ Mass attenuation coefficient of a compound or a mixture is approximated by a summation of a weighted average of its constituents:

$$\frac{\mu}{\rho} = \sum_{\mathrm{i}} w_{\mathrm{i}} \frac{\mu_{\mathrm{i}}}{\rho}$$

- w_{i} is the proportion by weight of the i-th constituent.
- μ_{i}/ρ is the mass attenuation coefficient of the i-th constituent.

1.4 PHOTON INTERACTIONS

1.4.2 Photon beam attenuation

- ☐ Attenuation coefficient μ has a specific value for a given photon energy $h\nu$ and absorber atomic number Z.

- ☐ The value for the attenuation coefficient $\mu(h\nu,Z)$ for a given photon energy $h\nu$ and absorber atomic number Z represents a sum of values for all individual interactions that a photon may have with an atom:

$$\mu = \sum_i \mu_i$$

1.4 PHOTON INTERACTIONS

1.4.3 Types of photon interactions with absorber

- ☐ According to the type of target there are two possibilities for photon interaction with an atom:
    - Photon - orbital electron interaction
    - Photon - nucleus interaction
- ☐ According to the type of event there are two possibilities for photon interaction with an atom:
    - Complete absorption of the photon
    - Scattering of the photon

1.4 PHOTON INTERACTIONS

1.4.3 Types of photon interactions with absorber

☐ In medical physics photon interactions fall into four groups:

- Interactions of major importance:
  - Photoelectric effect.
  - Compton scattering by free electron.
  - Pair production (including triplet production).
- Interactions of moderate importance:
  - Rayleigh scattering.
  - Thomson scattering by free electron.
- Interactions of minor importance
  - Photonuclear reactions (nuclear photoelectric effect)
- Negligible interactions:
  - Thomson and Compton scattering by the nucleus.
  - Meson production.
  - Delbrück scattering.

1.4 PHOTON INTERACTIONS

1.4.3 Types of photon interactions with absorber

Interaction	Symbol for electronic cross section	Symbol for atomic cross section	Symbol for linear attenuation coefficient
Thomson scattering	${}_{e}\sigma_{Th}$	${}_{a}\sigma_{Th}$	σ_{Th}
Rayleigh scattering	-	${}_{a}\sigma_{R}$	σ_{R}
Compton scattering	${}_{e}\sigma_{c}$	${}_{a}\sigma_{c}$	σ_{C}
Photoelectric effect	-	${}_{a}\tau$	τ
Pair production	-	${}_{a}\kappa_{pp}$	κ_{p}
Triplet production	${}_{e}\kappa_{tp}$	${}_{a}\kappa_{tp}$	κ_{t}
Photodisintegration	-	${}_{a}\sigma_{pn}$	σ_{pn}

1.4 PHOTON INTERACTIONS

1.4.3 Types of photon interactions with absorber

☐ TYPES OF TARGETS IN PHOTON INTERACTIONS

Photon-orbital electron interaction

- with bound electron
  - Photoelectric effect
  - Rayleigh scattering
- with “free” electrons
  - Thomson scattering
  - Compton scattering
- with Coulomb field of electron
  - Triplet production

Photon-nucleus interaction

- with nucleus directly
  - Photodisintegration
- with Coulomb field of nucleus
  - Pair production

1.4 PHOTON INTERACTIONS

1.4.3 Types of photon interactions with absorber

- Types of photon-atom interactions

Complete absorption of photon	**Photon scattering**
Photoelectric effect	Thomson scattering
Pair production	Rayleigh scattering
Triplet production	Compton scattering
Photodisintegration	

Photoelectric effect:

- ☐ Photon of energy $h\nu$ interacts with a tightly bound electron, i.e., with whole atom.
- ☐ Photon disappears.
- ☐ Conservation of energy and momentum considerations show that photoelectric effect can occur only on a tightly bound electron rather than on a loosely bound (“free”) electron.

1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

- ☐ Orbital electron is ejected from the atom with kinetic energy

  $$E_K = h\nu - E_B,$$

  where E_B is the binding energy of the orbital electron.

- ☐ Ejected orbital electron is called a photoelectron.

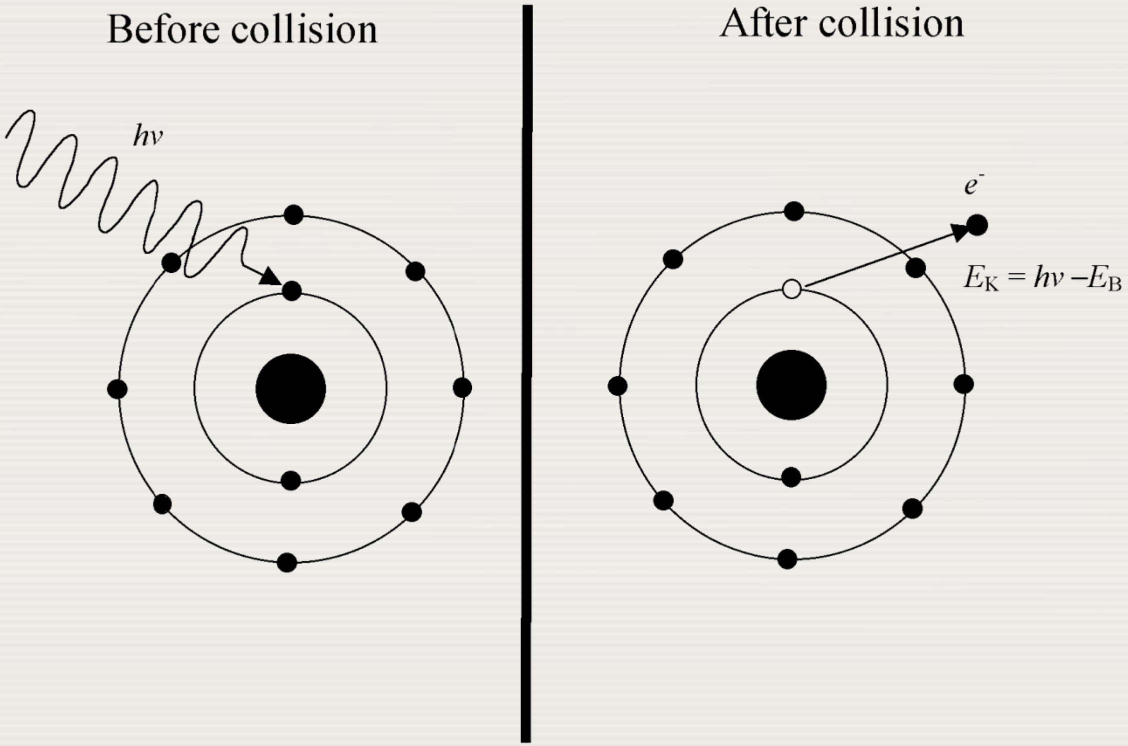
- ☐ When the photon energy $h\nu$ exceeds the K-shell binding energy $E_B(K)$ of the absorber atom, the photoelectric effect is most likely to occur with a K-shell electron in comparison with higher shell electrons.

1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

❑ Schematic diagram of the photoelectric effect

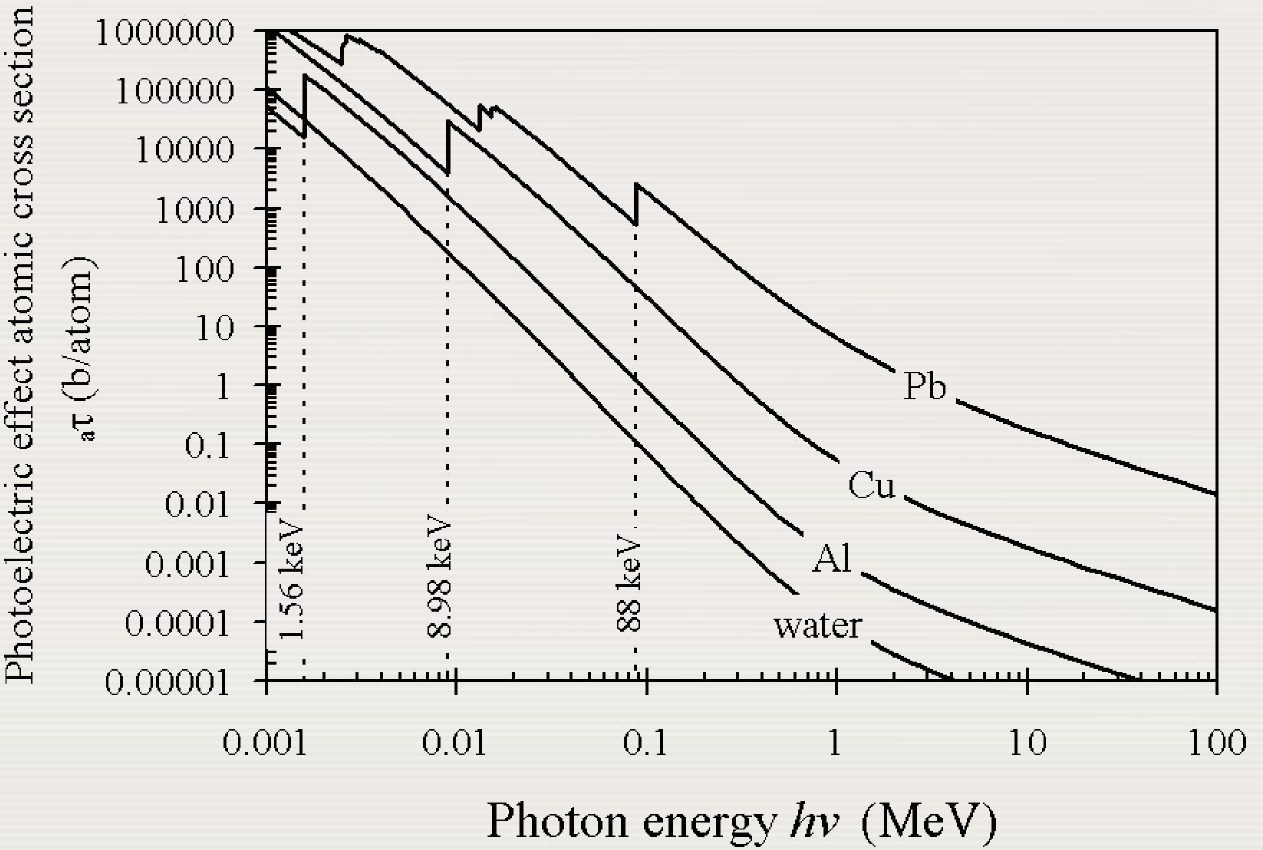
- A photon with energy $h\nu$ interacts with a K-shell orbital electron.
- Orbital electron is emitted from the atom as a photoelectron.



1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

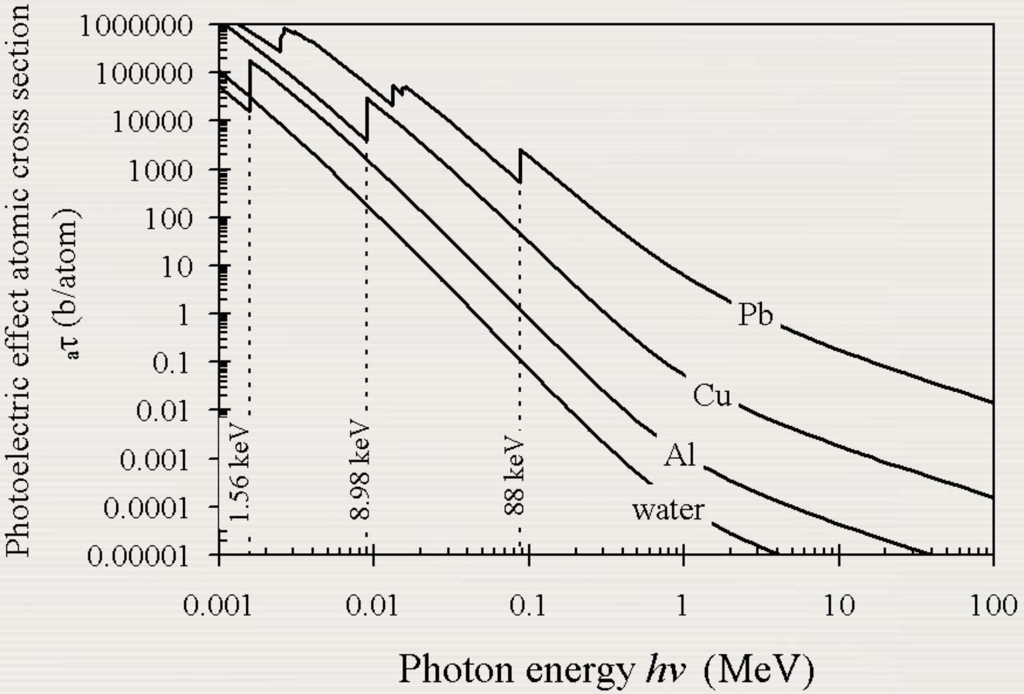
- Photoelectric atomic cross sections for water, aluminum, copper and lead against photon energy.



1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

- Atomic attenuation coefficient ${}_{a}\tau$ for photoelectric effect is proportional to $Z^4/(h\nu)^3$.
- Mass attenuation coefficient τ_m for photoelectric effect is proportional to $Z^3/(h\nu)^3$.

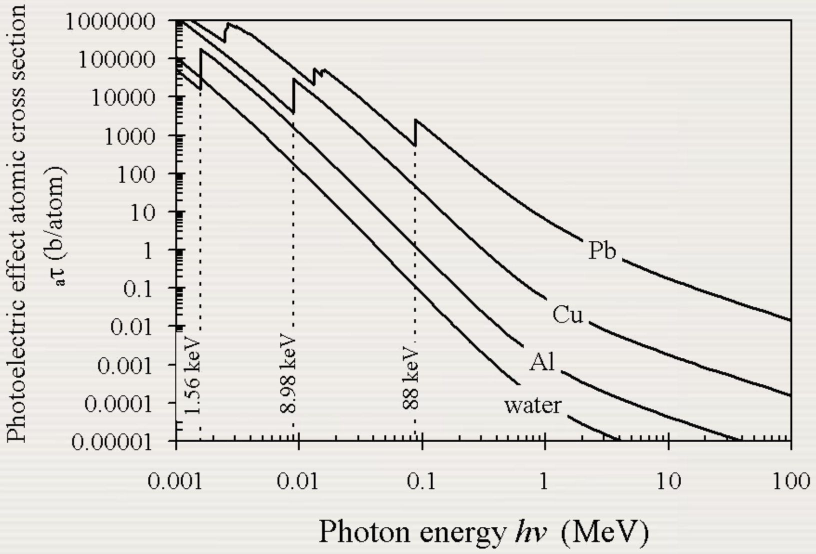


1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

- A plot of τ_m against $h\nu$ shows, in addition to a steady decrease in τ_m with increasing photon energy, sharp discontinuities when $h\nu$ equals the binding energy E_B for a particular electronic shell of the absorber.

- These discontinuities, called absorption edges, reflect the fact that for $h\nu < E_B$ photons cannot undergo photoelectric effect with electrons in the given shell, while for $h\nu \geq E_B$ they can.



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1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

- Average (mean) energy transferred from a photon with energy $h\nu > E_B(K)$ to electrons, $(\bar{E}_K)_{tr}^{PE}$, is given as:

$$(\bar{E}_K)_{tr}^{PE} = hn - P_K w_K E_B(K)$$

with

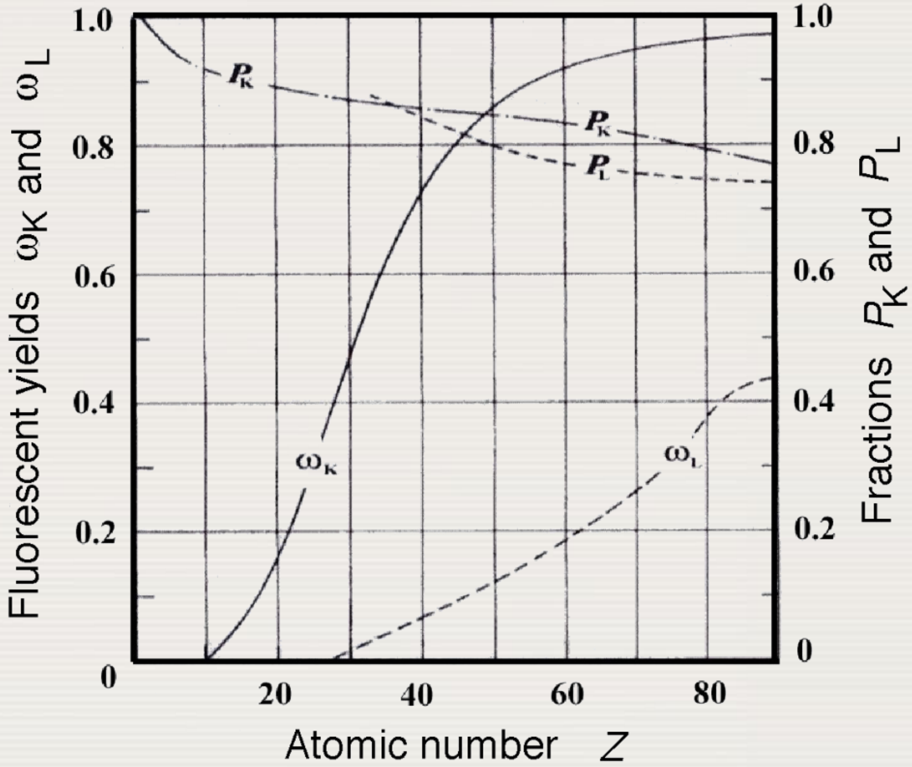
$E_B(K)$	binding energy of the K-shell electron (photoelectron).
P_K	fraction of all photoelectric interactions in the K shell.
ω_K	fluorescence yield for the K shell.

1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

Fluorescence yield w_X and function P_X

- Fluorescence yield w_X is defined as the number of photons emitted per vacancy in a given atomic shell X.
- Function P_X for a given shell gives the proportion of photoelectric events in given shell compared to the total number of photoelectric events in the whole atom.



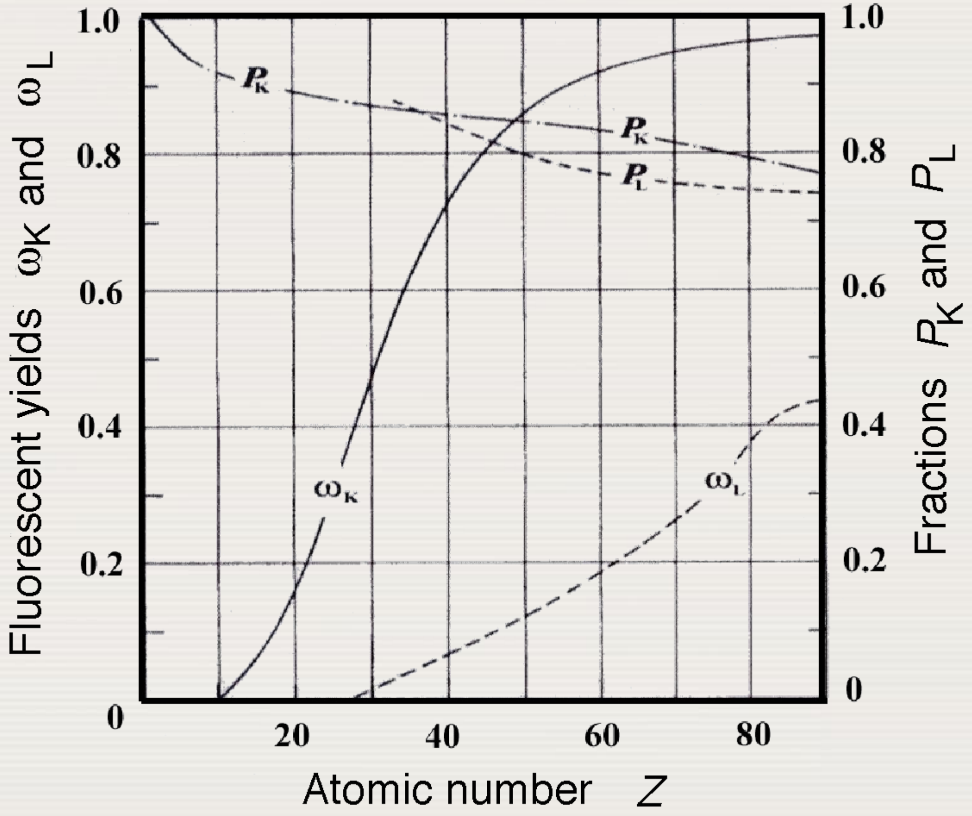
1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

- Fluorescence yields ω_K and ω_L and functions P_K and P_L

The range of P_K is from 1.0 at low atomic numbers Z to 0.8 at high atomic numbers Z of the absorber.

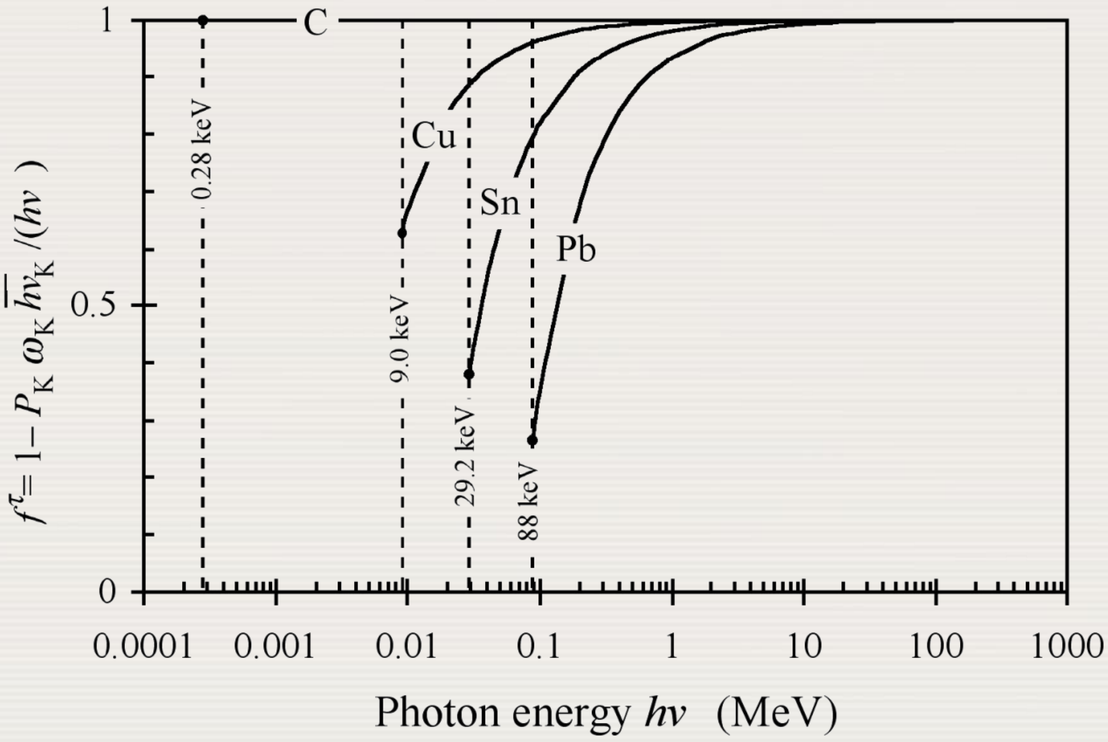
The range in ω_K is from 0 at low atomic numbers Z through 0.5 at Z = 30 to 0.96 at high Z.



1.4 PHOTON INTERACTIONS

1.4.4 Photoelectric effect

- Mean energy transfer fraction for photoelectric effect $\bar{f}_{PE}$ is:



$$\bar{f}_{PE} = \frac{hn - (\bar{E}_K)_{tr}^{PE}}{hn}$$

$$= 1 - \frac{P_K w_K E_B(K)}{hn}$$

1.4 PHOTON INTERACTIONS

1.4.5 Coherent (Rayleigh) scattering

Coherent (Rayleigh) scattering:

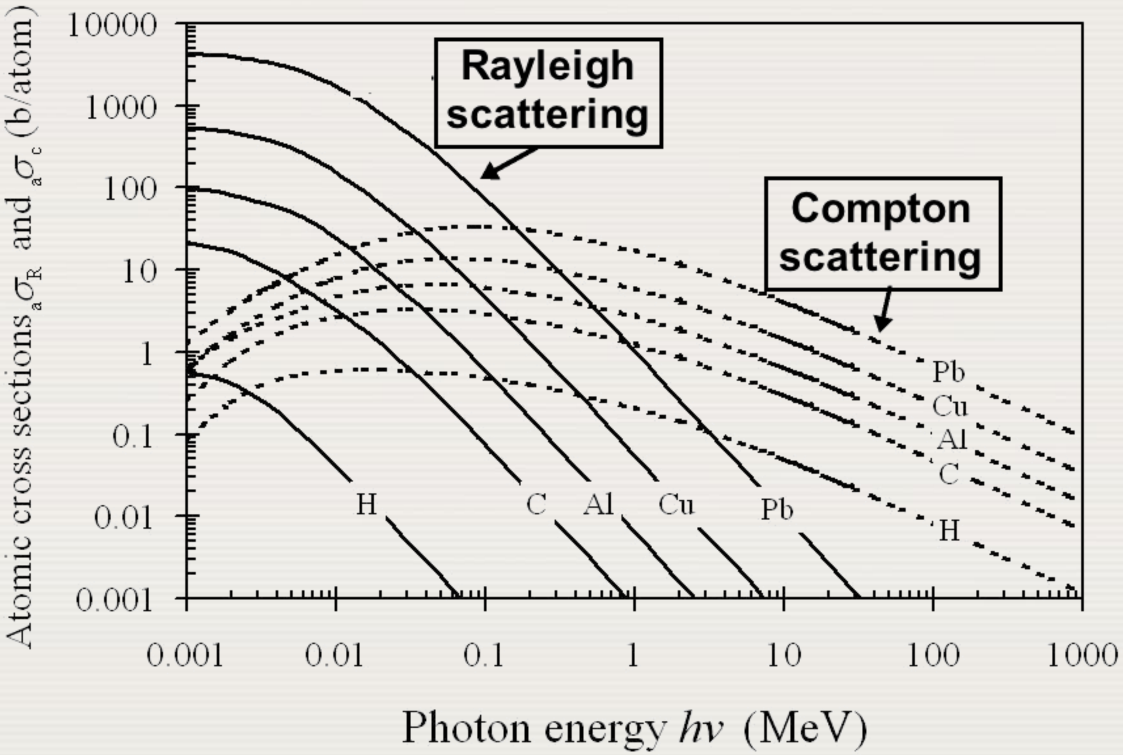
- In coherent (Rayleigh) scattering the photon interacts with a bound orbital electron, i.e., with the combined action of the whole atom.
- The event is elastic as the photon loses essentially none of its energy and is scattered through only a small angle.
- No energy transfer occurs from the photon to charged particles in the absorber; thus Rayleigh scattering plays no role in the energy transfer coefficient but it contributes to the attenuation coefficient.

1.4 PHOTON INTERACTIONS

1.4.5 Coherent (Rayleigh) scattering

❑ Coefficients for coherent (Rayleigh) scattering

- Atomic cross section is proportional to $(Z/h\nu)^2$.
- Mass attenuation coefficient is proportional to $(Z/h\nu)^2$.



1.4 PHOTON INTERACTIONS

1.4.6 Compton (Incoherent) scattering

Compton (incoherent) scattering

- In Compton effect (incoherent scattering) a photon with energy $h\nu$ interacts with a loosely bound (“free”) electron.

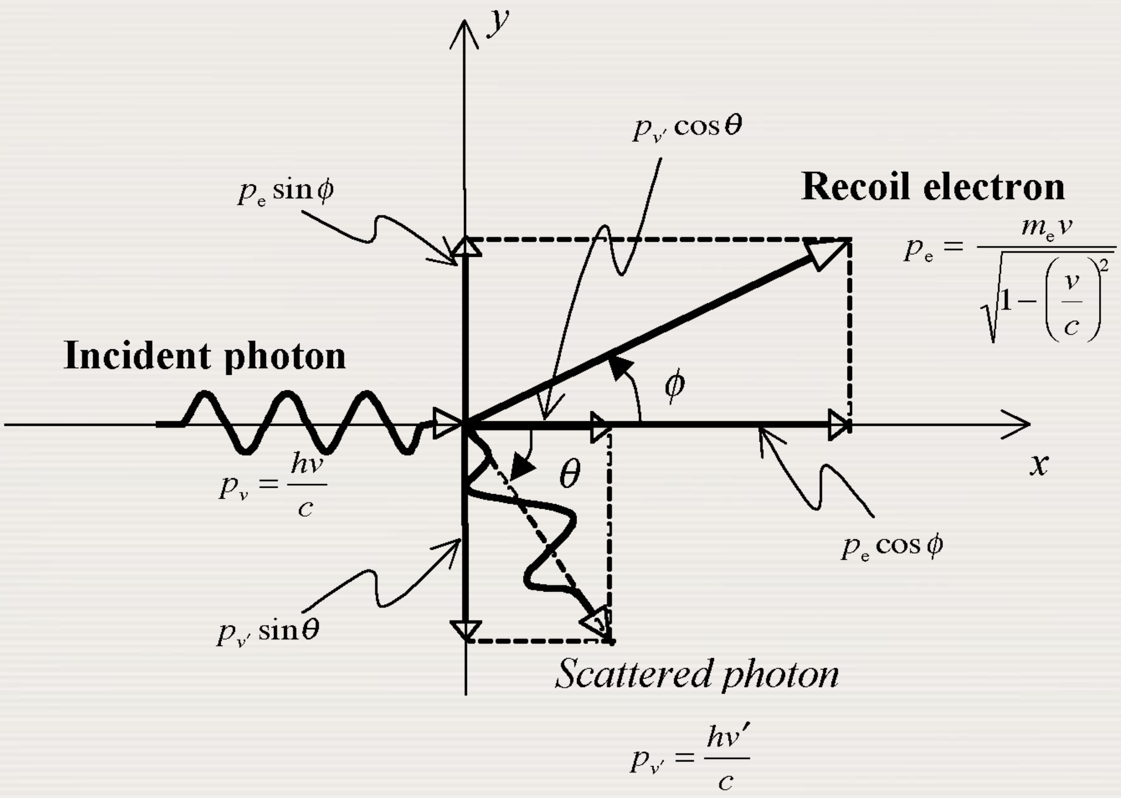
- Part of the incident photon energy is transferred to the “free” orbital electron which is emitted from the atom as the Compton (recoil) electron.

- Photon is scattered through a scattering angle θ and its energy $h\nu'$ is lower than the incident photon energy $h\nu$.

- Angle ϕ represents the angle between the incident photon direction and the direction of the recoil electron.

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering



Conservation of energy

$$h\nu + m_e c^2 = h\nu' + m_e c^2 + E_K$$

Conservation of momentum (x axis)

$$p_\nu = p_{\nu'}\cos\theta + p_e\cos\phi$$

Conservation of momentum (y axis)

$$0 = -p_{\nu'}\sin\theta + p_e\sin\phi$$

Compton expression:

$$\Delta\lambda = \lambda_C(1-\cos\theta)$$

$$\lambda_C = \frac{h}{m_e c} = 0.024\ \text{Å}$$

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Scattering angle θ and recoil angle ϕ are related as follows

$$\cot\phi = (1+\varepsilon)\tan\frac{\theta}{2} \qquad \varepsilon = \frac{h\nu}{m_e c^2}$$

- Relationship between the scattered photon energy $h\nu'$ and the incident photon energy $h\nu$ is:

$$h\nu' = h\nu\frac{1}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{h\nu}{m_e c^2}$$

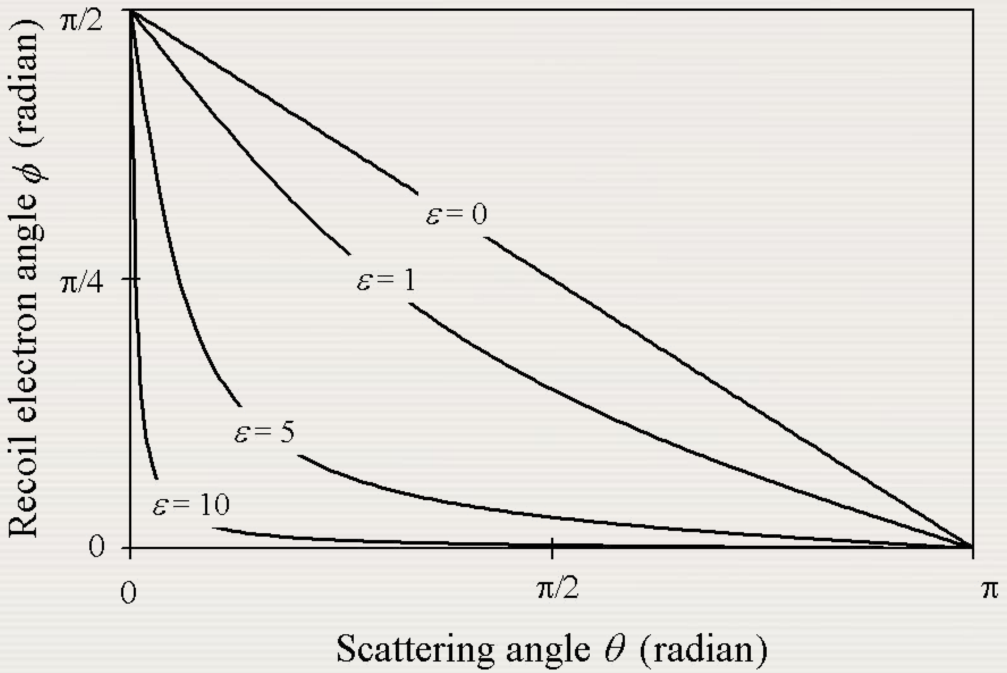
- Relationship between the kinetic energy of the recoil electron E_K and the energy of the incident photon $h\nu$ is:

$$E_K = h\nu\frac{\varepsilon(1-\cos\theta)}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{h\nu}{m_e c^2}$$

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Relationship between the photon scattering angle θ and the recoil angle ϕ of the Compton electron:



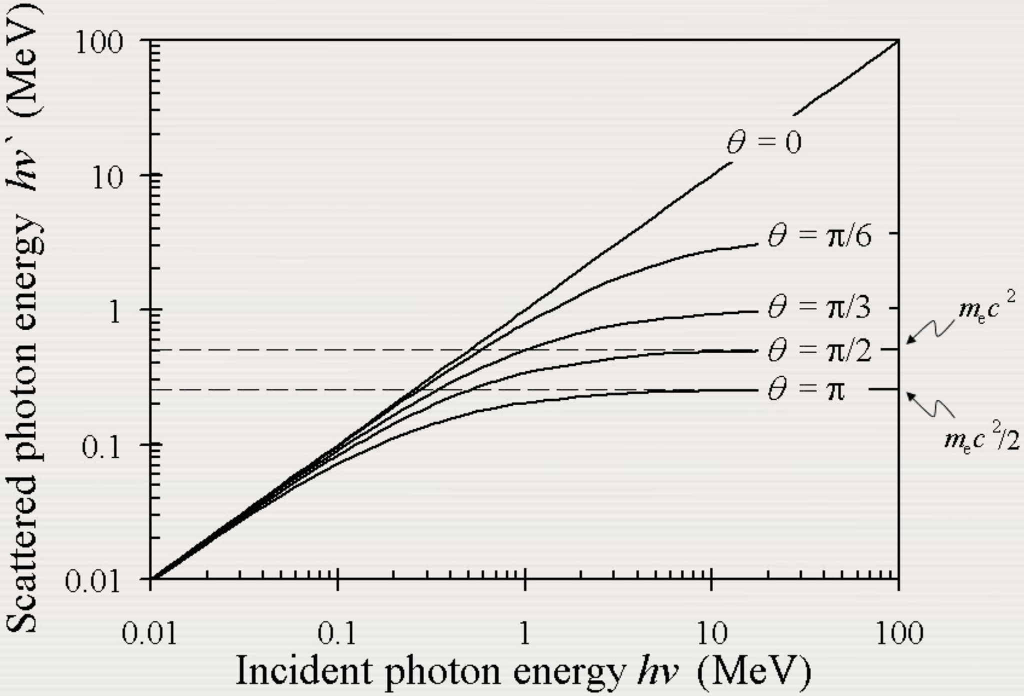
$$\cot\phi = (1+\varepsilon)\tan\frac{\theta}{2}$$

$$\varepsilon = \frac{h\nu}{m_e c^2}$$

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Relationship between the scattered photon energy $h\nu'$ and the incident photon energy $h\nu$:



$$h\nu' = h\nu \frac{1}{1+\varepsilon(1-\cos\theta)}$$

$$\varepsilon = \frac{h\nu}{m_ec^2}$$

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Energy of Compton scattered photons $h\nu'$ is expressed as:

$$h\nu' = h\nu \frac{1}{1+\varepsilon(1-\cos\theta)}$$

- Energy of photons scattered at $\theta = 90^{\circ}$

$$h\nu'(\theta = \pi/2) = \frac{h\nu}{1+\varepsilon} \qquad h\nu'_{\max}(\theta = \pi/2) = \lim_{h\nu\to\infty} \frac{h\nu}{1+\varepsilon} = m_e c^2 = 0.511\ \text{MeV}$$

- Energy of photons scattered at $\theta = \pi$

$$h\nu'(\theta = \pi) = \frac{h\nu}{1+2\varepsilon} \qquad h\nu'_{\max}(\theta = \pi) = \lim_{h\nu\to\infty} \frac{h\nu}{1+2\varepsilon} = \frac{m_e c^2}{2} = 0.255\ \text{MeV}$$

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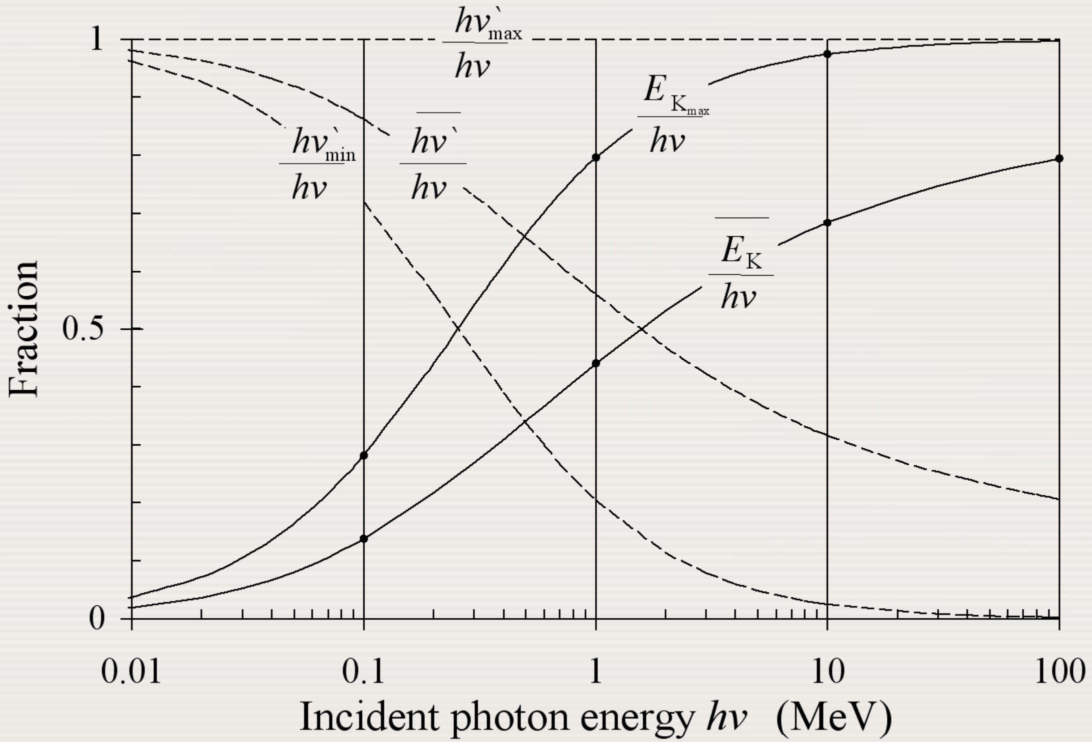
1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Maximum and mean fractions of incident photon energy $h\nu$ given to the scattered photon $hn¢$ and to Compton (recoil) electron E_K.



$$\frac{hn¢}{hn} = \frac{1}{1+e(1-\cos q)}$$

$$\frac{hn¢_{\max}}{hn} = \frac{hn¢}{hn}(q=0) = 1$$

$$\frac{E_K}{hn} = \frac{e(1-\cos q)}{1+e(1-\cos q)}$$

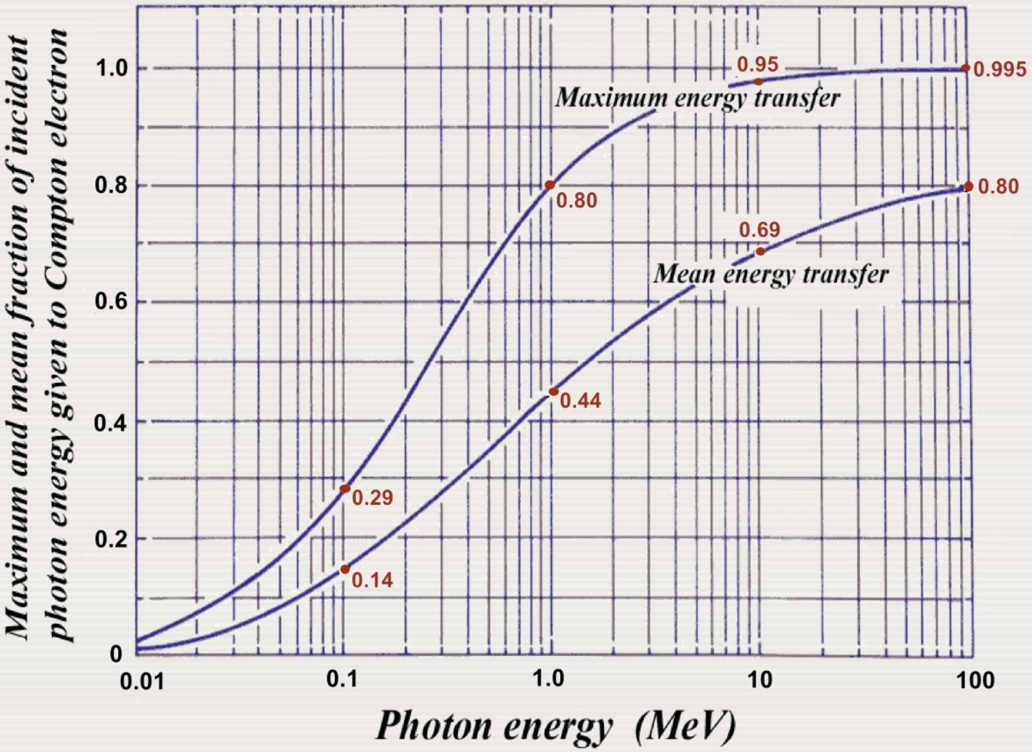
$$\frac{(E_K)_{\max}}{hn} = \frac{E_K}{hn}(q=p) = \frac{2e}{1+2e}$$

$$\frac{hn¢_{\min}}{hn} = \frac{hn¢}{hn}(q=p) = \frac{1}{1+2e}$$

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Maximum and mean energy transfer from the photon with energy $h\nu$ to Compton (recoil) electron (“Compton Graph #1”).
- Mean energy transfer fraction for Compton effect $\bar{f}_{\mathrm{C}}$.



$$\frac{E_{\mathrm{K}}}{h\nu} = \frac{\varepsilon(1-\cos\theta)}{1+\varepsilon(1-\cos\theta)}$$

$$\varepsilon = \frac{h\nu}{m_{\mathrm{e}}c^2}$$

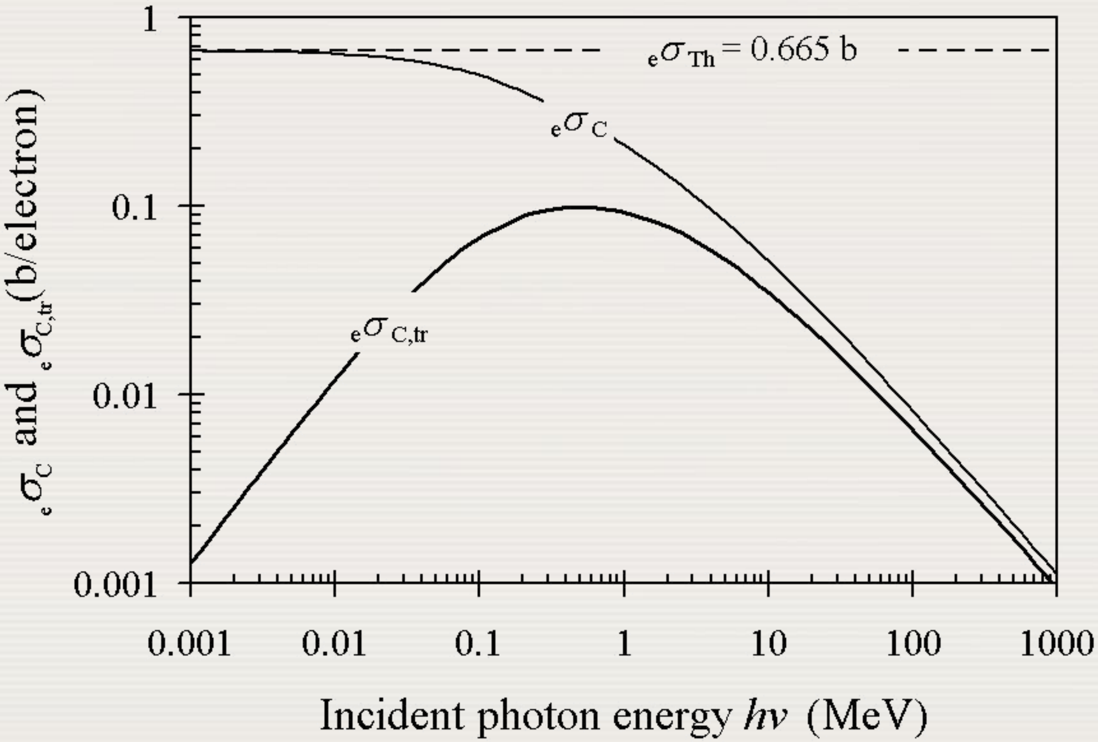
$$\frac{(E_{\mathrm{K}})_{\max}}{h\nu} = \frac{2\varepsilon}{1+2\varepsilon}$$

$$\bar{f}_{\mathrm{C}} = \frac{(\bar{E}_{\mathrm{K}})_{\mathrm{tr}}^{\mathrm{C}}}{h\nu}$$

1.4 PHOTON INTERACTIONS

1.4.6 Compton scattering

- Electronic Compton attenuation coefficient $_e\sigma_c$ steadily decreases with increasing photon energy $h\nu$.



$$({}_e s_C)_{tr} = {}_e s_C \,´\, f_C$$

$$\bar{f}_C = \frac{(\bar{E}_K)^C_{tr}}{hn}$$

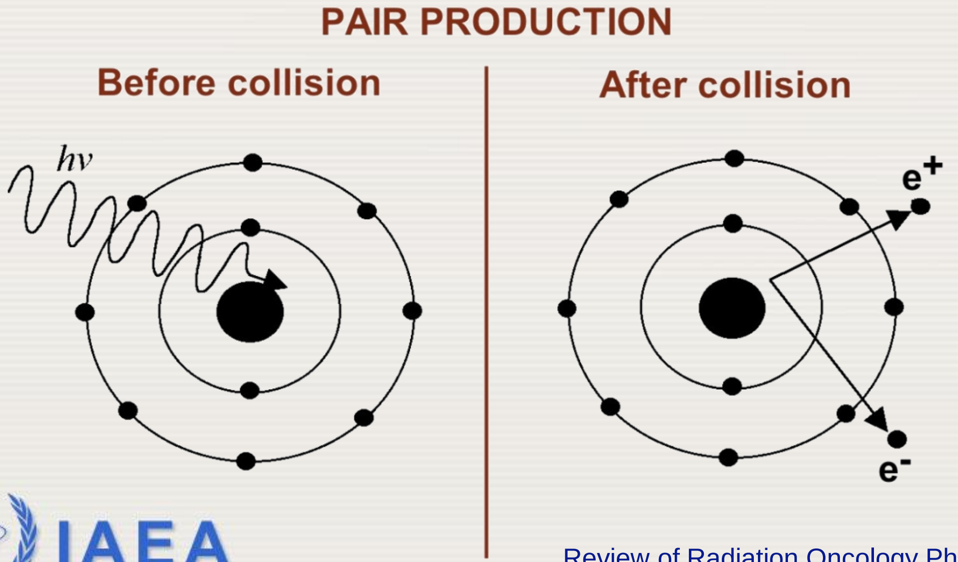
1.4 PHOTON INTERACTIONS

1.4.7 Pair production

Pair production (in field of the nucleus or orbital electron)

☐ In nuclear pair production

- Photon disappears.
- An electron-positron pair with a combined kinetic energy equal to $h\nu - 2m_ec^2$ is produced in the nuclear Coulomb field.
- Threshold energy $h\nu_{thr}$ for nuclear pair production is:



$$h\nu_{thr} = 2m_ec^2\left\{1+\frac{m_ec^2}{M_Ac^2}\right\} \approx 2m_ec^2$$

m_e electron mass

M_A mass of nucleus

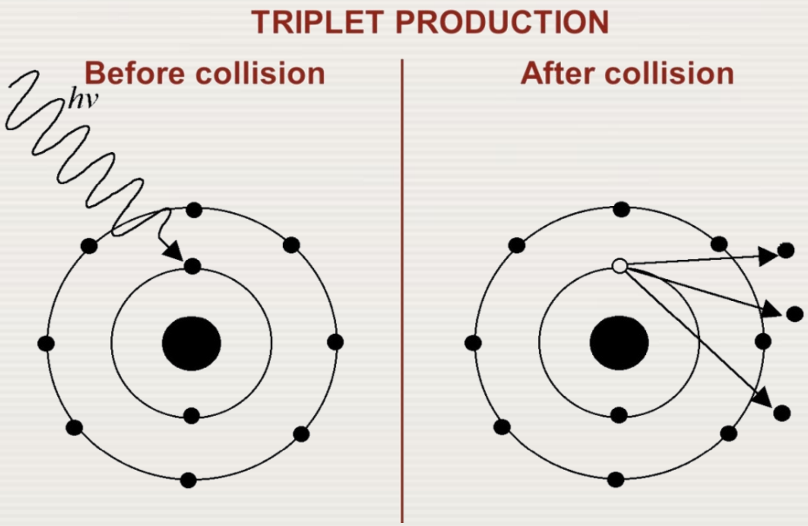
$m_ec^2 = 0.511\text{ MeV}$

1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- In triplet production (also called electronic pair production):
  - Photon disappears.
  - An electron-positron pair is produced in the Coulomb field of an orbital electron, and a triplet (two electrons and one positron) leave the site of interaction.
  - Threshold energy for triplet production is: $h\nu_{thr} = 4m_ec^2 = 2.04\ \text{MeV}$

$$m_ec^2 = 0.511\ \text{MeV}$$

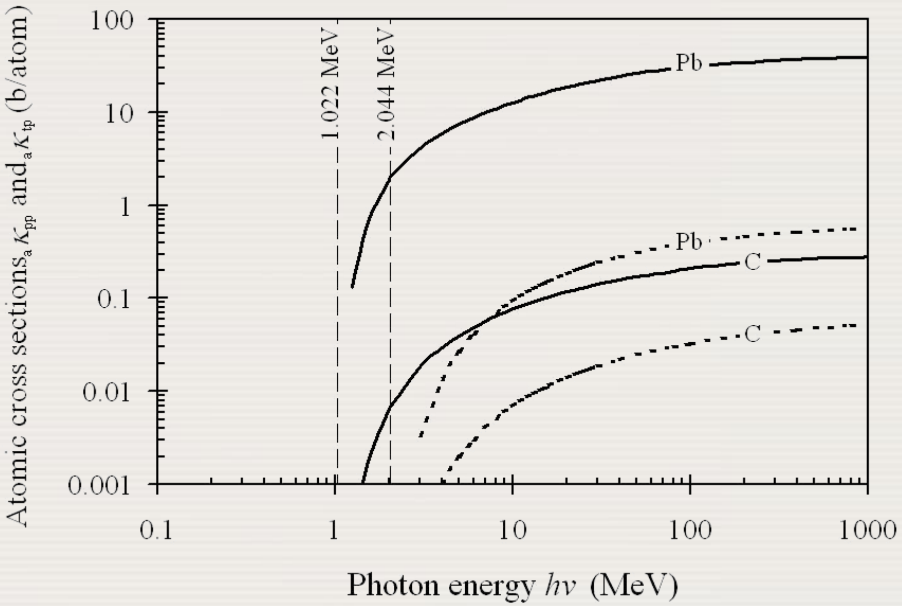


1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- Atomic cross sections for pair production and triplet production equal zero for photon energies below the threshold energy.

- Atomic cross section for pair production and triplet production increase rapidly with photon energy above the threshold energy.



Atomic cross sections for pair production: **solid curves**

Atomic cross sections for triplet production: **dashed curves**

1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- Atomic cross section for nuclear pair production ${}_{a}\kappa_{NPP}$ varies approximately as the square of the atomic number *Z* of the absorber.
- Atomic cross section for triplet production ${}_{a}\kappa_{TP}$ varies approximately linearly with *Z*, the atomic number of the absorber.

1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- Mass attenuation coefficient for nuclear pair production $(k/r)_{\mathrm{NPP}}$ varies approximately linearly with Z, the atomic number of the absorber.

- Mass attenuation coefficient for triplet production $(k/r)_{\mathrm{TP}}$ is essentially independent of the atomic number Z of the absorber.

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1.4 PHOTON INTERACTIONS

1.4.7 Pair production

☐ Attenuation coefficient for nuclear pair production exceeds significantly the attenuation coefficient for triplet production at same photon energy and atomic number of absorber.

☐ ${}_a\kappa_{TP}$ is at most about 30 % of ${}_a\kappa_{NPP}$ for $Z = 1$ and less than 1 % for high Z absorbers.

☐ Usually, the tabulated values for pair production κ include contribution of both the pair production in the field of the nucleus and the pair production in the field of electron, i.e.,

$${}_a\kappa = {}_a\kappa_{NPP} + {}_a\kappa_{TP}$$

1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- ☐ Total kinetic energy transferred from the photon to charged particles (electron and positron) in pair production is

$$h\nu - 2m_\mathrm{e}c^2$$

- ☐ Mass attenuation coefficient κ/ρ is calculated from the atomic cross section ${}_\mathrm{a}\kappa$ as follows

$$\frac{\kappa}{\rho} = {}_\mathrm{a}\kappa \frac{N_\mathrm{A}}{A}$$

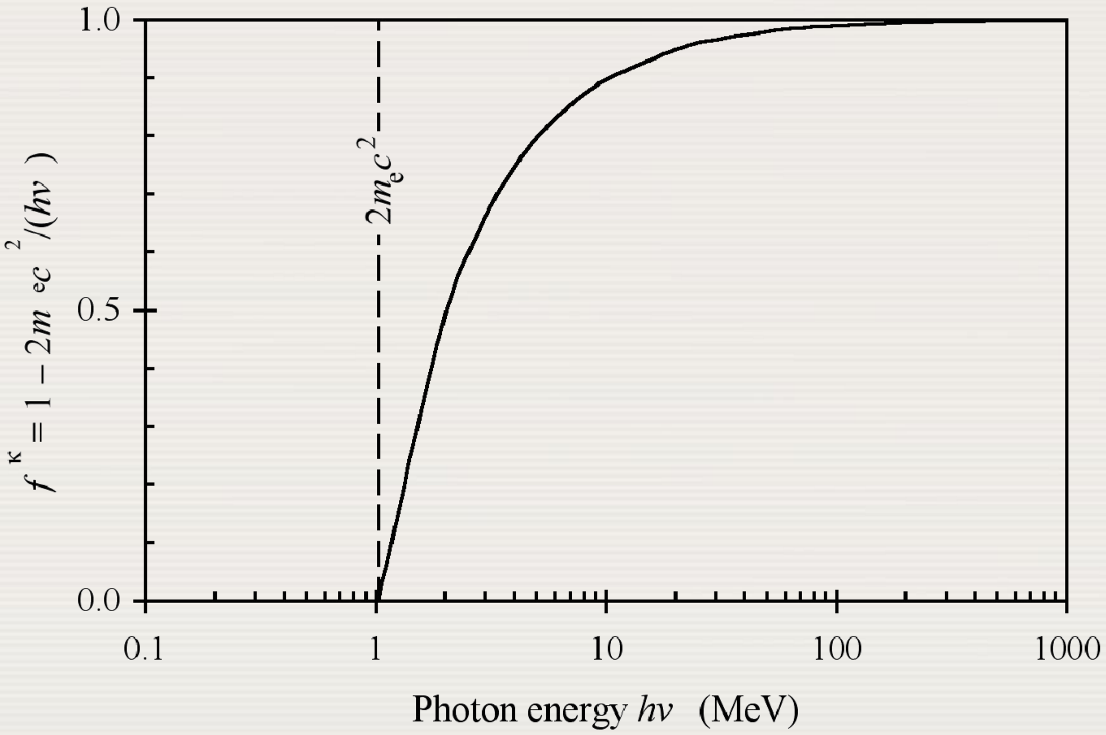
- ☐ Mass energy transfer coefficient $(\kappa/\rho)_\mathrm{tr}$ is:

$$\left(\frac{\kappa}{\rho}\right)_\mathrm{tr} = f_\mathrm{PP}\frac{\kappa}{\rho} = \frac{\kappa}{\rho}\left(1 - \frac{2m_\mathrm{e}c^2}{h\nu}\right)$$

1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- Mean energy transfer fraction for pair production $\bar{f}_{PP}$



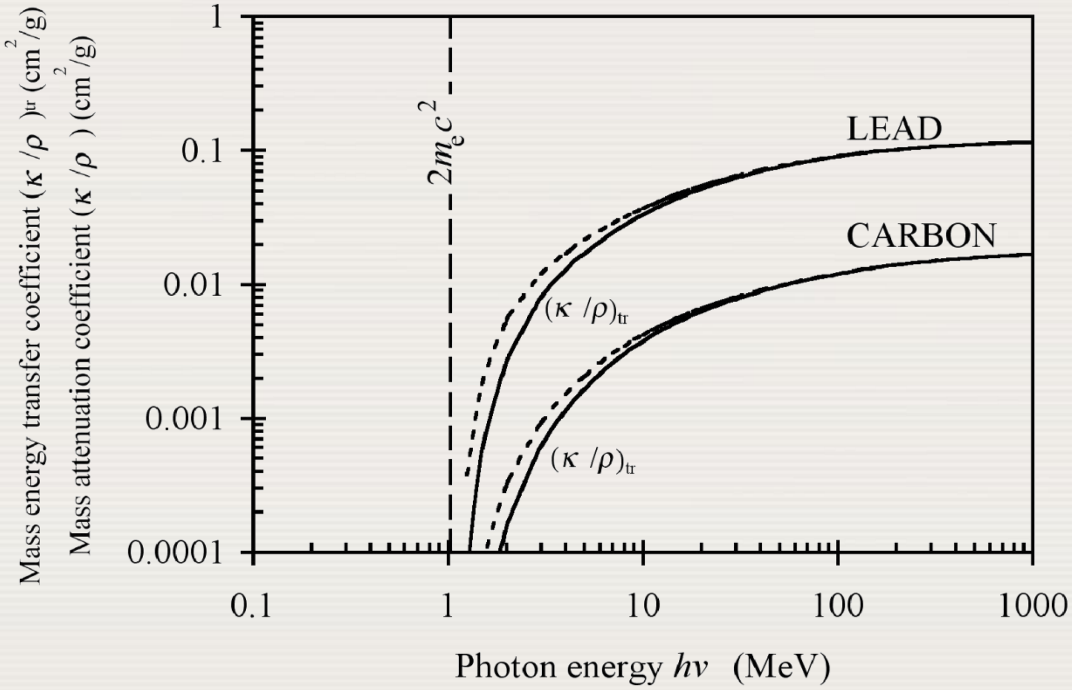
$$\bar{f}_{PP} = \frac{hn - \bar{E}_{tr}^{PP}}{hn}$$

$$= 1 - \frac{2m_e c^2}{hn}$$

1.4 PHOTON INTERACTIONS

1.4.7 Pair production

- Mass attenuation coefficient κ/ρ and mass energy transfer coefficient $(k/r)_{tr}$ for pair production against photon energy $h\nu$.



Mass attenuation coefficient: **dashed curves**

Mass energy transfer coefficient: **solid curves**

1.4 PHOTON INTERACTIONS

1.4.8 Photonuclear reactions

- ☐ Photonuclear reactions (nuclear photoelectric effect):
  - High energy photon is absorbed by the nucleus of the absorber.
  - A neutron or a proton is emitted.
  - Absorber atom is transformed into a radioactive reaction product.
- ☐ Threshold is of the order of ~10 MeV or higher, with the exception of the deuteron and beryllium-9 (~2 MeV).
- ☐ Probability for photonuclear reactions is much smaller than that for other photon atomic interactions; photonuclear reactions are thus usually neglected in medical physics.

1.4 PHOTON INTERACTIONS

1.4.9 Contribution to attenuation coefficients

- For a given $h\nu$ and Z:
  - Linear attenuation coefficient μ
  - Linear energy transfer coefficient μ_{tr}
  - Linear energy absorption coefficient μ_{ab} (often designated μ_{en})

are given as a sum of coefficients for individual photon interactions

$$\mu = \tau + \sigma_{\text{R}} + \sigma_{\text{C}} + \kappa$$

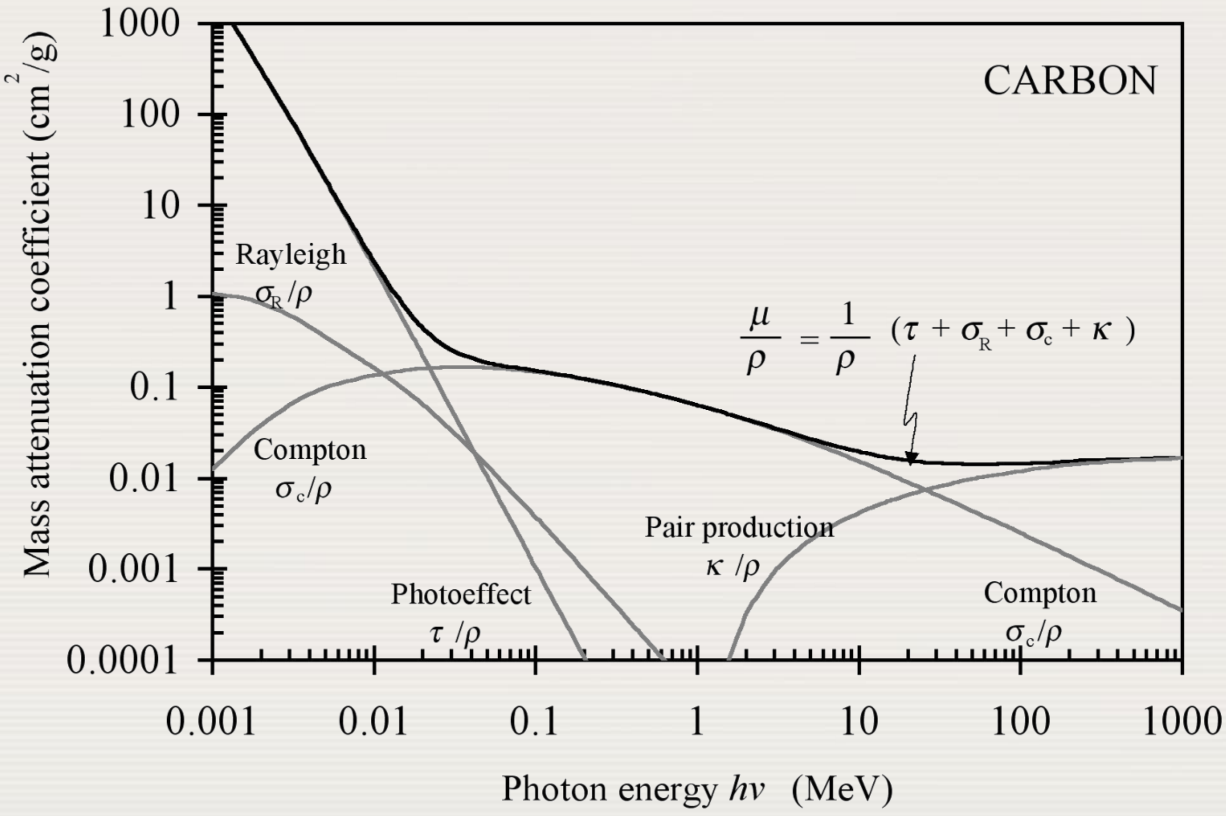
$$m_{\text{tr}} = t_{\text{tr}} + (s_{\text{R}})_{\text{tr}} + (s_{\text{C}})_{\text{tr}} + k_{\text{tr}} = f_{\text{PE}} t + f_{\text{C}} s_{\text{C}} + f_{\text{PP}} k$$

$$\mu_{\text{ab}} \equiv \mu_{\text{en}} = \mu_{\text{tr}}(1 - \bar{g})$$

1.4 PHOTON INTERACTIONS

1.4.9 Contribution to attenuation coefficients

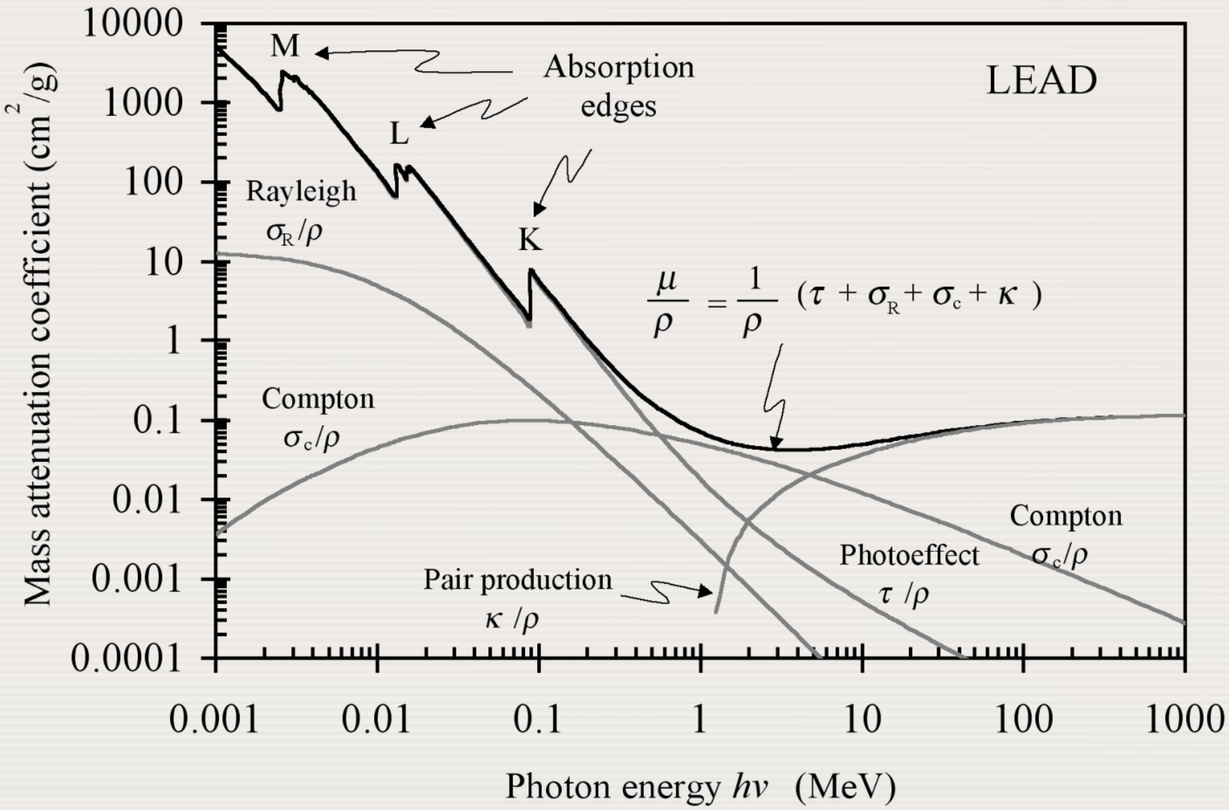
Mass attenuation coefficient against photon energy for carbon



1.4 PHOTON INTERACTIONS

1.4.9 Contribution to attenuation coefficients

Mass attenuation coefficient against photon energy for lead



1.4 PHOTON INTERACTIONS

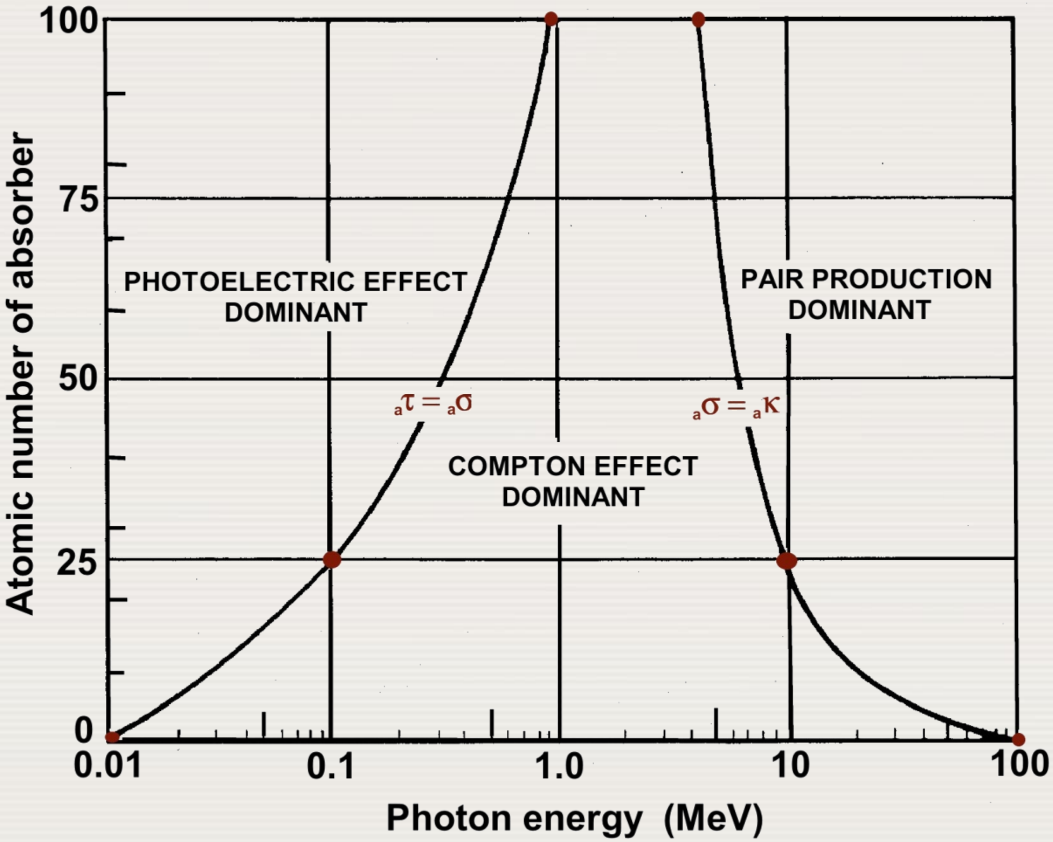
1.4.10 Relative predominance of individual effects

- ☐ Probability for a photon to undergo any one of the various interaction phenomena with an atom of the absorber depends:
  - On the energy $h\nu$ of the photon.
  - On the atomic number Z of the absorber.
- ☐ In general,
  - Photoelectric effect predominates at low photon energies.
  - Compton effect predominates at intermediate photon energies.
  - Pair production predominates at high photon energies.

1.4 PHOTON INTERACTIONS

1.4.10 Relative predominance of individual effects

- Regions of relative predominance of the three main forms of photon interaction with absorber.



1.4 PHOTON INTERACTIONS

1.4.11 Effects following photon interactions

- In photoelectric effect, Compton scattering and triplet production vacancies are produced in atomic shells through ejection of an orbital electron.
- Vacancies are filled with orbital electrons making transitions from higher to lower level atomic shells.
- Electronic transitions are followed by emission of characteristic x rays or Auger electrons; the proportion governed by the fluorescence yield.

1.4 PHOTON INTERACTIONS

1.4.11 Effects following photon interactions

- ☐ Pair production and triplet production are followed by the annihilation of the positron, which lost almost all its kinetic energy through Coulomb interactions with absorber atoms, with a “free” electron producing two annihilation quanta.
- ☐ The two annihilation quanta have most commonly energy of 0.511 MeV each, and are emitted at approximately 180^{o} to each other to satisfy the conservation of momentum and energy.
- ☐ Annihilation may also occur of an energetic positron with an electron and this rare event is referred to as annihilation-in-flight.

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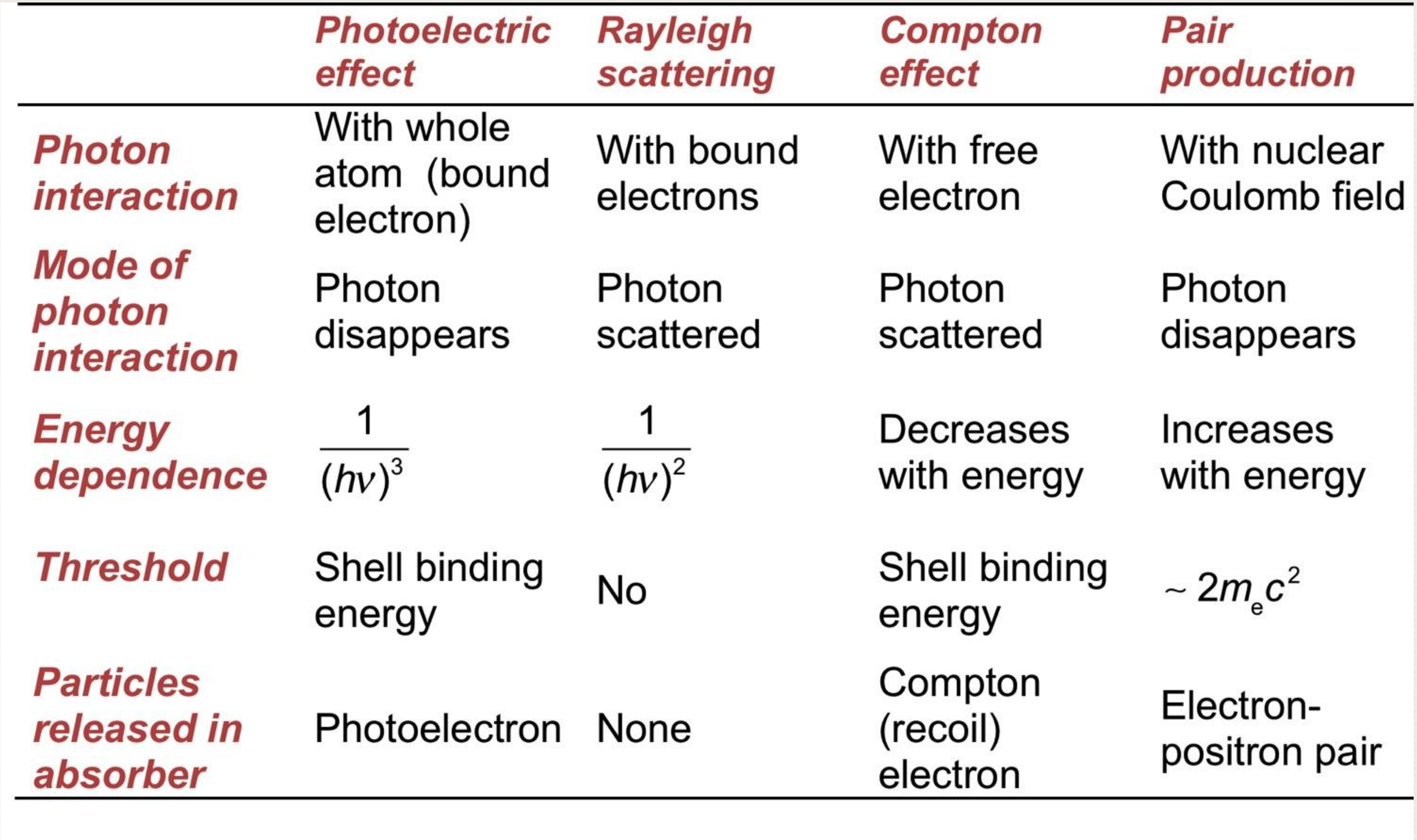
1.4 PHOTON INTERACTIONS

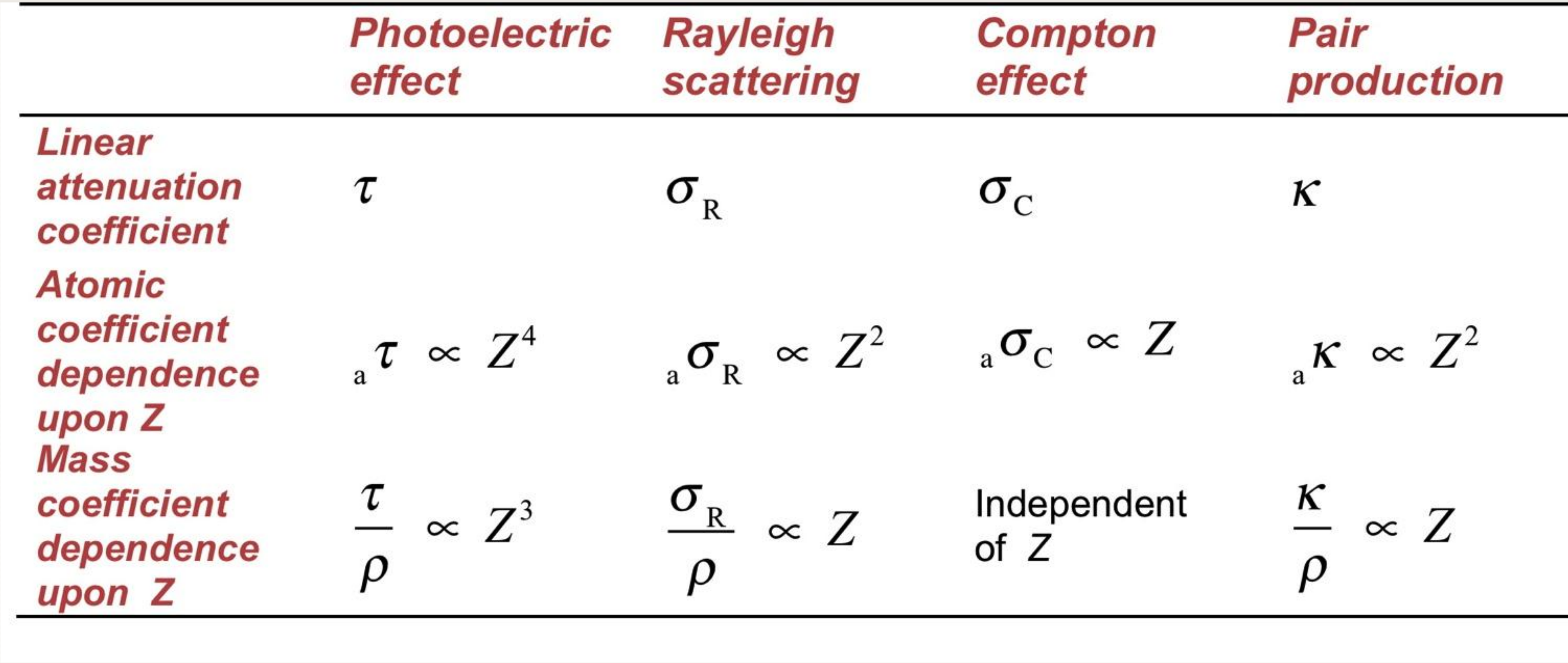
1.4.12 Summary of photon interactions



	Photoelectric effect	***Rayleigh scattering***	***Compton effect***	***Pair production***
Photon interaction	With whole atom (bound electron)	With bound electrons	With free electron	With nuclear Coulomb field
Mode of photon interaction	Photon disappears	Photon scattered	Photon scattered	Photon disappears
Energy dependence	$\frac{1}{(h\nu)^3}$	$\frac{1}{(h\nu)^2}$	Decreases with energy	Increases with energy
Threshold	Shell binding energy	No	Shell binding energy	$\sim 2m_ec^2$
Particles released in absorber	Photoelectron	None	Compton (recoil) electron	Electron-positron pair

1.4 PHOTON INTERACTIONS

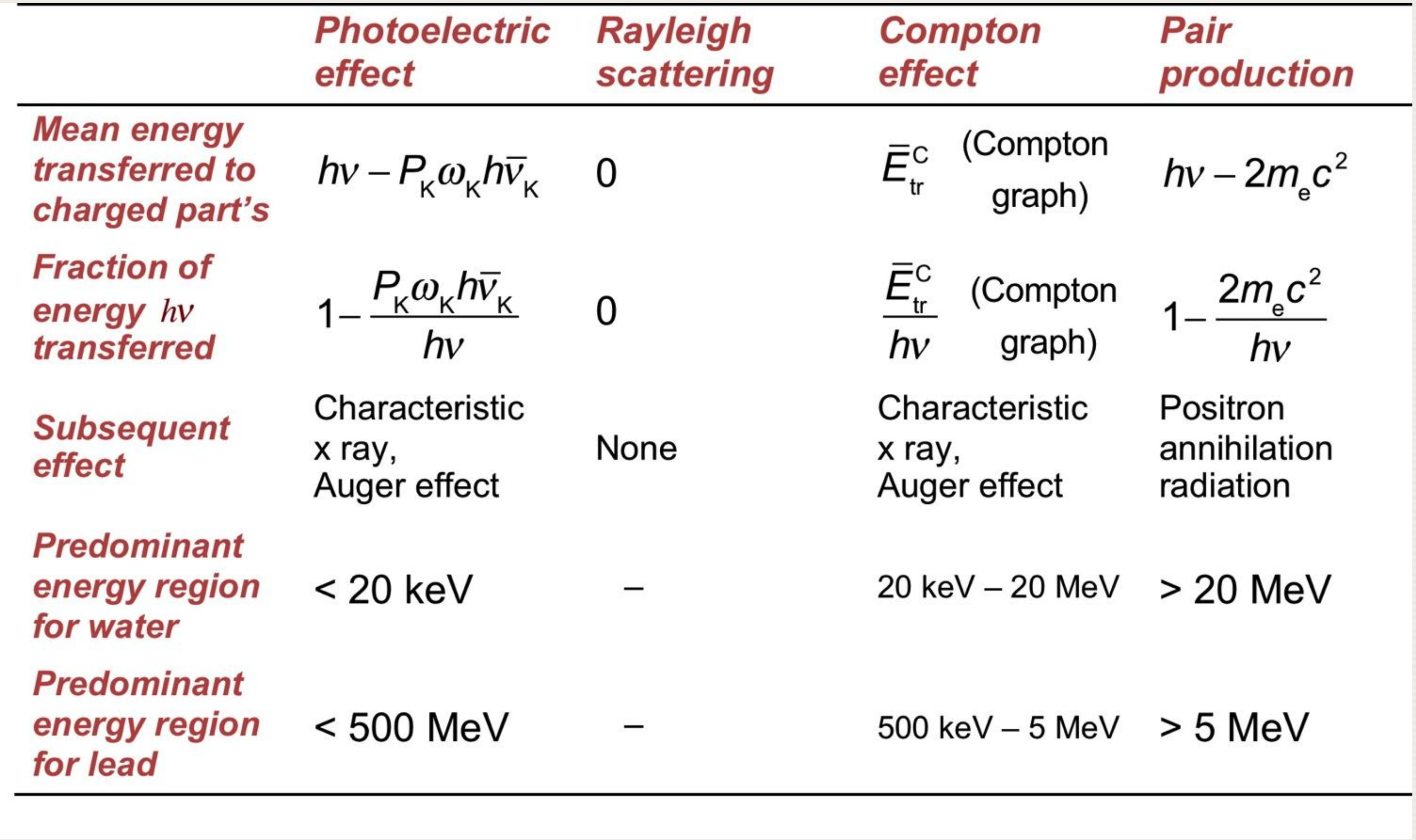
1.4.12 Summary of photon interactions



	Photoelectric effect	**Rayleigh scattering**	**Compton effect**	**Pair production**
Linear attenuation coefficient	τ	σ_R	σ_C	κ
Atomic coefficient dependence upon Z	${}_a\tau \propto Z^4$	${}_a\sigma_R \propto Z^2$	${}_a\sigma_C \propto Z$	${}_a\kappa \propto Z^2$
Mass coefficient dependence upon Z	$\frac{\tau}{\rho} \propto Z^3$	$\frac{\sigma_R}{\rho} \propto Z$	Independent of Z	$\frac{\kappa}{\rho} \propto Z$

1.4 PHOTON INTERACTIONS

1.4.12 Summary of photon interactions



	Photoelectric effect	**Rayleigh scattering**	**Compton effect**	**Pair production**
Mean energy transferred to charged part's	$h\nu - P_K \omega_K h\bar{\nu}_K$	0	$\bar{E}_{tr}^{C}$ (Compton graph)	$h\nu - 2m_e c^2$
Fraction of energy $h\nu$ transferred	$1 - \frac{P_K \omega_K h\bar{\nu}_K}{h\nu}$	0	$\frac{\bar{E}_{tr}^{C}}{h\nu}$ (Compton graph)	$1 - \frac{2m_e c^2}{h\nu}$
Subsequent effect	Characteristic x ray, Auger effect	None	Characteristic x ray, Auger effect	Positron annihilation radiation
Predominant energy region for water	< 20 keV	–	20 keV – 20 MeV	> 20 MeV
Predominant energy region for lead	< 500 MeV	–	500 keV – 5 MeV	> 5 MeV

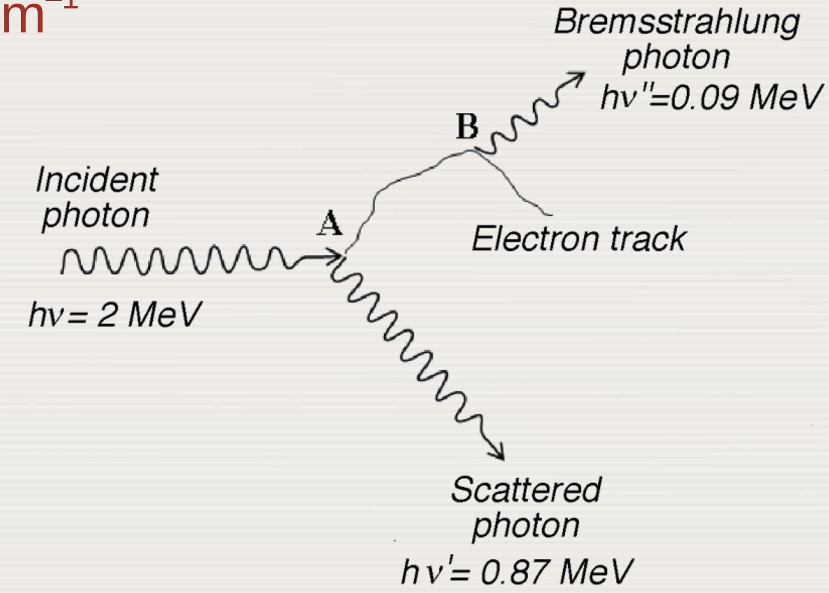
1.4 PHOTON INTERACTIONS

1.4.13 Example of photon attenuation

- For 2 MeV photons in lead (Z = 82; A = 207.2; ρ = 11.36 g/cm^3) the linear attenuation coefficients are as follows:
  - Photoelectric effect: $\tau = 0.055\ \mathrm{cm}^{-1}$
  - Coherent (Rayleigh) scattering: $\sigma_R = 0.008\ \mathrm{cm}^{-1}$
  - Compton scattering: $\sigma_C = 0.395\ \mathrm{cm}^{-1}$
  - Pair production: $\kappa = 0.056\ \mathrm{cm}^{-1}$
- Mean energy transferred to charged particles:

$$(\bar{E}_K)_{tr} = 1.13\ \mathrm{MeV}$$

- Mean energy absorbed in lead: $(\bar{E}_K)_{ab} = 1.04\ \mathrm{MeV}$



1.4 PHOTON INTERACTIONS

1.4.13 Example of photon attenuation

$$\tau = 0.055\ \text{m}^{-1} \qquad \sigma_{\text{R}} = 0.008\ \text{cm}^{-1} \qquad \sigma_{\text{C}} = 0.395\ \text{cm}^{-1} \qquad \kappa = 0.056\ \text{cm}^{-1}$$

- Linear attenuation coefficient:

$$\mu = \tau + \sigma_{\text{R}} + \sigma_{\text{C}} + \kappa = (0.055 + 0.008 + 0.395 + 0.056)\ \text{cm}^{-1} = 0.514\ \text{cm}^{-1}$$

- Mass attenuation coefficient:

$$\mu_{\text{m}} = \frac{\mu}{\rho} = \frac{0.514\ \text{cm}^{-1}}{11.36\ \text{g/cm}^3} = 0.0453\ \text{cm}^2/\text{g}$$

- Atomic attenuation coefficient:

$${}_{\text{a}}\mu = \left(\frac{\rho N_{\text{A}}}{A}\right)^{-1} \mu = \frac{207.2\ (\text{g/mol}) \times 0.514\ \text{cm}^{-1}}{11.36\ (\text{g/cm}^3) \times 6.022 \times 10^{23}\ (\text{atom/mol})}$$

$$= 1.56 \times 10^{-23}\ \text{cm}^2/\text{atom}$$

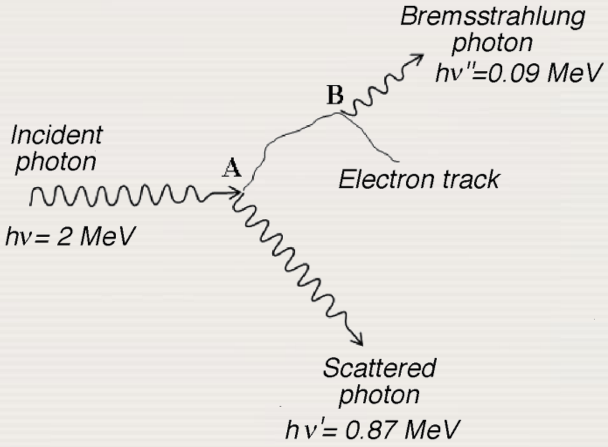
1.4 PHOTON INTERACTIONS

1.4.13 Example of photon attenuation

$$(\bar{E}_K)_{tr} = 1.13 \text{ MeV}$$

$$\mu_m = \frac{\mu}{\rho} = 0.0453 \text{ cm}^2/\text{g}$$

$$(\bar{E}_K)_{ab} = 1.04 \text{ MeV}$$



- ☐ Mass energy transfer coefficient:

$$\frac{\mu_{tr}}{\rho} = \frac{(\bar{E}_K)_{tr}}{h\nu}\frac{\mu}{\rho} = \frac{1.13 \text{ MeV} \times 0.0453 \text{ cm}^2/\text{g}}{2 \text{ MeV}} = 0.0256 \text{ cm}^2/\text{g}$$

- ☐ Mass energy absorption coefficient:

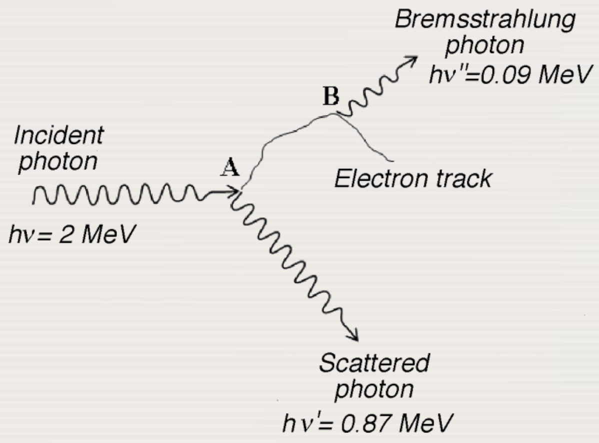
$$\frac{\mu_{ab}}{\rho} = \frac{(\bar{E}_K)_{ab}}{h\nu}\frac{\mu}{\rho} = \frac{1.04 \text{ MeV} \times 0.0453 \text{ cm}^2/\text{g}}{2 \text{ MeV}} = 0.0236 \text{ cm}^2/\text{g}$$

1.4 PHOTON INTERACTIONS

1.4.13 Example of photon attenuation

$$(\bar{E}_K)_{tr} = 1.13\ \text{MeV} \qquad \frac{\mu_{ab}}{\rho} = 0.0236\ \text{cm}^2/\text{g}$$

$$(\bar{E}_K)_{ab} = 1.04\ \text{MeV} \qquad \frac{\mu_{tr}}{\rho} = 0.0256\ \text{cm}^2/\text{g}$$



- Radiation fraction:

$$\bar{g} = \frac{(\bar{E}_K)_{tr} - (\bar{E}_K)_{ab}}{(\bar{E}_K)_{tr}} = 1 - \frac{(\bar{E}_K)_{ab}}{(\bar{E}_K)_{tr}} = 1 - \frac{1.04\ \text{MeV}}{1.13\ \text{MeV}} = 0.08$$

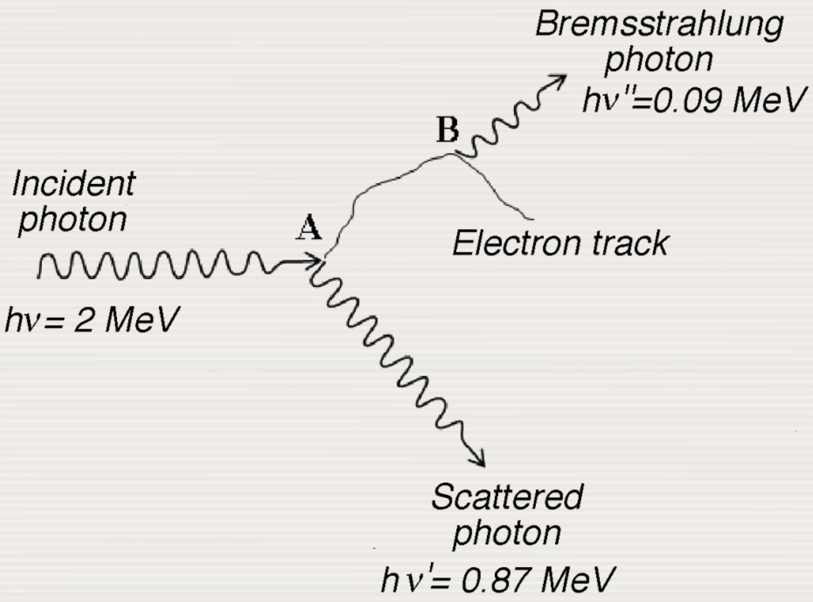
or

$$\bar{g} = 1 - \frac{\mu_{ab}/\rho}{\mu_{tr}/\rho} = 1 - \frac{0.0236\ \text{cm}^2/\text{g}}{0.0256\ \text{cm}^2/\text{g}} = 0.08$$

1.4 PHOTON INTERACTIONS

1.4.13 Example of photon attenuation

Conclusion:



☐ For a 2 MeV photon in lead on the average:

- 1.13 MeV will be transferred to charged particles (electrons and positrons).
- 0.87 MeV will be scattered through Rayleigh and Compton scattering.
- Of the 1.13 MeV transferred to charged particles:
  - 1.04 MeV will be absorbed in lead.
  - 0.09 MeV will be re-emitted in the form of bremsstrahlung photons.
- Radiation fraction $\bar{g}$ for 2 MeV photons in lead is 0.08.

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1.4 PHOTON INTERACTIONS

1.4.14 Production of vacancies in atomic shells

☐ There are 8 main means for producing vacancies in atomic shells and transforming the atom from a neutral state into an excited positive ion:

- Coulomb interaction (1) of energetic charged particle with orbital electron.
- Photon interactions
  - Photoelectric effect (2)
  - Compton effect (3)
  - Triplet production (4)
- Nuclear decay
  - Electron capture (5)
  - Internal conversion (6)
- Positron annihilation (7)
- Auger effect (8)

1.4 PHOTON INTERACTIONS

1.4.14 Production of vacancies in atomic shells

☐ *Note*: Pair production does not produce shell vacancies, because the electron-positron pair is produced in the field of the nucleus.

☐ Vacancies in inner atomic shells are not stable; they are followed:

- Either by emission of characteristic photons
- Or by emission of Auger electrons

and cascade to the outer shell of the ionized atom (ion).

☐ Ion eventually attracts a free electron from its surroundings and reverts to a neutral atom (ionic recombination).